Dual process in large number estimation under uncertainty

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Abstract

According to the dual process theory, there are two systems in the mind: an intuitive and automatic System 1 and a logical and effortful System 2. This study focused on the System 2 process for large number estimation. First, we constructed a process model of estimation. The model, corresponding to the problem-solving process, consisted of creating subgoals (System 2), retrieving values (System 1), and applying operations (System 2). Additionally, a knowledge network was used for the estimation process. Second, the results of an experiment based on our model showed that the deliberative System 2 process did not improve the value estimated by the intuitive System 1 process.

Keywords: Dual process theory, large number estimation, reasoning, problem solving, knowledge network.

Introduction

How many piano tuners are there in the world?

This is a well-known problem from a Google entrance examination. The question is a type of a Fermi problem that requires the estimation of a quantity that is difficult to measure directly. The estimation needs to be conducted on the basis of uncertain and limited information. In this study, we call this type of estimation "estimation under uncertainty." Many previous studies have investigated intuitive aspects and heuristics of this estimation. However, when tackling the Google problem, one tries to reach the correct answer systematically. In this study, we investigated a logical and deliberative process and its capability to estimate under certainty.

Evans (2003) and Kahneman (2011) argued that there are two systems in the mind: System 1 and System 2. System 1–also called the heuristic process–operates automatically and quickly, with little or no effort, and no sense of voluntary control. However its judgments and estimations are intuitive and biased. It selects and retrieves relevant information automatically. System 2 conducts a conscious and deliberate process in which a person approaches a goal step by step. Although the operations of System 2 are effortful and slow, they are rational and logical. In this study, we focus on the role of System 2 in estimation under uncertainty.

Tversky and Kahneman (1974)'s review of heuristics and biases in judgment under uncertainty is one of the most famous previous studies. They introduced three heuristics, cited in many studies of estimation: representativeness, availability, and anchoring (originally adjustment and anchoring). We estimate likelihood, frequency, or quantity based on representativeness of an instance or availability of information. For example, Brown and Siegler (1992) showed that, when estimating the population of 99 countries, the more knowledge that participants had about a country, the larger they estimated its population to be. System 1 uses such information unconsciously.

The anchoring effect implies a tendency to rely too heavily on prior information (the anchor). Kahneman (2011) discussed that two different mechanisms produced this effect, one for each system. First is selective accessibility. When System 1 assesses an anchor value, the accessibility of anchor-consistent information is selectively increased, which biases the judgment. Strack and Mussweiler (1997) showed that even an implausible anchor value produced an anchoring effect.

Second, Tversky and Kahneman (1974) originally suggested that a process of adjustment by System 2 produced the anchoring effect. Participants start estimating from an anchor value, assess whether it is too high or too low, and adjust it. An insufficient adjustment results in an estimated value biased toward the initial value. Epley and Gilovich (2001) demonstrated that the type of anchor value, whether self-generated or provided, affected which mechanism was dominant. Another factor, familiarity with a variable to be estimated, also affects the estimated value; estimation of a familiar variable is easy and accurate (Block & Harper, 1991). As stated above, while many previous studies have focused on simple heuristics and automatic processes, the deliberative System 2 process has not been sufficiently studied.

To investigate the process of estimation under uncertainty, we assume that it represents a kind of problem solving. We focus on means-ends analysis, a problem-solving strategy (Newell & Simon, 1972). Given a current state and a goal state, an operation that will reduce the difference between the two states is applied to the current state. When a goal is not immediately attainable, we break the problem down into smaller problems by creating a subgoal. A similar process is expected in estimation under uncertainty. A goal state is one in which the value of a target variable is known. Subgoals would be created because it is difficult to reach the goal state directly. Some operations for estimating values would be observed.

The first purpose of this study was to construct a process model of estimation under certainty, including the deliberative process of System 2. The second purpose was to investigate whether a value estimated by the System 1 process is improved by the System 2 process. This investigation is important because a value estimated by the System 1 process is susceptible to bias.

Experiment 1

The purpose of Experiment 1 was to construct a process model of estimation. We observed an estimation process with no manipulation.

Methods

Participants Twenty undergraduates in Nagoya University participated in Experiment 1.

Problems We prepared five problems that required estimation of a quantity that was either difficult or impossible to measure directly. The following variables were estimated (with answers in parentheses): Student problem, undergraduate students in Japan (2,570,000); Passenger problem, passengers using Narita Airport per day (74,011); Dog problem, dogs kept in Japan (11,861,000); Doctor problem, doctors in Japan (295,049); Cell problem, cells in the human body (10 trillion).

Procedure Before starting on a problem, the participants were asked to state the value of the target variable in ten seconds without deliberation (pre-estimated value). They then estimated freely using a paper and pen until they reached a satisfied value. They were instructed to think aloud while estimating, and the whole process was videotaped. The value estimated at the end of the process (post-estimated value) was used for analysis.

Results

Accuracy For comparison of accuracy between the pre- and post-estimated values, absolute Order of Magnitude Error (OME; Brown, 2002) was computed as follows.

$$\mathrm{OME} = |\log_{10}(\mathrm{EstimatedValue}/\mathrm{ActualValue})| \quad (1)$$

The smaller the OME score, the closer was the estimated value to the correct value. One participant who estimated the number of passengers per year was excluded from the Passenger problem analysis. We calculated OME scores for the pre- and post-estimated values for each problem. Three participants whose OME scores deviated more than 3 standard deviation from the average were excluded: one from the Student problem and two from the Passenger problem. Figure 1 shows the average OME scores of the pre- and post-estimated values for each problem. The average OME score was marginally or significantly lower for the post-estimated value than that for the pre-estimated value for all problems (Students $t(18) = 4.314, p < .001$; Passengers $t(16) = 3.176, p = .006$; Dogs $t(19) = 3.760, p = .001$; Doctors $t(19) = 2.022, p = .057$; Cells $t(19) = 2.090, p = .050$). The estimated value was improved through the estimation process.

Protocol Analysis We used the records of the Student problem to analyze the estimation process. The participant excluded from the analysis of accuracy was included. Utterances were categorized as "retrieval" or "operation" according to the following criteria.

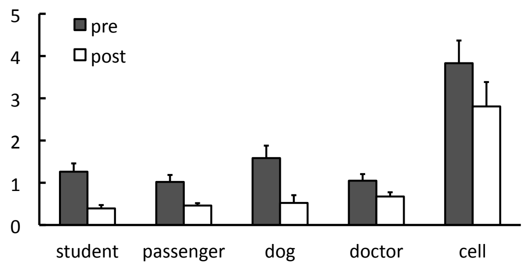

Figure 1: OME scores of pre- and post-estimated value for each problem.

Utterances in which the participants retrieved the value of a certain variable were categorized as "retrieval." The following example shows the "retrieval" of Japanese population.

> Participant 9: How many million in Japan now? 100 million, 100 and ten million? 120 million? OK, assuming that it's 120 million ··· .

"Retrieval" is one of the automatic operations of System 1. We did not subdivide this process further because we focused on System 2.

The utterances in which the participants acquired a new value from the existing values belong to one of the following two "operations": "calculation" or "adjustment." First, the cases in which a new value combining the existing values was calculated were categorized as "calculation." For example, participant 12 multiplied the number of undergraduate students per grade in a university by the duration of university (four years) to calculate the number of undergraduate students in a university.

> Participant 12: Assuming that there are 2,000 students in a grade. ··· there are from first to fourth grade. 2,000 times four is 8,000 students.

The participants also used "calculation" to verify the value. In the following example, the participant "calculated" the ratio of undergraduates to all Japanese in order to investigate the validity of the calculated value.

> Participant 19: (Her calculation indicated that the number of undergraduate students in Japan is 2,200,000.) The population in Japan is around one hundred million. So about 10%, 10%? Less than 10%, ··· it may be OK.

Second, we categorized the utterances in which the participants assessed whether a retrieved or calculated value was too high or too low, and then adjusted it, as "adjustment." Participant 15 estimated the number of undergraduate students per grade in a university from that in Nagoya University and then "adjusted" it to 2,000 because Nagoya University is one of the biggest universities in Japan.

> Participant 15: In Nagoya University ··· about 3,000 per grade. But it's a large one. So it's about 2,000 in the average university.

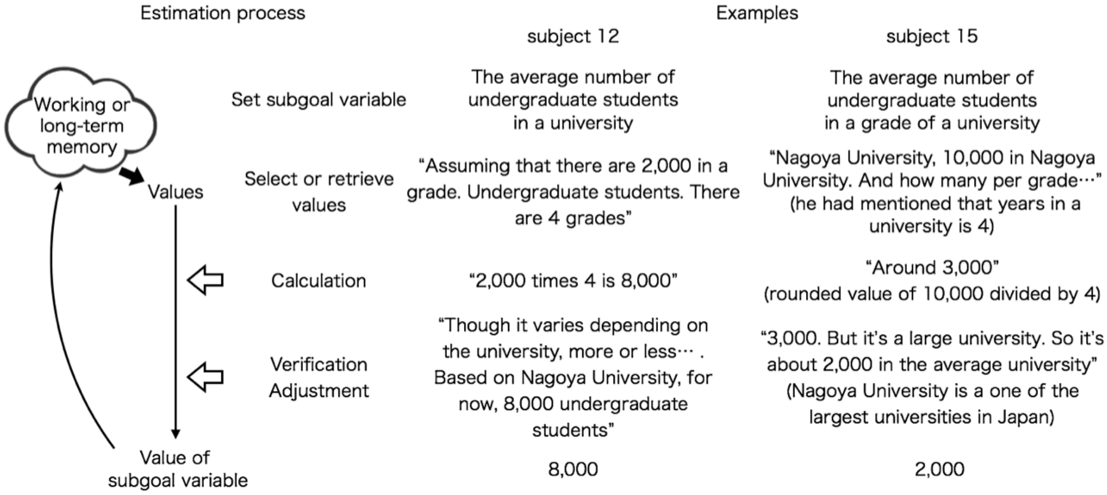

Figure 2: The process observed in Experiment 1 and the examples.

Table 1 shows the number of "retrievals" and "operations" ("calculation" and "adjustment") that appeared in the estimation process. All participants set more than one subgoal and used "calculation" to estimate the target value. Participants 4, 7, and 8 relied on "retrieval" or "adjustment" rather than on "calculation." Other participants utilized "retrieval," "calculation," and "adjustment" systematically. The following process was observed in common. Participants first set a subgoal to be estimated. Next, they "retrieved" or selected the relevant values and used them for the "calculation." The calculated value was verified and "adjusted" if needed. This process was followed recursively until the value of the target variable was reached. Figure 2 shows the process observed in Experiment 1 and the examples.

Table 1: Number of retrievals and operations appearing in the estimation process.

	Operation		Retrieval
	Calculation	Adjustment	
sub 1	4	0	13
sub 2	5	0	9
sub 3	5	0	7
sub 4	1	3	3
sub 5	8	0	8
sub 6	8	5	17
sub 7	2	1	2
sub 8	1	1	5
sub 9	6	0	5
sub 10	7	1	10
sub 11	5	1	6
sub 12	3	1	7
sub 13	8	0	19
sub 14	7	1	8
sub 15	6	2	6
sub 16	4	1	16
sub 17	9	0	8
sub 18	5	1	11
sub 19	9	3	10
sub 20	6	0	10

Discussion

The results of Experiment 1 showed that the estimated value was improved through deliberative estimation. We also constructed a process model of estimation under uncertainty, which included creating subgoals, "retrieval," "calculation," verifying values, and "adjustment."

We could construct a network structure using the variables utilized as subgoals. Figure 3 shows the network of the variables observed in the Student problem. Each node represents a variable, and nodes that are related in the "calculation" are linked. The participants utilized a part of the network for estimation. They created subgoals by tracing links from the target variable to further variables, and then traced back to the target variable by using "calculation."

Note that, unlike a calculation problem, the participants always verified the estimated values. They would also utilize the network structure for the verification. They judged validity on the basis of a consistency of values in the network, as well as a direct comparison with their own knowledge. For example, in Figure 3, groups A and B both included the variable "the number of undergraduate students per grade." If the value calculated in group A was inconsistent with those in group B, an "adjustment" was applied to the value to maintain consistency of values in the network.

Experiment 2

The purpose of Experiment 2 was to investigate whether a value estimated by the System 1 process was improved by the System 2 process. First, we observed a transition of the estimated values. Second, we investigated whether the transition pattern was changed by manipulating factors related to the System 2 process, on the basis of the model in Experiment 1.

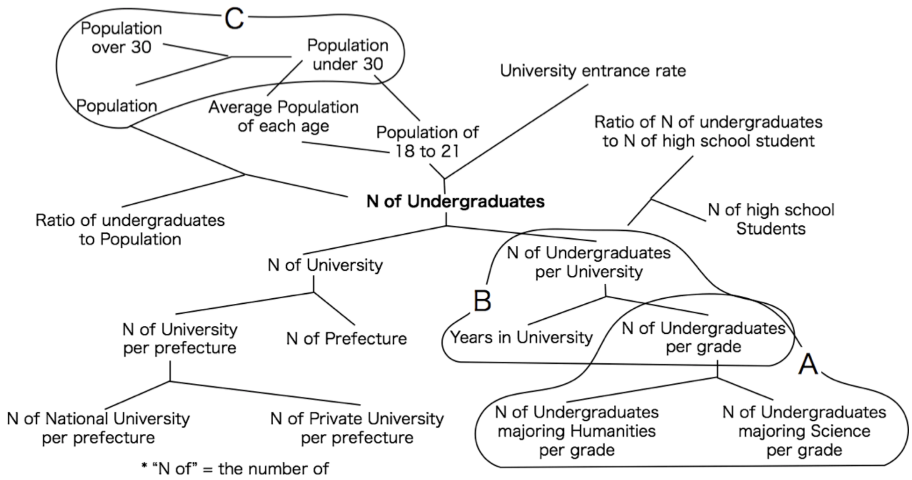

Figure 3: Network of the variables observed in the Student problem in Experiment 1.

Task

We developed an interface to control the process of estimation under uncertainty. We used the Student, Doctor, and Passenger problems. We first constructed a variable network for each problem such as that shown in Figure 3. The participants in Experiment 1 estimated values from further (peripheral) variables to the target variable. In Experiment 2, the participants stated the values of variables selected by the experimenters on the basis of each network.

Figure 4 shows an example screenshot of the task display in a trial used in Experiment 2 (Global-Relevant condition). The participants estimated and entered the three values indicated at the center of the display. Each problem consisted of three blocks and each block consisted of three to five trials. To observe the transition of an estimation value, we asked for the estimated value of the target variable after each block. Additionally, on the basis of the model developed in Experiment 1, we manipulated the three factors that affected accessibility of the network.

The first factor was the "three values factor"; whether the three variables to be estimated were related to one another. In the related condition, they were linked to one another in the network. For example, each triplet in each group A, B, and C in Figure 3 was related. In the unrelated condition, three values were not directly linked. In the related condition, the participants could easily apply "calculation," which ensured consistency among the values of the three local variables.

The second factor was the "trial sequence factor"; whether successive trials were connected directly in the network. In the connected condition, successive triplets had one common variable. In addition, the triplets were presented from the further (peripheral) variable to the target variable. For example, group A in Figure 3 was presented followed by group B. In the unconnected condition, successive triplets were selected randomly, regardless of their distance from the target variable in the network, and had no common variable. For example, group B in Figure 3 was presented followed by group C. In the connected condition, the participants could apply "calculation" in the same order as that in the process model in Experiment 1. Additionally, it was easy to maintain consistency within the whole network because they could use one of the values in the triplet for "calculation" in the next triplet.

The third factor was the "history factor"; whether the history window was displayed. In the displayed condition, the history window was displayed, as in the example shown in Figure 4, whereas in the non-displayed condition it was not. The history window, which supported the participants to maintain the calculated values, decreased the cognitive load.

Table 2 shows the levels of each factor in the following four conditions: Global-Relevant, Local-Relevant, Irrelevant, and No-History. In the Global-Relevant condition, the estimation process was almost the same as that observed in Experiment 1, except that the experimenters had decided the subgoals in advance. The difference between the process model and an estimation process increased in the Local-Relevant condition and more in the Irrelevant condition. In the No-History condition, the participants only retrieved a value of each variable randomly. In summary, the situation was suitable for the operations of System 2 in the order Global-Relevant, Local-Relevant, Irrelevant, No-History. We investigated the influence of the System 2 process by comparing the estimated val-

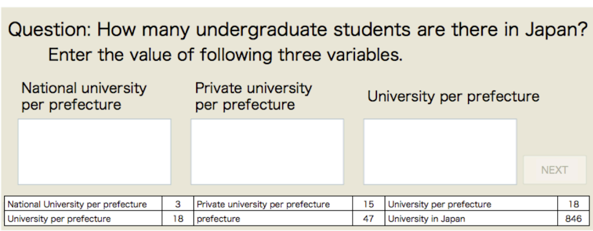

Figure 4: Example screenshot of the task display in a trial used in Experiment 2 (Global-Relevant condition).

All sentences and words were presented in Japanese. A problem statement was presented at the top of the display. Participants entered three values of the presented variables in boxes at the center of the display. The table at the bottom of the display shows the history.

Table 2: Levels of each factor in the four conditions.

	Trial sequence	Three values	History
Global-Relevant	Connected	Related	Displayed
Local-Relevant	Unconnected	Related	Displayed
Irrelevant	Unconnected	Unrelated	Displayed
No-History	Unconnected	Unrelated	Non-displayed

ues in each condition.

Before starting on each problem, the participants estimated the value of the target variable in 15 s without deliberation (pre-estimated value). During estimation using our interface, they entered the estimated value of the target variable after each of blocks 1, 2, and 3. To investigate the transition of an estimated value of a variable other than the target variable, they entered an estimated value of the intermediate variable at the same time.

Predictions

In the initial stage of estimation, there was no value for "calculation" or "adjustment." The participants needed to "retrieve" values for the operations. This meant that System 1, which "retrieved" relevant information automatically, governed the early estimation process. However, in the later stage, System 2 mainly governed the process since "calculation" and "adjustment" could be conducted on the already calculated or retrieved values.

For the above reasons, the difference between the pre-estimated and estimated values for block 1 reflected the result of the System 1 process. The variables presented in our task were subgoals, which were more familiar than the target variable. Considering that a value of a familiar variable could be retrieved with little error, the estimated value for block 1, which was estimated on the basis of such familiar values, would be more accurate than the pre-estimated value.

In the latter stage, there were two predictions, according to whether an estimated value is improved by the System 2 process. If System 2 improves the estimated value, the estimated value would be more accurate in block 2 than in block 1 and in block 3 than in block 2, owing to the accumulated "calculations" and "adjustments." By contrast, if System 2 does not improve the estimated value, the estimated value in block 1 would not change in blocks 2 and 3.

In addition, there were two predictions about the estimated values in each condition. If System 2 improves the estimated value, the largest improvement would be observed in the estimated values in the Global-Relevant condition, which is the most suitable for the System 2 process. In other conditions, improvement would decrease in the order Local-Relevant, Irrelevant, No-History. By contrast, if System 2 does not improve the estimated value, there would be no difference in the estimated values among the four conditions.

Method

Participants Eighty-four undergraduates participated in Experiment 2. Each participant was assigned to one of the four conditions.

Procedure Experiment 2 was conducted in small groups of a maximum of six participants. The pre-estimated value and estimated values for each block were collected as we mentioned above.

Results

Three participants (one each in the Local-Relevant, Irrelevant, and No-History conditions) who were not able to complete all three problems were excluded from the analysis.

We used an average OME score across the three problems. Figure 5 shows the average OME scores in each condition. We conducted a 4 (accessibility: Global-Relevant, Local-relevant, Irrelevant, No-History) × 4 (answer: pre-estimated, estimated value in blocks 1, 2, 3) ANOVA on the OME scores of the target variable. The main effect of the answer factor was significant ($F(3,231) = 31.645, p < .001$): all estimated values for blocks 1, 2, and 3 were significantly closer to the correct answer than the pre-estimated value ($ps < .001$). There was no significant difference among the three estimated values. The main effect of the accessibility factor ($F(3,77) = .959, p = .416$) and the interaction of two factors ($F(9,231) = .990, p = .430$) did not reach significance. For the intermediate variable, the main effect of the answer factor was also significant ($F(3,231) = 9.221, p < .001$): all estimated values for blocks 1, 2, and 3 were significantly closer to the correct answer than the pre-estimated value

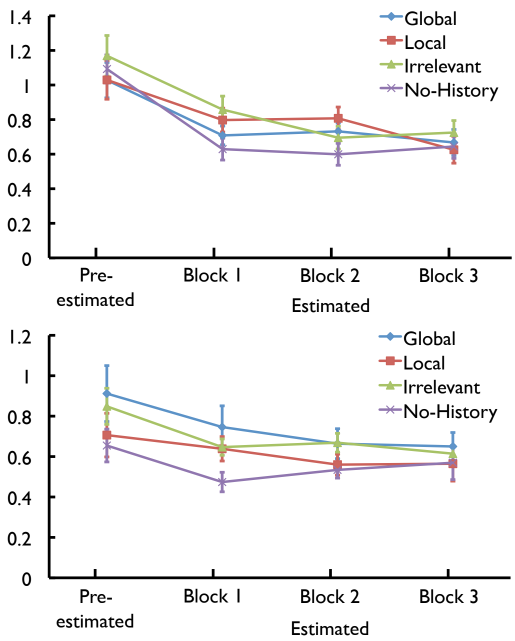

Figure 5: OME scores in each condition.
The upper graph shows transitions of OME scores for the target variable and the lower graph shows transitions of OME scores for the intermediate variable.

($ps < .01$). There was no significant difference among the three estimated values. The main effect of the accessibility factor ($F(3,77) = 1.785, p = .157$) and the interaction of the two factors ($F(9,231) = .714, p = .644$) did not reach significance.

Discussion

The results of Experiment 2 suggest that the System 2 process does not improve the value estimated by the System 1 process. There was no significant difference among the estimated values in blocks 1, 2, and 3, nor among the four conditions. To consider the possibility that the difference between the pre-estimated and estimated values in block 1 was too large to detect the difference between the estimated values in blocks 1, 2, and 3, we excluded the pre-estimated value from the analysis. The data in the No-History condition, which was extremely unsuitable for the operations of System 2, were also excluded. We conducted a 3 (condition: Global-Relevant, Local-relevant, Irrelevant) $\times$ 3 (block: estimated value in blocks 1, 2, 3) ANOVA on the OME scores of the target variable. The main effect of the block factor was significant ($F(2,116) = 5.607, p = .007$): the estimated value for block 3 was significantly closer to the correct answer than that for block 1 ($p = .025$). The main effect of the accessibility factor did not reach significance ($F(2,58) = .261, p = .772$), and the interaction of the two factors was marginally significant ($F(4,116) = 2.426, p = .061$). An improvement by the System 2 process was observed, although it was not as large as that produced by the System 1 process. These results suggest that System 1 estimated a value roughly and then System 2 made fine adjustments to the value.

General discussion

This study had two aims. First, to construct a process model of estimation under certainty, and second, to investigate whether a value estimated by the System 1 process is improved by the System 2 process.

In Experiment 1, we observed the process of logical deliberation and constructed a process model, which consisted of creating subgoals and applying operations and corresponded to the problem-solving process proposed by Newell and Simon (1972). Additionally, the process used the knowledge network. As Kahneman (2011) pointed out, System 2 works on data that are retrieved in an operation of System 1. During the estimation process, especially in the initial stage, we observed many "retrievals" of a value, about which the participants frequently said "for now" or "assuming that $\cdots$." They used retrieved values as hypothetical values to be adjusted through the estimation process. These results suggest that the process of estimation under uncertainty does not just repeat calculations of retrieved values, but decides the most plausible value of the target variable using the network. Specifically, the participants always verified consistency between variables, and, wherever necessary, adjusted the values.

The results of Experiment 2 indicated that the System 2 process does not improve the value estimated by the System 1 process. Considering that the value produced by System 1 was rather good, the participants would consider the value as sufficiently valid. In other words, they did not find any need for adjustments through the verification of consistency in the network. The additional analysis suggested that System 2 produced fine adjustments. There have been some previous studies of the relationship between System 1 and System 2 where there was conflict between the two systems (Evans, 2007). This study showed the interdependent relationship of two systems: System 2 works on data retrieved by System1 and the retrieved data are adjusted by System 2.

The manipulations based on the process model had no effect on the estimated values. This result also supports the conclusion that the System 2 process does not improve the estimated value. Another possible reason is that even the Global-Relevant condition is insufficient for executing the enough System 2 process. The participants created subgoals and approached the target variable along paths that they selected *by their own will*. However, in Experiment 2, the experimenters decided all subgoals and the presentation order. This would impair the effect of a deliberative System 2 process. In future work, we need to develop an appropriate interface to support the deliberative process to improve an intuitively estimated value.

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