

# Counting Practice with Pictures, but not Objects, Improves Children's Understanding of Cardinality

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## Abstract

When counting, the final word used to tag the final item in a set represents the cardinality, or total number, of the set. Understanding of this concept serves as a foundation for children's basic mathematical skills. However, little is known about how the early learning environment can be structured to help children understand this important concept. The current study examined the effects of the representational status of to-be-counted items on preschoolers' understanding of cardinality. Children ( $M$  age = 3 years, 6 months) were randomly assigned to receive counting practice with either physical objects or pictures over five practice sessions. Children's counting skill and understanding of cardinality were assessed at pretest and posttest. Results revealed that only children in the picture condition increased their understanding of cardinality from pretest to posttest. These results suggest that picture books are better than physical objects at supporting children's understanding of cardinality.

**Keywords:** cognitive development; counting; learning; mathematics

Counting is a foundational skill. It provides the basis for learning to add and subtract, as well as for other basic arithmetic skills (Aunio & Niemivirta, 2010; Fuson, 1988;

Stock et al., 2009). According to the NCTM (2000), teaching young children to "count with understanding and recognize 'how many' in sets of objects" is one of the earliest ways to get them on the path toward developing a good number sense.

Counting is also of theoretical interest because it is the first formal math system that young children learn, and it takes children a surprisingly long time to master it (Le Corre, et al., 2006; Sarnecka & Carey, 2008; Wynn, 1990). Research suggests that children go through a relatively predictable developmental progression as they construct an understanding of counting. First, they pass through the "one-" knower level, where they can reliably identify and give one object, but not more. Then several months later they become "two-" knowers and then "three-" knowers, and some studies have even found "four-" knowers (Le Corre et al., 2006). Finally, children become "cardinal principle" (or CP) knowers (Le Corre et al., 2006). Prior to this development, children construct understanding of the numbers "one," "two," and "three" quite slowly over the course of several months. In contrast, CP-knowers, seemingly all at once, develop understanding of any number

in their count list (Le Corre et al., 2006; Sarnecka & Carey, 2008; Wynn, 1990).

Although this conceptual leap typically happens around 3 1/2 to 4 years of age (Sarnecka & Gelman, 2004; Wynn, 1990), there are large individual differences in the age at which children become CP-knowers, with some children demonstrating understanding at age three and some not until age five (Gunderson & Levine, 2011; Sarnecka & Lee, 2009). These early individual differences matter for future success. Indeed, research has shown that early math knowledge prior to the start of kindergarten is one of the strongest predictors of future academic achievement (Duncan et al., 2007). Moreover, early difficulties in mathematics are not easily overcome with schooling. Instead, children who struggle early on tend to lag increasingly behind their peers in the acquisition of more complex math knowledge over time (Jordan et al., 2009).

Given the foundational role of understanding of cardinality in children's future achievement, it is surprising that relatively few studies have investigated how the learning environment shapes children's understanding. The assumption must be that the learning environment matters, as preschool teachers, curricula, and Standards focus on teaching cardinality (Common Core; NCTM, 2000; Greenes et al., 2004; Sarama & Clements, 2009). A few pieces of empirical evidence also support the idea that different types of input lead to differences in understanding. For example, Levine and colleagues (Gunderson & Levine, 2011) showed that the quantity and quality of math talk used at home in the toddler months predicts preschoolers' understanding of cardinality. Ramskar et al. (2011) found that the way children are introduced to the to-be-counted sets affects their understanding of cardinality. Children learn more when the to-be-counted objects are labeled first and then quantified (e.g., "Balls. There are two.") than when they are quantified first and then labeled (e.g., "There are two balls."). In another study, Mix et al. (2012) showed that counting practice in which the cardinality of the set is labeled first and then counted immediately after is more effective than other types of counting practice at promoting understanding of cardinality. These studies suggest that the learning environment affects children's understanding of cardinality. However, to our knowledge, Ramskar et al. and Mix et al. are the only to experimentally investigate how variations in the learning environment affect understanding of cardinality.

Given the lack of research on this topic, there are many different types of variations in the learning environment ripe for study. In the present study, we focused on whether the entities used during counting practice matter. Specifically, we considered whether it is better for children to practice counting with pictures of objects versus using the actual physical objects themselves. We focused on this feature of the environment for several reasons. First, it is highly controllable. Second, concreteness of instructional materials is a hot topic in cognitive development and educational research. And finally, it is not intuitively obvious which

materials teachers and parents should use when teaching children to count. Mix et al. used picture books in their study. At the same time, teachers tend to prefer to use physical objects in their lessons on counting.

There are some theoretical reasons to expect objects to be better than pictures. First, objects might simply be more engaging and motivating than pictures, and they might help children stay engaged in the task. They are also more manipulatable, and being able to manipulate objects during learning tasks may benefit children's comprehension and memory (Glenberg et al., 2004). Physical objects might also encourage pointing, touching, and moving gestures, which have been shown to facilitate counting accuracy. Alibali and DiRusso (1999) suggested that pointing and touching promote counting accuracy because they help children keep track of the items they've counted and coordinate tagging the objects and saying the number words in one-to-one correspondence. Such gestures allow children to use the external environment to help them segment the set into individuals that have and have not been counted, so children do not have to hold all of the information in working memory. Objects may be particularly helpful in this sense because children can physically move objects that have already been counted away from objects that are still left to count. Thus, we predicted that children might gain more counting skill after practicing counting with objects than after practicing counting with pictures.

There are also strong theoretical reasons to expect pictures to be better than objects for promoting understanding of cardinality in particular. Pictures have greater representational status than objects. Representational status refers to the ease with which something can represent something else. According to DeLoache et al.'s (1998) dual representation hypothesis, the more interesting an object is in its own right, the more difficult it is for children to think of it as a representation of something else. DeLoache (1991) has shown that pictures are inherently less interesting as objects in their own right than are other symbols and are more easily understood as representations of something else.

The higher representational status of pictures compared to objects comes with at least two benefits: first, it means that pictures are less distracting than objects. They are not very interesting as objects in their own right, so they do not prompt a lot of actions or play. DeLoache et al. (1998) have shown that pictures quickly become objects of "contemplation and communication" by children as young as 19 months old. Second, Gelman et al. (2005) have shown that children tend to think about objects depicted in pictures in terms of their group membership, rather than as individuals. The research of Mix et al. (2012) and Ramskar et al. (2011) indicates that children's understanding of cardinality benefits when their attention is drawn to the group, or set, as a whole instead of to the individual objects. This evidence suggests that pictures may be especially useful for facilitating understanding of cardinality. Thus, we predicted that children in the picture condition would

construct a better understanding of cardinality than children in the object condition.

In sum, the research literature pointed to somewhat opposing predictions. On one hand, it suggested that objects would be better than pictures for facilitating counting skill. On the other hand, it suggested that pictures would be better than objects for helping children construct an understanding of cardinality. We tested these ideas in a training experiment with three and four year olds.

## Method

### Participants

Participants were recruited from two childcare centers located on two college campuses in the Midwestern United States. Tuition at the childcare centers is based on a sliding scale, and 30% of children receive some form of reduced tuition. Fifty-seven children participated in this study. Five children did not complete all of the practice sessions. One child participated in all sessions but did not actually engage in any of the tasks. Of the remaining 51 children who completed the pretest and five practice sessions, an additional 12 did not have complete data for one or more of the pre-post measures because of refusal to participate, uncodable performance, or experimenter error. Thus, the final sample contained 39 children (25 girls, 14 boys;  $M$  age = 3 years, 6 months; 81% White, 12% Asian, 4% Black or African American, 4% Hispanic or Latino).

### Design

The design was a pretest-intervention-posttest design. Children were randomly assigned to one of two counting interventions: the picture intervention or the object intervention. They completed measures to assess their counting skill and understanding of cardinality immediately before and after receiving the intervention.

### Measures

**Count disks** This task was used to assess children's counting skill. It was a modified version of the task used in Mix et al. (2012). Children were shown 20 one-inch disks affixed on poster board in a line and spaced one-inch apart. The disks alternated in color to help children keep track of their count. The experimenter pointed to the leftmost disk and asked the children to count all the disks starting there. The largest number children reached without error was considered their highest count. The task ended once children indicated they were finished counting.

**Give-a-number** This task was used to assess children's understanding of cardinality. It is a commonly used measure (Le Corre et al., 2006; Sarnecka & Carey, 2008; Wynn, 1990). Children received a pile of 15 disks, and their goal was to give a monkey puppet a specified number of objects. After the child gave the puppet a number of objects, the puppet said, "Thanks," and the experimenter asked the

child: "Does Monkey have  $n$ ?" If the child agreed that the puppet had the correct number, then the next trial began. If the child disagreed that the puppet had the correct number, the experimenter prompted the child to give the correct amount by saying: "But Monkey wanted  $n$ . Can you make it so that he has  $n$ ?"

Children always were asked to give one object on the first trial. Subsequent trials were based on children's performance. If children gave the correct number of objects, they were asked to give the next consecutive number ( $n + 1$ ). If children gave the incorrect number of objects, they were asked to give the preceding number ( $n - 1$ ). Trials continued in this manner until children failed on a given number twice. If children succeeded on all numbers 1-6, then the experimenter started again with one object and repeated the sequence of trials as described. A child was classified as a "knower" of the highest number of objects (out of 6) he or she gave correctly twice.

### Experimental Conditions

**Picture Intervention** All children in the picture intervention used counting "books" during counting practice. These books were three-ring binders with pictures of one to nine objects on each page. These objects came from three categories: animals, vehicles, and food. The three types of animals were elephants, lions, and hippos. The three types of vehicles were boats, planes, and buses. The three types of foods were strawberries, carrots, and bananas. During practice-only sessions, children viewed one entire picture book that contained two different pages for each of the quantities one through nine (18 pages total). During practice-and-testing sessions, children viewed one entire picture book that contained one page for each of the quantities one through nine (9 pages total). Thus, counting practice during practice-only sessions was twice as long as the counting practice during practice-and-testing sessions.

The training procedure was based on the intervention that Mix et al. (2012) found was most effective for promoting children's understanding of cardinality. On each page of the picture books, the experimenter first labeled the set's quantity (e.g., "Look, this page has three cars. Can you say it with me? Three cars."). Next, the experimenter immediately counted the same set (e.g., "Let's count them 1, 2, 3!"). Children then were asked to count and label the items. They were told: "Now it's your turn. How many cars are there?" They were asked to point to each object as they labeled it with a number name. Children were given feedback on both their counting and labeling of the set. For example, if a child correctly labeled the set, but counted incorrectly, the experimenter demonstrated the correct count (e.g., "Right, there are three cars, but watch: 1, 2, 3."). If a child labeled the set without counting, the experimenter requested the counting procedure (e.g., "Right! Three cars. Can you count them?"). If a child counted the objects without labeling the set, the experimenter requested a label (e.g., "Right! 1, 2, 3. So, how many are there?"). If the child

failed to point to the objects, the experimenter reminded them to point to each object (e.g., “Right! 1, 2, 3. Can you point to them like this: 1 [point], 2 [point], 3 [point]?”).

**Object Intervention** The object intervention was identical to the picture intervention in all ways except the materials used during the counting practice sessions. Children in the object intervention used the physical, plastic versions of the objects depicted in the books in the picture intervention. Instead of counting two-dimensional pictures of objects in books, children in the object intervention counted three-dimensional plastic objects that were placed on a white work mat. Special care was taken to ensure that the objects were similar in size and layout to the picture counterparts. As in the picture intervention, the to-be-counted items were presented simultaneously. As the experimenter set up the objects in the same orientation and position as they were depicted in the picture books, she hid the objects with a piece of cardboard. Once the objects were placed, she moved the cardboard so the child could see all objects presented simultaneously.

## Procedure

Children were randomly assigned to one of two practice interventions (picture or object), and they participated in five sessions. The sessions were held one week apart in a quiet room. Children were seated in a chair and looked down at the to-be-counted entities in both conditions to ensure that none of the objects would be blocking one another or obscured from children’s view. In the first session, children completed the pretest followed by their first counting session with modeling and feedback provided by the experimenter. The next three sessions (sessions 2, 3, and 4) were purely counting practice sessions with modeling and feedback provided by the experimenter. In the fifth session, children first completed their final counting session with modeling and feedback provided by the experimenter and then completed the posttest. During the pretest and posttest, children completed the measures to assess their counting skill and understanding of cardinality (described above). Children did not receive feedback on any of the pretest or posttest measures.

## Results

To assess the effect of condition on pre-to-post change in counting skill, we conducted a mixed ANOVA with condition (picture or object) and age (3 or 4) as the between-subjects variables, test (pre or post) as the within-subjects variable, and highest number correctly counted on the count disk task (out of 20) as the outcome. The main effects of test and age were both significant. Children performed better on the posttest ( $M = 10.68$ ,  $SE = 0.95$ ) than on the pretest ( $M = 7.91$ ,  $SE = 0.89$ ),  $F(1, 35) = 5.25$ ,  $p = .03$ , partial eta squared = .13. Four year olds performed better ( $M = 10.86$ ,  $SE = 0.96$ ) than three year olds ( $M = 7.73$ ,  $SE = 1.00$ ),  $F(1, 35) = 5.10$ ,  $p = .03$ , partial eta squared = .13. None of the other main effects or interactions were statistically significant (all

$p$ ’s  $> .10$ ), including the two-way interaction between condition and test that we would expect to see if children’s counting skill improved more in one condition than the other,  $F(1, 35) = 0.74$ ,  $p = .74$ , partial eta squared  $< .01$ .

To assess the effect of condition on pre-to-post change in children’s understanding of cardinality, we conducted a mixed ANOVA with condition (picture or object) and age (3 or 4) as the between-subjects variables, test (pre or post) as the within-subjects variable, and knower level on the give-a-number task (out of 6) as the outcome. The main effects of test and age were both significant. Children performed better on the posttest ( $M = 3.95$ ,  $SE = 0.29$ ) than on the pretest ( $M = 3.42$ ,  $SE = 0.34$ ),  $F(1, 35) = 5.46$ ,  $p = .03$ , partial eta squared = .14. Four year olds performed better ( $M = 4.52$ ,  $SE = 0.41$ ) than three year olds ( $M = 2.86$ ,  $SE = 0.43$ ),  $F(1, 35) = 7.82$ ,  $p = .01$ , partial eta squared = .18. As hypothesized, the interaction between condition and test was significant,  $F(1, 35) = 5.07$ ,  $p = .03$ , partial eta squared = .13. Figure 1 presents the average change in knower-level from pre-to-post as a function of condition. As shown in the figure, children in the picture condition improved their knower-level from pretest ( $M = 3.12$ ,  $SE = 0.47$ ) to posttest ( $M = 4.14$ ,  $SE = 0.41$ ), but children in the object condition did not (pretest  $M = 3.74$ ,  $SE = 0.49$ ; posttest  $M = 3.76$ ,  $SE = 0.42$ ). None of the other main effects or interactions were statistically significant (all  $p$ ’s  $> .10$ ).

Conclusions were the same when we used ANCOVA to examine whether children in the picture condition had a better understanding of cardinality than children in the object condition did at posttest, adjusting for pretest knowledge,  $F(1, 35) = 4.26$ ,  $p = .047$ , partial eta squared = .11. Conclusions were also the same when we categorized children as cardinal principle knowers or not based on Negen, Sarnecka, and Lee’s (2012) approximation of Lee and Sarnecka’s (2010) model. The percentage of children who were classified as CP-knowers increased from pretest to posttest in the picture condition (from 33% to 57%), but not in the object condition (from 33% to 33%),  $\chi^2(1, N = 39) = 4.92$ ,  $p = .03$ .

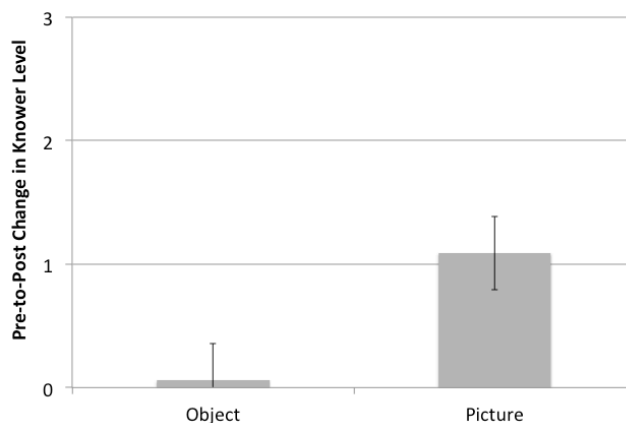


Figure 1: Pre-to-Post Change in Knower-Level on the Give-a-number Task by Condition.

## Discussion

We compared the effects of counting practice with picture books versus physical objects on children's counting skill and understanding of cardinality. Contrary to our hypothesis, we did not find evidence that the two conditions differentially affected children's counting skill. However, as hypothesized, we did find that the two conditions differentially affected children's understanding of cardinality. Children who practiced counting with picture books improved their understanding of cardinality, but children who practiced counting with objects did not.

Similar to Mix et al.'s (2012) study, children's performance on the give-a-number task increased by one knower-level after participating in a picture book intervention. This finding is impressive given that previous research suggests that children slowly increase their knower-level on this task with many months in between acquisition of new knower-levels (Le Corre et al., 2006; Wynn, 1990). Note that children in the object condition also practiced counting in the same way that Mix et al.'s study found was beneficial for understanding of cardinality, yet they showed no gains in understanding by posttest. This finding may suggest that simply linking counting and set size during counting practice with objects was not enough to improve children's understanding of cardinality. However, it is important to note that Mix et al. designed their intervention for use with picture books, and that may be the context in which it works best.

Despite receiving counting practice that was identical to the object condition in all ways but the specific items counted, only children in the picture condition improved their understanding of cardinality. We have suggested that the benefits of the picture books may be due to the higher representational status of pictures versus objects. But why should the representational status of the counted objects affect the construction of children's understanding of cardinality? Previous studies have suggested that one major benefit of learning math concepts with objects that have high representational status is that they are less distracting than objects with low representational status (DeLoache, 2000; Uttal et al., 1997). Preliminary analyses of the videos from the current study are providing support for this idea.

The videos are also suggesting that children in the object condition may have been generally more distracted than children in the picture condition. They may have been likely to exhibit behaviors that disrupted the counting session like talking about things that were unrelated to the counting task. They may also have been more likely than children in the picture condition to carry out task-irrelevant behaviors on the objects themselves (e.g., moving the toy animals on the table, pretending to eat the toy strawberries, driving the toy bus on the table). A consequence of disruptions such as these is that it may make it more difficult for the instructor to present a given trial in the highly structured way (i.e., labeling set and then immediately counting in close temporal continuity with no interruptions) that is thought to benefit children's understanding of cardinality. It also may

have caused children to focus their attention on the objects at the expense of noticing cardinality as an important attribute of the sets. Indeed, children's focus on playing with the objects may be one reason why we did not find the predicted difference between conditions in counting skill. If children were focused on moving the objects in play-relevant ways during the counting practice, then they would not have been focused on moving the objects in the counting-relevant ways that might help them segment the set and keep track of their count.

Because objects with higher representational status, like pictures, have been shown to decrease children's focus on the individual objects as objects and increase children's focus on the objects as members of a group (Gelman et al., 2005), we hypothesized that pictures would also be more likely than objects to focus children's attention on the set during the counting practice sessions. We reasoned that this attention to the set would help children improve their understanding of cardinality, thus serving as another potential benefit of using pictures for counting. A more detailed analysis of the children's speech and their use of set-focused language (e.g., plural form) may provide evidence to support this idea.

Results suggest that educators' money may be better spent on counting books than on physical counters. Counting books have the advantages over physical counters of teaching children the concept of cardinality while also improving children's counting skill as much as physical counters. It is important to note, however, that the picture books that were used in the current study were created for this study and not simply selected from among popular existing counting books. Existing counting books differ in many respects that may make them more or less effective as materials for counting practice. The picture books used in the current study contained pages that featured only the set with no other pictorial details, words, or Arabic numerals. Thus, the effectiveness of these picture books may not generalize to all counting books. Future studies should analyze counting books for aspects that may differentially benefit children's understanding of cardinality.

Although we had to make our own picture books for the picture condition, we were able to use objects for the object condition that are widely available and sold as counters in teaching supply stores. This means that our objects were similar to objects used for counting in the real world. At the same time, however, it also means that the objects were brightly colored and toy-like. We have shown in a previous study that objects that are toy-like hinder children's counting performance (Petersen & McNeil, 2012). Thus, it is possible that children would have benefitted more from practice with blander, less toy-like physical objects.

The benefits of counting book practice from this study were impressive given that children only received five short practice sessions. It is possible that gains in understanding of cardinality would be even more impressive over continued practice with counting books. A question for future research is whether differences in actual counting

practice in the home and preschool environments might account for differences in children's understanding of cardinality.

Overall, the current study contributes to our understanding of the malleable factors in the early learning environment that affect children's understanding of a foundational mathematical concept, cardinality. More generally, the current study adds to the growing evidence that seemingly small variations in the materials children use in learning situations affects the knowledge they construct. These results can also provide information to educators who must decide which materials they should bring into their classrooms to best teach their students the concept of cardinality.

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