The Hands That Guide the Thinking: Interactivity in Mental Arithmetic

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Abstract

Whether it is in mining distal cultural influences or using more proximal artefacts, problem solving in the wild routinely scaffolds on the basis of interacting with resources outside the head. Individuals often gesture, point or use objects as an aid to solving quotidian arithmetic problems. Interactivity has been linked to better performance in problem solving, possibly due to a more efficient allocation of attentional resources and better distribution of cognitive load. Previous research suggests an interplay between the cognitive and motor system whereby the later can lighten the strain on working memory capacity (Goldin-Meadow, Nusbaum, Kelly & Wagner, 2001; Carlson, Avraamedes, Cary & Strasberg, 2007; Vallée-Tourangeau, 2013). In attempting to simulate these moves made in the world, different levels of interactivity were examined with a series of mental arithmetic problems. Participants were also profiled in terms of attitude to varying problem presentations as an assessment of their engagement in the task. The integration of artifacts, such as tokens or a pen, provided individuals with the possibility to explore the opportunities afforded by a dynamic modification of the problem. Mental arithmetic performance was more accurate and more efficient under these conditions. Participants also felt more positive about and better engaged with the task when they could reconfigure the problem presentation through interactivity. These findings underscore the importance of engineering task environments that support distributed problem representation and adequate levels of interactivity that creates a dynamically shifting topography of action affordances.

Keywords: Interactivity; Mental arithmetic; Problem solving; Distributed Representation; Task engagement.

Introduction

Mathematical problems are embedded in everyday life in a variety of different shapes and forms. When confronted with an arithmetic task, people often rearrange the physical display by interacting with the environment. They might move coins while counting their money, note subtotals with a pen or use their hands to gesture, point or count (Kirsh, 1995; Neth & Payne, 2001).

Mental arithmetic tasks often entail strategic thinking and deliberate information processing, which require time and effort (Vallée-Tourangeau, 2013). Besides basic, well-rehearsed sums, computations are generally said to pose a relatively high cognitive load on an individual's internal resources, such as working memory (Ashcraft, 1995; DeStefano & Lefevre, 2004). Numbers are held, added and manipulated in order to solve the problem employing different working memory subsystems, including storage, retrieval and allocation of attentional resources. Dependent on the complexity and length of a mental arithmetic task, the demands of finding a solution may impose a relatively low or high cognitive load, potentially imposing substantial demands on working memory capacity. This capacity may, however, be stretched or reduced by certain internal or external factors, which can subsequently paint a misleading profile of an individual's true arithmetic capabilities (Ashcraft & Moore, 2009).

Interactivity

The internal cognitive and physical resources deployed to tackle a problem may be taxed by various features of the task—such as time pressure, level of difficulty, and fatigue. Reasoners naturally recruit artefacts and use the physical space to make thinking easier and more efficient. Increased levels of interactivity have been linked to better performance, possibly due to a stronger focus of attention and better distribution of cognitive load (Goldin-Meadow et al., 2001; Carlson, Avraamides, Cary & Strasberg, 2007; Vallée-Tourangeau, Sirota & Villejoubert, 2013). Previous research implicates an interplay between the cognitive and motor system whereby the later can lighten the strain on working memory capacity reducing the expenditure of internal resources (Goldin-Meadow, Alibali & Church, 1993).

Improved effectiveness, indicated by increased accuracy and speed, has also been related to movement execution, such as nodding and pointing (Goldin-Meadow et al., 2001), as well as manipulations of the problem's spatial arrangement (Vallée-Tourangeau, 2013). So it seems that the shaping and re-shaping of the problem presentation can help surpass the original limitations of working memory capacity by lowering the expense of internal resources necessary to solve the task and guide attention. This could subsequently increase efficiency.
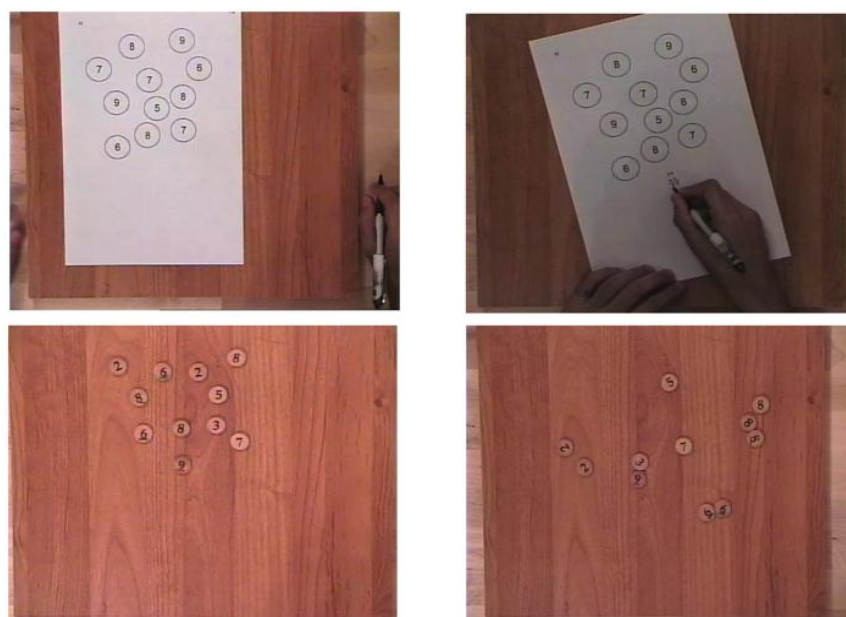
Figure 1: The board on the top left is an example of a standard template used for all four conditions. The board on the top right shows the participant undertaking the pen-paper condition. The board on the bottom left is an example of the wooden tokens in preparation for the participant. On the bottom right the same board after the participant has completed the task. Note the congenial groups of the numbers.

Thus, interacting with the environment and utilizing artefacts can increase efficiency by distributing the storage and computational demands of the task across resources internal and external to the reasoner. Such distributed cognitive processes shift the cognitive load from the reasoner onto a system in which she is embedded (Vallée-Tourangeau, 2013).

Mathematical tasks are frequently assessed in terms of accuracy and efficiency. Accuracy measures the precision of the calculated solution in relation to the correct answer. Efficiency, on the other hand, involves a relation between invested effort and resulting performance (Vallée-Tourangeau, 2013). Yet, it is not only the problem itself or its complexity that impacts how accurately or efficiently an individual performs in a mathematical task. The presentation of a problem can guide behaviours and strategic choices in the path to a solution (Vallée-Tourangeau et al., 2011). Embedded in this problem presentation are the varying possibilities for interaction, the dynamic loop of information and action flowing between a person and the outside world, the nature of these interactions having the potential to direct strategic choices (Neth & Payne, 2001; Kirsh 2013). Kirsh (1995) describes an organizing activity that recruits external elements such as the hands, coins and pen and paper to reduce cognitive load as a complimentary strategy to the internal processes of cognition. In turn this coupling of the mental and external space configures a distributed thinking system.

Attitude Toward the Task. Student engagement in performing academic tasks may be an influential factor in learning and achievement, with the suggestion that the activity by which learning is experienced may provide a stimulus for this engagement (Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003). It is also possible that a

task that offers a student a sense of connection to the real world is more likely to maximize student engagement (Newmann, Wehlage, & Lamborn, 1992). Furthermore Schiefele and Csikszentmihaly (1995) discuss the importance of the affective experience on performance, while engaging in mathematics in the classroom. Positive emotions elicited by the task experience may contribute to increased problem-solving capacities (Shernoff et al., 2003).

The Current Experiment

Previous research has investigated gesturing (Goldin-Meadow et al., 2001), interactivity and additions utilising a PC interface (Neth & Payne, 2001), interactivity and working memory (Vallée-Tourangeau, 2013) and simple coin counting strategies (Kirsh, 1995). Results indicated that interactivity potentially influences the ability to solve problems. However, the picture is piecemeal, and no study as yet has compared a wide range of different types of interactive behavior using artifacts. Consequently, the current experiment explored the role of interactivity in adult participants using tangible artefacts with which the problem presentation can be modified as participants complete the arithmetic task. Thus the external problem presentation tracks the dynamic interface between the agent's internal representation and the world. Previous research on the role of interactivity in mathematical reasoning and learning has generally presented material either on paper on a computer display. Interactivity and the potential to re-shape the problem presentation was manipulated in terms of four conditions. In the first, participants added a sequence of single-digit numbers with their hands down and in a second they were allowed to point at the numbers. Thus in these two conditions, the problem presentation can not be modified, but participants

can engage in some complementary actions in the latter. In the other two conditions participants could re-shape the problem presentation. For the third, they were given a pencil and could recast the sum as they saw fit in arriving at a total. In the fourth, the sums were presented as a randomly arrayed set of wooden tokens that participants were invited to move to arrive at the correct sum. Across these four different levels of interactivity, performance was measured in terms of accuracy and efficiency. Not only did we expect accuracy to be influenced by interactivity, but efficiency should be related to the degree to which participants can modify the problem presentation as they compute the totals. We defined efficiency in terms of the degree of accuracy relative to the resources invested in completing the sum; the latter was operationalized as the time taken to do the sum. We expected that interactivity conditions that made it possible for participants to manipulate the problem presentation in a manner that reflected and complemented their internal processing to yield the highest level of accuracy and efficiency.

Method

Participants

Sixty participants (40 females, mean age 23.32, *SD* = 4.41) were recruited for this experiment.

Materials and Measures

Arithmetic Task. All participants were presented with five sets of numbers in four conditions and asked to calculate the sum of the numbers. Therefore each participant calculated 20 sums over the experimental session. They were requested to calculate each set as quickly and accurately as possible. Each set consisted of 11 single-digit numbers. For the purpose of the present study, single-digit numbers between one and nine were first categorized as low (1-4) or high (5-9) in order to generate the range of possible sums in a more principled manner. Four groups of sums were created: Group I (5 low, 6 high), Group II (only high), Group III (3 low, 8 high) and Group IV (4 low, 7 high). A set of only low numbers was not included to reduce ceiling effects. The sets of sums presented to participants consisted of two sums from group I and three from groups II –IV. Each of these groups was assigned to one of the four interactivity conditions, and this assignment was counterbalanced across conditions. As a result each participant was presented with a unique set of sums in each condition.

Sums were configured with templates consisting of 11 circles (1.2cm) covering between half and ¾ of a side of A4, that was delineated by a 39cm by 34cm varnished wooden board on which participants carried out all the additions (see Fig 1). By altering the order of the templates within each group, the visual presentation of the sums was additionally randomized for each set. For the tokens condition, templates of tracing paper were created with the same configurations of the constituent numbers as the paper version of the other conditions. Wooden tokens were placed in the corresponding position and the tracing paper was removed before the start of the task. The numbers were not revealed to the participant until all tokens were in place. Performance was measured in terms of accuracy (the correct answer), absolute error (the absolute deviation from the correct answer) latency (time taken until answer verbalised) and efficiency.

Efficiency was calculated as a ratio of the proportion of correct answers for a given problem set over the proportion of time invested in solving that set (out of the longest time the slowest participants invested in solving that set). For each of the four conditions, participants were first ranked according to their averaged latencies. The average of the slowest 25% served as a reference point and represented the maximum effort one could expend in that condition. Thus the efficiency ratio denominator was a given participant's latency over the average latency for the slowest quartile; the numerator was that participant's proportion correct solutions in that condition. For example, a participant in a given condition may have solved three out of the five sums, for a proportion .6 correct. In turn, the participant's average latency for completing the five sums in that condition might have been 30 seconds. If the average latency for the slowest quartile was 40 seconds, then that participants invested 75% (30/40) of the total possible time for completing the sums in that condition. The efficiency ratio for that participant would then be .6/.75, or .8.

Level of Interactivity. Interactivity was manipulated in terms of four experimental conditions; namely (i) static, (ii) pointing, (iii) pen-paper and (iv) tokens. Tokens were used as a close representation of coins as everyday artefacts; the decision to use tokens rather than coins was made in order to maintain the simplicity of the sums by using the tokens numbered 1-9. In the static condition, participants were asked to compute the sum mentally with their hands flat on the table. In the pointing condition, there were no restrictions on movement, other than to exclude the use of the pen to make notes. Hence, participants were allowed to use their fingers to point to the numbers that composed the sum. In the pen and paper condition, participants were given a pen and were allowed to write on the sheet provided by the experimenter containing the number set. Finally, in the tokens condition, the sums were presented in the form of round numbered wooden tokens (1cm in diameter, with black digits 1-9), which could be moved by the participants. The format of the presentation was visually constant and the material was always presented on the same surface.

Attitude Toward Task Assessment (ATTA). Shernoff et al. (2003) used the Effective Sample Method (ESM) to measure a number of factors including affective experiences. This affective experiences component of the ESM questionnaire was used as the basis for a scale, Attitude Toward Task Assessment (ATTA) designed to assess the engagement of participants in the tasks undertaken in this study. The primary purpose of ATTA was to assess the affect of abstract versus concrete methods in mental arithmetic rather than an individual's preference for or reliance on external aids, such as a calculator or pen and paper in daily life.

A scale composed of eight items was created to assess an individual's attitude towards completing the sums in each experimental condition. The eight items asked
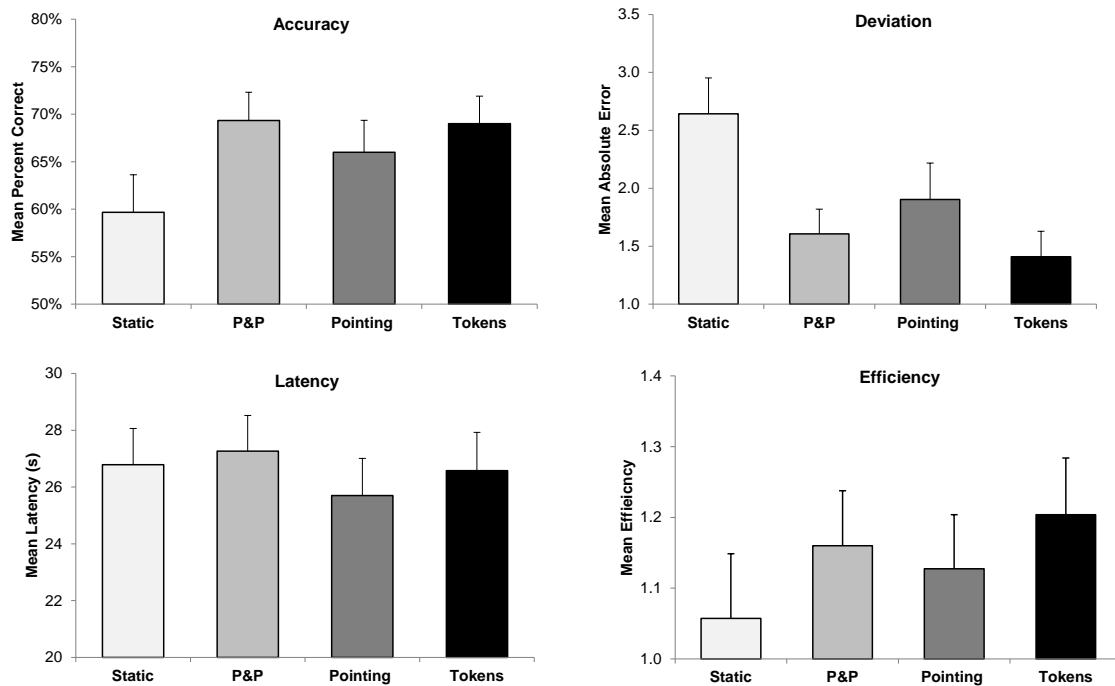
Figure 2: Mean percentage correct sums (top right panel), absolute calculation error (top left panel), latency to solution (bottom left panel) and mean calculation efficiency (bottom right panel) in the four experimental conditions. Error bars are standard errors of the mean.

participants to rate how easy, pleasurable, fun, threatening, stressful, tiresome or effortful the task was and how motivated they were to perform well in the task. Each item was scored on an 8-point Likert scale, labeled from zero (definitely not) to seven (definitely yes). Total scores could range from zero to 56-the higher the score the more positive the attitude toward the task. Each participant completed the same ATTA scale four times once following each of the four conditions. The alpha reliability of the eight-item scale for each experimental condition indicated that the scale had good reliability (Static, Cronbach's α = .80; Pen-paper, Cronbach's α = .77; Pointing, Cronbach's α = .78; Tokens, Cronbach's α = .77).

Results

Accuracy

The mean number of correct answers, as shown in the top left panel of Figure 2, was greatest in the tokens (M = .69, SD = .22) and the pen-paper (P&P, M = .69, SD = .23) conditions. The pointing condition (M = .66, SD = .26) indicated slightly less accurate calculations, with the static condition resulting in the weakest performance (M = .60, SD = .30). A one-factor repeated measures analysis of variance (ANOVA) indicated a significant difference between the conditions, $F(3,177)$ = 3.12, p = .027, η^2 = .050. Post-hoc tests revealed a significant difference between the static and the pen-paper conditions (p = .006) and the static and tokens conditions (p = .020).

Absolute Error

Deviation from the correct answer was greatest in the static condition (M = 2.64, SD = 2.39); the pointing (M = 1.90, SD = 2.43) and pen-paper (M = 1.61, SD = 1.65)

conditions produced lower deviations than the static condition while the lowest deviations from the correct answer were observed in the tokens condition (M = 1.41, SD = 1.69; see top left panel of Fig. 2). The one-factor repeated measures ANOVA revealed a significant difference between interactivity conditions, $F(3,177)$ = 6.34, p < .001, η^2 = .097, with post-hoc tests indicating a significant difference between the pen-paper and the tokens conditions when compared to the static condition (p = .005, p < .001 respectively).

Latency

The latency data are shown in the bottom left panel of Figure 2. Participants generally took about the same amount of time to complete the task across the four conditions (static M = 26.79, SD = 9.88; pen-paper M = 27.26, SD = 9.73; pointing M = 25.70, SD = 10.09; tokens M = 26.58, SD = 10.41). The main effect of interactivity in the one-way repeated measures ANOVA was not significant, F < 1.

Efficiency

As illustrated in the bottom right panel of Figure 2, performance was most efficient in the tokens (M= 1.20, SD = .62) and the pen-paper conditions (M = 1.15, SD = .60) with the static (M = 1.05, SD = .71) and the pointing (M = 1.12, SD = .59) conditions being least efficient. The main effect of interactivity, however, was not significant, F (3,177) = 1.39, p = .247

Attitude Toward the Task

The attitude of participants was more positive toward the pen-paper (M = 37.98, SD = 8.38) and the tokens (M = 37.78, SD = 8.94) conditions, than the pointing condition (M = 34.12, SD = 8.76) and least favourable for the static

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condition (M = 31.63, SD = 9.13; see Fig. 3). The main effect of interactivity was significant, $F(3,117)$ = 17.07, p < .001, η^2 = .231. Post-hoc tests further identified highly significant differences between the static and the tokens conditions and the static and pen-paper conditions (p < .001 for both conditions). The static and pointing conditions were also significantly different (p = .025). Feelings toward the pointing condition differed significantly from those in the pen-paper (p < .001) and tokens conditions (p = .008). The correlation between ATTA scores and efficiency was positive, although only marginally significant in the pen and paper and tokens conditions: Static, $r(58)$ = .197, p = .131; pen and paper, $r(58)$ = .260, p = .045; pointing, $r(58)$ = .174, p = .185; and tokens, $r(58)$ = .248, p = .056.
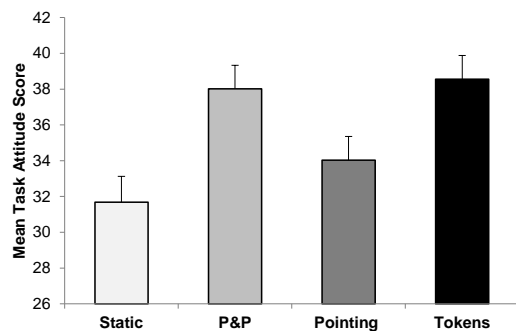

Figure 3: Mean attitude toward task assessment score in the four interactivity conditions. Error bars are standard errors.

Discussion

The present experiment was designed to explore the effects of different levels of interactivity on arithmetic performances for single-digit additions. The degree of engagement with and attitude towards completing the task as a function of the level and nature of interactivity was also investigated. The results indicated that the use of artefacts enhanced performance in simple arithmetic problems, supporting the hypothesis that interactivity benefits performance. When participants were given the opportunity to use artefacts such as tokens, accuracy improved and deviation from the correct answer decreased. The increase in interactivity generally required no more time to announce an answer, confirming similar findings in previous research (Neth & Payne, 2011; Vallée-Tourangeau, 2013). Accuracy increased with greater interactivity, while latencies remained unchanged; as a result, performance tended to be most efficient in the tokens condition. Although the overall main effect of interactivity on efficiency was not significant, the difference between the tokens and static condition was nearly significant, $t(59)$ = 1.92, p = .060 Conversely, when participants were asked to rely primarily on internal cognitive resources, as in the static condition, accuracy was impaired. In the higher interactivity conditions participants were given the opportunity to recruit external resources to aid in calculating the answer. The opportunity to engage with the environment enabled the distribution of cognitive load, augmented working memory resources and delegated the control of attention in part to the dynamic environment that cued the next action. In addition, the possibility of modifying the physical presentation of the problem, enabled participants to reconfigure the problem. This improved the cognitive congeniality of the problem (Kirsh, 1995), but also provided a more dynamic set of action affordances that supported more efficient problem solving. Contrary to previous literature (Goldin-Meadow et al., 2001) the opportunity to gesture, point or even use the fingers to count in the pointing condition did not offer any significant benefit to the performance in solving the simple maths sums. This is possibly due to the focus of Goldin-Meadow et al. (2001) on memory rather than accuracy of results. Thus the beneficial effect of interactivity on reasoning is not simply a function of off-loading working memory, but also reflects better executive function skills that are cued and prompted by the shifting affordances offered by a dynamic problem environment.

The data on the participants' attitude towards completing the sums in the different conditions paralleled the impact of interactivity on performance. Conditions involving external resources, pens or tokens, seemed to elicit a more positive, engaged attitude towards the simple arithmetic problems, than the restricted, static condition. Of course, participants were also more accurate in the interactive conditions. But the more positive attitudes towards the problems cannot be attributed to task success since participants were not given feedback about their performance, that is, after announcing each sum, the experimenter did not tell the participants whether their answer was right or wrong. However, results also showed that as the ATTA scores increased efficiency increased, with marginally significant correlations in the tokens and pen and paper. These being the two conditions in which participants exerted some control over the problem configuration. This suggests that engagement with the task tended to encourage more efficient performance. These findings are in keeping with the notion that higher levels of personal involvement positively affect performance (Shernoff et al., 2003). Also, changing the visual display may ease the task and thereby lighten the cognitive load, which increases effectiveness and alters attitudes (Vallée-Tourangeau et al., 2013).

In calculating simple arithmetic sums, an individual presented with the opportunity to use a complimentary strategy, such as manipulating tokens, is embedded in a distributed cognitive environment. Studying systems rather than individuals poses theoretical and methodological challenges. Theoretically, the nature of the problem representation and the trajectory of the solution as it evolves from an embryonic to a fully formed answer, should perhaps be understood as being distributed and configured in terms of a transaction between the participants' internal resources and the shape and nature of the resources in the external environment. What a participant is 'thinking' is not independent of the state of the environment, and as the environment is shaped by the participants, understanding that environment is not independent from the participant. The methodological implications of this transactional perspective are important. Of course, systems can be more complex, and composed of a much wider range of functional elements,

which challenges the traditional toolkit of experimental cognitive psychologists designed to deal with a cognitively sequestered individual in a laboratory environment that generally prevents interactivity. But beyond issues of complexity and computational promiscuity (Wilson & Clark, 2009), a participant-environment transactional link specifies a more qualitative idiographic cognitive science supported by an observational toolkit that can code at a much smaller time scale the evolution of a problem representation and its solution (for an excellent example of how such a toolkit can be developed, see Steffensen, 2013). Finally, adapting the cognitive psychologist's laboratory to permit the physical manipulation of a problem presentation offers a more representative window onto thinking outside the laboratory. To be sure, people can simulate and think in their head without physically interacting with the outside world (although this internal cogitation may well reflect the internalization of much interactivity); but they often "go to extraordinary lengths to *avoid* having to resort to (…) fully environmentally detached reflection(s)" (Clark, 2010, p. 24, emphasis in the original). The data presented here reveals the importance of engineering task environments in the lab that support distributed problem representations to better understand the engagement of individuals as they explore and manipulate the external world to solve problems.

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