To react or not to react? A double-well potential model of event-driven human control

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Abstract

Understanding how humans control unstable systems is central to many research problems, with applications ranging from quiet standing to aircraft landing. Increasingly much evidence appears in favor of event-driven control hypothesis: human operators are passive by default and only start actively controlling the system when the discrepancy between the current and desired system states becomes large. The present study proposes a cognitive model describing the transitions between the passive and the active phase of control behavior. The model is based on the concept of random walk in double-well potential widely employed in physics. Unlike the conventionally used model of fixed threshold, the proposed model is intrinsically stochastic and thus conforms to the physiological interpretation of the threshold as a probabilistic rather than deterministic notion. The model is studied numerically and is confronted to the experimental data on virtual stick balancing. The results confirm the validity of the model and suggest that the double-well potential can be used in modeling human control behavior in a diverse range of applications.

Keywords: Stochastic modeling; control behavior; stick balancing; intermittency

Control of unstable systems underlies many critical procedures performed by human operators (e.g., manipulation of industrial machinery, aircraft landing), as well as numerous routines that all of us face in daily life (e.g., standing upright, riding a bicycle, carrying a cup of coffee). Eliciting and modeling the basic mechanisms of human control can help us to understand the nature of such processes, and in the end, hopefully, to reduce the risks associated with human error.

Continuous control models describe human actions well in many situations (Gawthrop, Loram, Lakie, & Gollee, 2011). On the other hand, increasingly much evidence appears in favor of more general concept —*intermittent* control (Gawthrop et al., 2011; Loram, Gollee, Lakie, & Gawthrop, 2011; Balasubramaniam, 2013; Milton, 2013). The latter implies that human control is discontinuous, repeatedly switching on and off instead of being always active throughout the process.

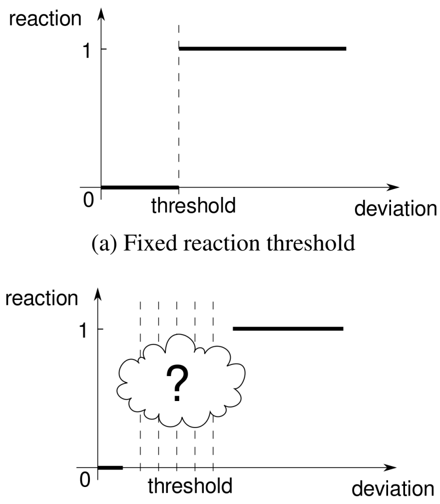
Intermittency has long been attributed to a general class of human control processes (Craik, 1947). However, despite being recognized for decades, human control intermittency is still far from being completely understood. The event-driven control hypothesis has recently become the most widely employed explanation of intermittency of human control (Gawthrop et al., 2011; Milton, 2013). Event-driven models build up on the fact that human operators cannot detect small deviations of the controlled system from the goal state. Therefore, the control is switched off as long as the deviation remains below a certain threshold value (Fig. 1a). Whenever the deviation exceeds the threshold, the control is switched on so that the system is driven back to the goal state.

The existing models based on the standard, deterministic threshold mechanism can explain many features of experimentally observed dynamics. Possibly that is why the nature of the threshold as some precise, fixed number has rarely been questioned in the literature on human control. In the real control process, where the control switches on and off many times, would the operator always react to precisely the same deviation? If not, then how diverse are the actually implemented threshold values?

The concept of threshold is not deterministic but probabilistic, as evidenced by psychophysics (Gescheider, 1997). In principle, the perception threshold is characterized not by a fixed value, but by a probability distribution of the stimulus magnitude allowing one to recognize the stimulus. However, the magnitude corresponding to the 50 % chance of recognizing the stimulus is commonly used as a simple shortcut for the perception threshold. Indeed, ignoring the variability of the threshold may be completely plausible as long as this variability is low enough. In such cases the fluctuations of the threshold would have a minor effect on the system dynamics and may be neglected. However, we argue that in controling unstable objects human operators can disregard not only the small deviations that cannot be perceived, but also the deviations significantly exceeding the perception threshold.

In contrast to psychophysical experiments, in controling unstable objects many factors other than the magnitude of stimulus (i.e., deviation from the goal) affect the actual threshold value triggering human response. For instance, if the control process lasts for a relatively long time, the mental expenses for staying perfectly aware of the smallest deviations may be unbearable for the operator. In this case, even the deviation that can otherwise be clearly perceived may be neglected due to energy considerations. Another relevant factor is the limited ability of the operator to precisely manipulate the unstable system. Even a highly skilled operator cannot accurately compensate for small, barely detectable deviations. In order not to destabilize the system by the imprecise interruption, the operator may prefer to wait until the deviation becomes large enough. As a result, the corrective movements need not be thoroughly planned and implemented. These and some other factors may cause high variability of the actual threshold triggering human control (Fig. 1b).

In the present paper we propose a stochastic model capturing the probabilistic nature of human control. The model



(a) Fixed reaction threshold

(b) Probabilistic reaction threshold

Figure 1: Two basic models of human reaction threshold

introduces two mechanisms, "how to react" and "when to react", which jointly determine the operator control behavior. We hypothesize that it is the stochasticity of the second mechanism that mainly causes the random fluctuations in the dynamics of unstable systems under human control. This stochasticity is captured in the model by means of the double-well potential concept inherited from physics. To confirm the validity of the model, we confront it with the experimental data on overdamped stick balancing. Our results suggest that the stochastic mechanism of control triggering may be a key to understanding the dynamics of human-controlled systems.

Methods

Ten right-handed healthy volunteers (six male, four female, aged from 20 to 61, median 26) participated in the experiments. Three subjects had previously participated in the preliminary experiments involving the same task (Zgonnikov, Lubashevsky, & Mozgovoy, 2012, 2013). Seven other participants had had no prior experience in either virtual or real stick balancing. On the computer screen the subjects saw a vertically oriented stick and a moving cart rigidly connected with the base of the stick (Fig. 2). The task was to maintain the upright position of the stick by moving the platform horizontally via computer mouse.

The stick dynamics were simulated by numerically solving the following ordinary differential equation

$$\tau_\theta \dot{\theta} = \sin\theta - \frac{\tau_\theta}{l} \upsilon \cos\theta\,, \qquad (1)$$

where θ is the angular deviation of the stick from the vertical position, $\dot{\theta}$ is the angular velocity of the stick, and υ is the velocity of the cart controlled by the operator via computer mouse. The parameter τ_θ defines the time scale of the stick motion, and the stick length l de facto determines the typical magnitude of the cart displacements required for keeping the stick upright. The parameter values used in the experiments are $\tau_\theta = 0.3$, $l = 0.4$.

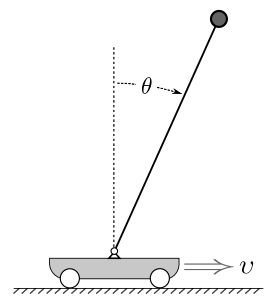


Figure 2: One-degree-of-freedom overdamped inverted pendulum.

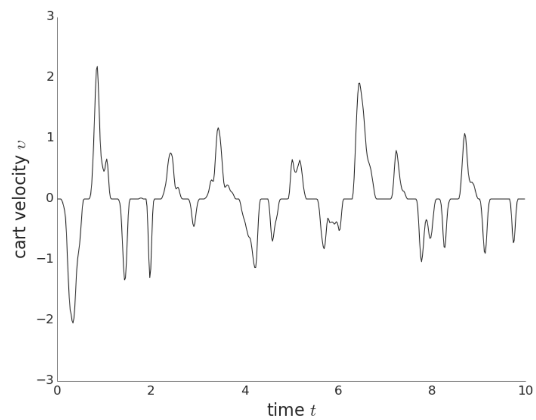


Figure 3: Typical cart velocity pattern of the overdamped stick balancing obtained experimentally. The trajectory represents the randomly selected 10-second time fragment of balancing without stick falls.

The experiment consisted of one-minute practice period and three five-minute recorded trials separated by two three-minute rest periods. The distance between the monitor and the subject eyes was about 70 cm, the stick length on the screen was about 10 cm. The screen update frequency was 60 Hz. The horizontal position of mouse cursor on the screen was sampled with frequency of 50 Hz.

Results

Pronounced intermittent control patterns were found in all subjects. The observed intermittency is illustrated by the typical cart velocity dynamics (Fig. 3). The subjects control the stick intermittently: they spend substantial portion of time in the passive control phase. The fragments of active control are most often isolated and unimodal.

The typical phase trajectory of the stick balancing in the $\theta - \dot{\theta}$ phase plane provide important insights into the system dynamics (Fig. 4). Based on the phase trajectory it is easy to reconstruct the typical pattern of the observed operator be-

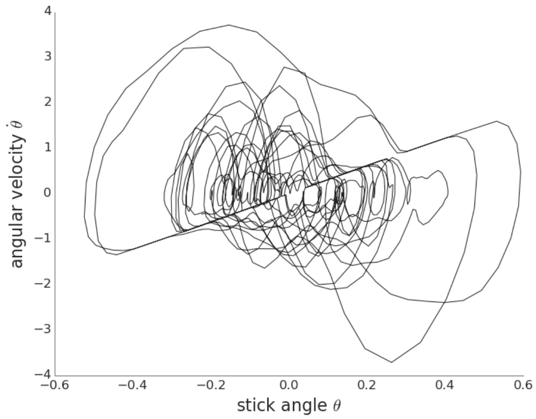


Figure 4: Typical phase trajectory of the overdamped stick balancing obtained experimentally. The trajectory represents the randomly selected 30-second time fragment of balancing without stick falls.

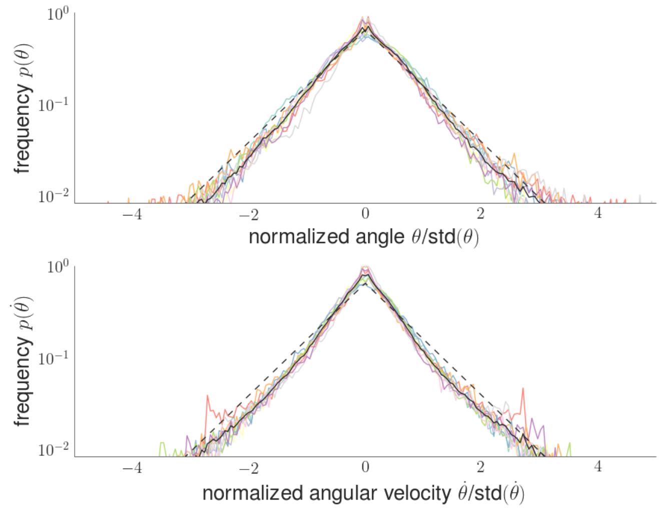


Figure 5: Experimentally obtained distributions of stick angle and angular velocity. The angle and angular velocity values are normalized with respect to their standard deviations: $\theta \to \theta/\text{std}(\theta)$, $\dot{\theta} \to \dot{\theta}/\text{std}(\dot{\theta})$. Colored lines represent the distributions for each subject. Solid black lines represent the average distributions calculated based on the aggregated data for all the subjects. Dashed lines represent the Laplace distributions (zero mean, unit variance) for reference.

havior. Given that the initial deviation of the stick from the vertical position is small, the operator takes no action, so the stick falls on its own for some time. Then, the operator takes the control over the system, moving the cart to compensate for the deviation. The corrective movements are generally imprecise: the operator practically never drives the stick close to the upright position. Substantial errors are usually corrected straight away. On contrary, in the case of small to moderate error the operator usually halts the control for some time after the initiated cart movement is completed, even if the resulting deviation from upright position is evident.

To check whether the diversity in the subjects' age and previous task experience leads to fundamentally different properties of the system dynamics, we analyzed the statistical distributions of the system state variables. The distributions of the stick angle θ and the angular velocity $\dot{\theta}$ are extremely similar (up to scale) for all ten subjects (Fig. 5). The angle has approximately Laplacian distribution, however, the distribution is bimodal with a narrow gap (width of order 0.1 std(θ)). The angular velocity distribution is similar in profile to the angle distribution. Both the θ and $\dot{\theta}$ distributions are substantially non-Gaussian, which confirms that the observed control behavior is highly nonlinear.

The remarkable similarity of the distributions across the experimental group (up to scale) may indicate that all the subjects employ the same nonlinear mechanisms in controling the stick.

Model

To infer the key mechanisms governing the intermittent control behavior observed in the experiments, we construct a model for the dynamics of overdamped stick controlled by human operator. We single out two control mechanisms that are hypothesized to be crucial in the given setting. We propose a mathematical model that captures only these specific mechanisms, neglecting the factors of supposedly minor importance. Finally, we confront the model with the experimental data to verify our hypothesis about the mechanisms of human control in the analyzed task.

The overdamped stick dynamics are governed by Eq. (1). For simplicity, prior to constructing the control model we linearize it near the vertical position $\theta = 0$,

$$\tau_\theta \dot{\theta} = \theta - \frac{\tau_\theta}{l}\upsilon, \quad (2)$$

where the cart velocity υ is controlled by the operator.

We hypothesize that two mechanisms jointly determine the operator control behavior. The first mechanism determines the control dynamics in the active control phase, regulating the magnitude of control based on the deviation of the system from the desired state ("how to react"). The second mechanism is responsible for detecting the events that should be responded to ("when to react"). Within this framework, we suppose the first mechanism to operate as an *open-loop controller* intermittently activated by the second mechanism, which is based on the idea of *stochastic control triggering*. The dynamics of the two mechanisms are independent, although coupled. Appealing to the idea of the phase space extension (Zgonnikov & Lubashevsky, 2014), we decompose the term υ representing human control in Eq. (2) into two separate phase variables:

$$\upsilon = u\xi, \quad (3)$$

where u describes the dynamics of the "how-to-react" mechanism and ξ represents the "when-to-react" mechanism.

"How-to-react" mechanism

Human control is often characterized by open-loop, preprogrammed corrective actions, rather than closed-loop feedback strategies (Ben-Itzhak & Karniel, 2008; Gawthrop et al., 2011). In the current context it may imply that once the operator launches a hand movement to compensate for the detected stick deviation, this movement is not interrupted until fully executed. Indeed, many variations of such a strategy are possible. For example, our experimental results suggest that if the operator generates a highly imprecise movement, this movement is likely to be corrected early despite still being executed. However, in the model we capture only the basic pattern of open-loop control: once the deviation is detected, the system is driven to some vicinity of the goal state by a single preprogrammed, short control effort.

In the analyzed task the operator accelerates the cart on the screen in response to the current state of the system, θ and u. The basic pattern of the operator actions in the active control phase can be easily captured by the following equation

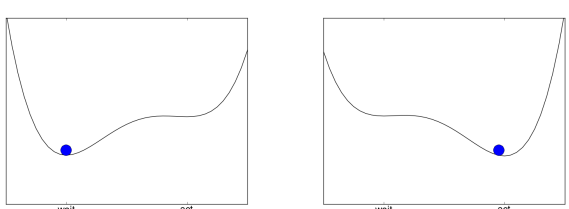
$$\dot{u} = \gamma\theta - \sigma u, \tag{4}$$

where γ and σ are non-negative constant coefficients.

"When-to-react" mechanism

In modeling the "when-to-react" mechanism we employ the concept of event-driven intermittency: the control is triggered whenever the system essentially deviates from the goal state. How particularly the operator determines whether the deviation is essential or not is however a non-trivial question.

We hypothesize that in controlling an unstable system the operator can be either in the "act" state or in the "wait" state, which is determined in part by the current deviation from the goal, and in part by stochastic factors. Appealing to physics, we capture the switching between these two states by the random walk in double-well potential. The potential landscape changes depending on the current system state (Fig. 6), and the random fluctuations allow for the probabilistic mechanism of switching.



(a) Small deviation (b) Large deviation

Figure 6: Double well potential of human mental state during a control process

The new phase variable ξ describes the dynamics of the mental state of the operator. Whenever the operator feels that the current value of deviation can be neglected, ξ remains equal or close to zero. In contrast, $\xi = 1$ when the operator decides to actively control the system. The dynamics of ξ is intrinsically stochastic: the larger the deviation, the higher the probability that ξ "switches on". Such behavior can be naturally captured by the following equation

$$\tau_\xi \dot{\xi} = -\frac{\partial H}{\partial \xi} + \varepsilon H \zeta, \tag{5}$$

where τ_ξ defines the time scale of the mental state dynamics, H is the Hamiltonian describing the energy landscape and $\varepsilon H \zeta$ is a multiplicative white noise. The Hamiltonian H should be of such form that captures the basic properties illustrated in Fig. 6: the system energy should be low in the "wait" state when the current deviation is small, and, on the opposite, when the deviation is large, the "act" state should be the most probable. Here we employ the following ansatz implementing these properties:

$$H(\xi,\theta) = \frac{\xi^4}{4} - (1+a)\frac{\xi^3}{3} + a\frac{\xi^2}{2} + \frac{1-a}{12}, \tag{6}$$

where $a = a(\theta)$ causes the energy landscape to change depending on the system state. Specifically, function $a(\theta)$ should be chosen in such a way that $a \approx 1$ if $\theta \approx 0$, and $a \approx 0$ otherwise. We use the following expression (although any function with the similar profile can be used):

$$a = \frac{\theta_{\text{th}}^2}{\theta_{\text{th}}^2 + \theta^2}, \tag{7}$$

where θ_{th} is the characteristic angle threshold value.

Equations (2–7) form the model capturing the "how-to-react" and the "when-to-react" mechanisms and their interaction in the context of overdamped stick balancing.

Response delay

Both components of the proposed framework reflect in principle complex cognitive operations which take some time in the real control process. However, in the overdamped stick balancing the operator reaction delay effects are supposed to be of minor importance, given the event-driven control hypothesis. During the time required for the two mechanisms to process the initial deviation $\theta(t_0)$ this deviation increases by a factor depending on the response delay T and the time scale of the uncontrolled stick motion τ. Solving the initial value problem for Eq. (2) where $\upsilon = 0$, we get $\theta(t_0 + T) = \theta(t_0)e^{T/\tau}$. The response delay thus affects the difference between the angle initially detected as worth reacting to, $\theta(t_0)$, and the angle the operator actually reacts to, $\theta(t_0 + T)$. However, as long as T remains in some sense less than τ, the response delay cannot affect the basic patterns of the system dynamics, in contrast, to, e.g., the standard, underdamped stick balancing. This allows us to conclude that in the overdamped stick balancing the operator response delay does not substantially affect the system dynamics and therefore may be omitted in the model.

Simulation results

Here we report the preliminary analysis of the model (2–7) and confront the model with the experimental data. Indeed, the proposed model still requires the detailed scrutiny, as well as thorough comparison to the data from human subjects; these analyses will be reported elsewhere.

We study numerically the basic properties of the system (2–7) by simulating its dynamics using the explicit order 1.5 stochastic Runge-Kutta method (Roessler, 2005). The values of system parameters used in simulations are: $\tau_\theta = 3, l = 1, \gamma = 1, \sigma = 1, \tau_\xi = 0.05, \varepsilon = 0.7, \theta_{\text{th}} = 0.2$.

The phase trajectory of system (2–7) shows that both the described mechanisms are actually captured by the model (Fig. 7). The initially perturbed system moves along the $\tau_\theta\dot{\theta} = \theta$ manifold with the cart velocity u close to zero. This motion regime represents the passive control phase. As the angle θ increases, the system may escape from the vicinity of the manifold $\tau_\theta\dot{\theta} = \theta$, thereby switching from the passive to the active control phase. This transition is induced by the random factor $\varepsilon H \zeta$, so it occurs at probabilistically determined angle. The trajectory of the system during the active control phase represents the single corrective movement aimed at driving the stick to the vicinity of the vertical position. This point is highlighted by the cart velocity dynamics: the initially generated corrective movement is not corrected once started (Fig. 8).

The simulated phase trajectory of system (2–7) represented in Fig. 7 resembles the experimentally obtained phase trajectories (Fig. 4). Naturally, many features of the experimental data are not captured by the model. For instance, the model always overshoots the goal position by design, while human subjects often undershoot substantially (Fig. 4). At the same time, this and other mismatches between the experimental and simulted trajectories are apparently of minor importance, as demonstrated by the analysis of the statistical distributions (Fig. 9).

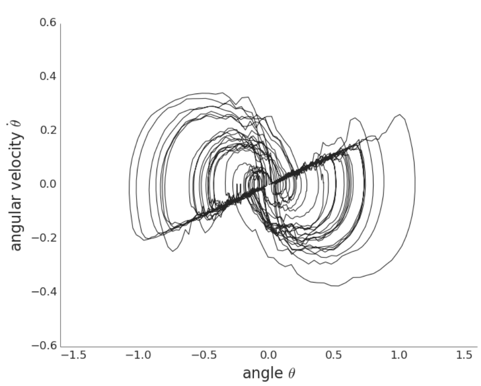


Figure 7: Phase trajectory of stick motion generated by model (2–7). Values of parameters used for simulations are: $\tau_\theta = 3, l = 1, \gamma = 1, \sigma = 1, \tau_\xi = 0.05, \varepsilon = 0.7, \theta_{\text{th}} = 0.2$.

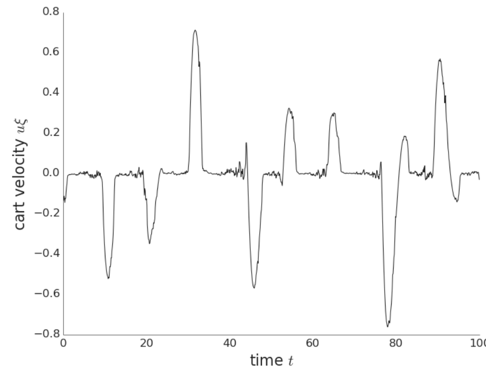


Figure 8: Cart velocity generated by model (2–7). Values of parameters used for simulations are: $\tau_\theta = 3, l = 1, \gamma = 1, \sigma = 1, \tau_\xi = 0.05, \varepsilon = 0.7, \theta_{\text{th}} = 0.2$.

We compare the distributions of θ and $\dot{\theta}$ of the stick balancing by human subjects and the distributions produced by system (2–7) in a numerical simulation with duration of 1000 time steps. Both the stick angle and angular velocity distributions of the human-controlled stick are well captured by the model (2–7) (Fig. 9).

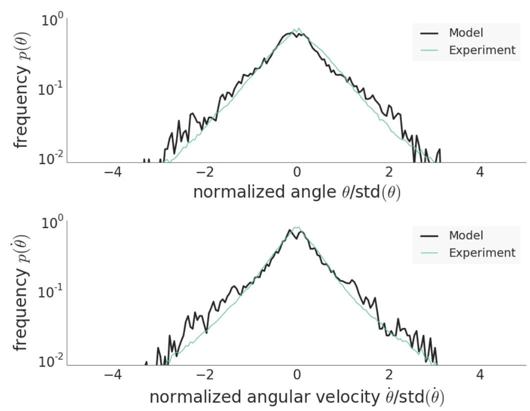


Figure 9: Stick angle and angular velocity distributions exhibited by system (2–7) and the average distributions obtained experimentally. The model distributions are calculated based on the data obtained numerically for $T = 1000$ time units. The parameter values used for simulations are $\tau_\theta = 3, l = 1, \gamma = 1, \sigma = 1, \tau_\xi = 0.05, \varepsilon = 0.7, \theta_{\text{th}} = 0.2$. The angle and angular velocity values are normalized with respect to their standard deviations: $\theta \to \theta/\text{std}(\theta)$, $\dot{\theta} \to \dot{\theta}/\text{std}(\dot{\theta})$.

Discussion

This paper highlights that the stochastic control triggering mechanism is an essential, possibly crucial component of human control. We found that in overdamped stick balancing the subjects exhibited clear intermittent control patterns. The resulting distributions of the stick angle and the cart velocity do not depend (up to scale) on the subject's age, balancing experience and skill. The universality of the distributions implies that the mechanism underlying the human control in the present task do not vary from subject to subject, but instead are rather fundamental. We hypothesize that this mechanism operates as a threshold-based open-loop control, where the threshold is defined in a stochastic manner. The model implementing the hypothesized mechanism matches the experimental data. The phase trajectory exhibited by the model imitates the basic motion pattern of the overdamped stick under human control. Most importantly, the statistical distributions produced by the model match those obtained experimentally. Overall, our results imply that the stochasticity of the threshold mechanism plays a decisive role in human control at least in the considered task, and possibly in a wide class of human-controlled processes.

Conventional approach to modeling human control is to approximate the basic control algorithm implemented by human central nervous system (CNS) in a deterministic way, usually as a linear feedback with a threshold element. In virtually all available human control models the stochasticity of the system dynamics is typically expressed by adding the additive or multiplicative noise to the control signal. The noise term is thought to capture the cumulative effect of all the factors unaccounted for in the basic control law. Such noise is often called "sensorimotor", which reflects the assumption that the major sources of uncertainty in human control are the sensory and motor systems. Our results provide strong evidence that besides the noisiness of the sensory and motor systems, the processing of the input signal by the CNS is intrinsically stochastic on its own. Consequently, the stochastic control triggering mechanism may be one of the key aspects of human control. Indeed, one may imagine many situations where the stochasticity of the threshold would be a minor factor, e.g., due to the overall high degree of uncertainty in the control system. On the other hand, it is completely possible that the effects of the threshold stochasticity would be amplified, not diminished, by other factors. Hence, regardless the practical considerations, one has to be aware of this stochasticity.

According to our hypothesis, in balancing the overdamped stick the operator continuously observes the external process (i.e., stick motion), and decides when and how exactly to interrupt it given the current circumstances. Similar processes (although typically in much more complex environments) are studied within the field of dynamic decision making, which focuses on the processes "which require a series of decisions, where the decisions are not independent, where the state of the world changes, both autonomously and as a consequence of the decision maker's actions, and where the decisions have to be made in real time" (Brehmer, 1992). Similarly to the overdamped stick balancing, in arguably any dynamic process involving human as a decision maker the procedure of detecting the deviations from the desired situation is stochastic in its nature. For instance, in car following, air traffic control or organizational management a system state either may be classified as acceptable with some probability, or may trigger the active behavior of a human observing the system. Thus, the concepts and models elaborated in the investigations of the event-driven human control may also prove useful in understanding human behavior in a wide variety of dynamic processes.

Acknowledgments

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