Early Understanding of Intensive Properties of Matter: Developmental and Cultural Differences

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Abstract

Many properties of substances/materials are intensive, and children are widely believed to have difficulties with reasoning about intensive quantities. Here we used a novel judgment method, together with a cross-cultural paradigm to study 4- to 9-year-olds' understanding of the intensity/ concentration (sweetness) of sugar-water solutions. UK children knew from the youngest age that intensity increases with amount of solid, and a significant, but small effect for decreasing amount of liquid appeared by age 7, a couple of years prior to the age typically reported. Hong Kong children were more advanced, with strong liquid effects and the normative concentration pattern from the youngest age. Problems with intensive quantity reasoning may not reflect a universal cognitive limitation, but seem to depend on cultural experience. This has implications for children's chemistry reasoning and education.

Keywords: intensive properties, chemistry, reasoning, cognitive development.

Introduction

We have learnt much about young children's understanding of objects, but far less is known about how they understand the substances/materials of which these objects are composed. Matter has extensive and intensive properties, those that depend on amount (e.g. weight, height, volume) and those that hold independent of amount, for every bit of homogenous matter (e.g., density, strength, temperature, colour, taste). Many properties we consider basic characteristics of substances/materials are of the latter, intensive kind. Here we consider children's understanding of the taste intensity of mixed substances (sweetness of solutions of sugar in water). Prior work concludes that children have little understanding of intensive properties, including sweetness, until the late primary years. This study re-opens the case, applying an Information Integration approach, previously successful in eliciting understanding in other domains deemed difficult for children, and using a cross-cultural paradigm, studying children in the UK and Hong Kong.

Prior work on intensive properties: Which cup is sweeter? Much work documents that children have great difficulty with intensive quantities. Some attribute this to conceptual problems, such as insufficient differentiation of extensive and intensive concepts (Jäger & Wilkening, 2001; Piaget & Inhelder, 1974; Smith, Carey, & Wiser, 1985) or logical incongruence of intensive concepts (Kloos, 2007). Another set of accounts focuses on measurement of intensive quantities as ratio of two extensive quantities, e.g., sweetness depends on the ratio of sugar to water, or density depends on the ratio of mass to volume. In this view, children's problems reflect difficulties with proportional thinking (Nunes, Desli, & Bell, 2003; Stavy, Strauss, Orpaz, & Carmi, 1982). Most studies used choice tasks, e.g., children chose which of two cups of water was sweeter, one half full or one full, both with the same quantity of sugar added. Children up to 9 or 10 years either said that both were equally sweet, containing the same amount of sugar, or that the full glass was sweeter, mistaking the indirect, diluting effect of the water for a direct effect (Stavy et al., 1982). Nunes et al. (2003) had similar findings for a range of intensive properties, but also reported that problems with intensive quantities go beyond those with indirect variables: Children were worse on intensive than extensive problems even when both involved indirect variables. The reasons remain unclear, but it is clear that in choice paradigms children have problems until fairly late.

Information Integration Approach: How sweet is this cup? In other domains, choices turn out to be insensitive measures of children's competence. When a densely spaced row of pennies is pushed further apart, children say that the longer row has more – centration and lack of number conservation (Piaget, 1965). But when making graded judgments of numerosity on graphic, not numerical scales, as used within Information Integration Theory (Anderson, 1981, 1996), children as young as 3 take both length and density of the row into account (Cuneo, 1982). When asked to choose which of two plates with red winner and clear loser marbles is better for winning in a blind draw, many 8-year-olds still choose based on the number of winners only (Siegler, Strauss, & Levin, 1981). But when making judgments of how easy it is to win in a blind draw, even 4-year-olds consider both winners and losers (Anderson & Schlottmann, 1991). This may come about because choice tasks tend to elicit analytic, system II forms of reasoning, with dimension-by-dimension comparison (Siegler et al., 1981), while graded judgment tasks tap into more synthetic, intuitive system I forms of thinking (Schlottmann & Wilkening, 2011). This intuitive understanding can be dramatically more advanced than children's analytic understanding.

Here, we use an Information Integration approach to study children's concepts of taste intensity, sweetness, as a function of both amounts of solid/sugar and liquid/water used to create a solution, in order to see whether this approach can assess intuitions of intensive quantities that prior, more analytic methods may have missed.

Cultural Differences in Mathematics/Science: We studied children's understanding of solution intensity in the UK and Hong Kong. It is well documented that East Asian students typically outperform Western students in maths and science (Geary, Bow-Thomas, Liu, & Siegler, 1996; IEA, 2013; Leung, 2002; Stevenson et al., 1990). There is also a growing body of research on young children: Chinese pre- and primary schoolers count or add numbers or solve novel mathematical problems at a level 1 to 2 years above comparable American children (Geary et al., 1996; Siegler & Mu, 2008). The most recent Trends in International Mathematics and Science Study (IEA, 2013) finds advantages from 4th grade, e.g., Hong Kong students outperformed their English peers in maths/science. This Asian advantage persists up to university level (Bao et al., 2009; Huang & Waxman, 1995; IEA, 2013). It is attributed to many factors, including instruction differences, greater investment of Asian parents in their children's education and parental expectation of academic success (Kao, 1995; Schneider & Lee, 1990), the dominance of pre-entrance exams in Asian schools (Bao et al., 2009), different attitudes towards extra-curricular activities, and more time spent on homework (Leung, 2002; Schneider & Lee, 1990). We study Hong Kong and UK children here to see whether intensity understanding is also culturally variable. If Hong Kong children are more advanced than UK children, difficulties with intensity understanding, typically attributed to inherent cognitive factors, also depend on cultural experience.

Method

Participants

A total of 203 children participated. In Hong Kong, 116 children were recruited from church groups, including 36 4- and 5-year-olds (*M* = 5 years 1 month, *Range* = 4 years to 5 years 11 months, 30 girls), 44 6- and 7-year-olds (*M* = 7 years, *Range* = 6 years to 6 years 11 months, 30 girls), and 36 8- and 9-year-olds (M = 8 years 10 months, *Range* = 8 years to 9 years 11 months, 22 girls). In the UK, 87 children were recruited at London state schools including 28 4- and 5-year-olds (*M* = 5 years, *Range* = 4 years 6 months to 5 years 5 months, 15 girls), 32 6- and 7-year-olds (*M* = 7 years 1 month, *Range* = 6 years 1 month to 7 years 7 months, 16 girls), and 27 8- and 9-year-olds (*M* = 8 years 10 months, *Range* = 8 years 5 months to 9 years 8 months, 15 girls).

Design

Each child judged sweetness of all mixtures created from small, medium and large sugar volumes, factorially combined with small, medium and large water volumes. UK and 48 Hong Kong children judged two replications, 18 trials in total, to improve analysis at the individual level. Initially there were two separate smaller Hong Kong datasets using similar procedures and materials, however, children in the first set judged only one replication. The datasets showed no differences in preliminary analyses, so were combined. Overall, the study thus had a 2 (culture) x 3 (age) x 3 (solid volume) x 3 (liquid volume) mixed model design.

Materials

Children used a graphic sweetness scale with 17 wooden dowels increasing in height from 2.5 to 18.5cm, with each stick 1cm taller than the previous one. Similar scales have been used successfully with young children before (e.g., Schlottmann & Wilkening, 2011) Children pointed to a stick to indicate how sweet each mixture would be. Small jars, containing only water or only sugar sat by the ends of the scale, to remind children of scale direction

Experimental trials involved 3 water flasks holding 25ml to 1l of water, and 3 vials, holding .8g to 20g of sugar, with different containers/amounts used for instruction.

Figure 1: Measures of solid (sugar, left) and liquid (water, right), and the stick scale

Procedure

Children were tested individually in English/Chinese at their school/church by experimenters with English/ Chinese as first language. Initially, children were shown how sugar dissolves in water, then asked, "what happened?" and "what would happen if we added more sugar/water to this?" to probe baseline understanding. Children were then introduced to the scale, with long sticks for sweeter and short sticks for less sweet mixtures, and asked to place jars with only sugar/only water by the scale to check understanding. During instruction, children made four mixtures (fewer in some Hong Kong children), combining small and large amounts of sugar and water. Only one ingredient, water or sugar, changed from one trial to the next, so that sweetness increased from the previous trial when sugar was added, while it decreased when water was added. Prior to the sugar dissolving completely, these mixtures first turned cloudy, depending on the sugar/water ratio, which provided another, temporary, visual cue to intensity. On each trial, children were asked, "Now, how sweet is this mixture? Which stick do you think it belongs with?", and corrections were made if changes in children's ratings from one trial to the next were not in the expected direction. To prevent memorizing of responses, different amounts were used than in experimental trials.

Four practice trials followed in which children saw the separate ingredients, but did not mix them, rating how sweet the mixture would be, as in experimental trials. Feedback was only given if children reversed the scale or only used the extreme sticks, but typically not needed. The 18 experimental trials, in individually randomized order, followed immediately, with no feedback at all.

Results

Group Analysis

Figure 2 shows the mean judgments of sweetness (solution intensity) at 3 ages (rows) and in 2 cultures (columns). Each panel shows the mean judgments as a function of amount of solid (horizontal) and amount of liquid (curve factor). Overall, children gave meaningful judgments from the youngest age, but age and culture differences are also evident in Figure 2.

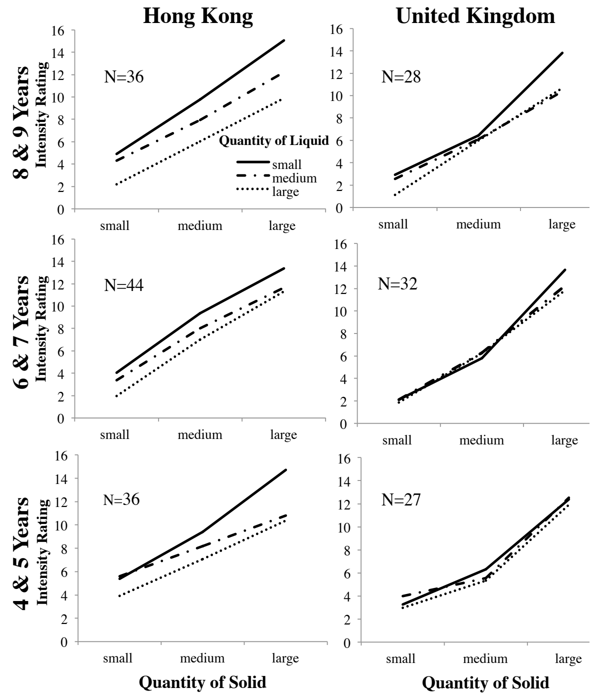

Figure 2: Mean judgments of sweetness/intensity as a function of liquid and solid

The oldest Hong Kong children (top left) show a pattern as normatively expected. Intensity ratings increase with amount of solid (curve slope), and decrease with amount of liquid (curve separation). Curves diverge towards the right, as predicted by the normative proportion model, $\text{concentration}_{solid} = \text{volume}_{solid}/(\text{volume}_{solid} + \text{volume}_{liquid})$. These judgments reflect appropriately how water dilutes concentration of the mixture.

The other panels all have positive slope as well, indicating that children of both cultures and all ages understood that sweetness increases with amount of solid, $F(1.68, 329.95) = 784.58$ in the overall ANOVA. (Unless specifically noted, all F values are reported at $p<.05$, here and in subsequent analyses, with Greenhouse-Geisser corrections if needed). The solid effect differed by age, $F(3.35, 329.95) = 4.30$, and culture, $F(1.68, 329.95) = 11.27$, being slightly smaller for the youngest age and for Hong Kong children. The liquid effect, $F(1.49, 293.42) = 48.03$, differed between cultures and ages as well, $F(1.49, 293.42) = 11.27$ and $F(2.97, 293.42) = 2.94$, respectively: Hong Kong children show more curve separation than UK children (left versus right panels), and older show more curve separation than younger children (top to bottom). The divergent curve pattern led to an overall liquid x solid interaction, $F(3.66, 720) = 10.04$, with bilinear trend, $F(1, 197) = 12.52$, a more stringent test of the fanning. This divergence was more pronounced for older and Hong Kong children, leading to a liquid x solid x culture x age interaction, $F(7.32, 720) = 2.50$. Finally, the main effect of culture, $F(1, 197) = 23.98$, of little interest, showed slightly higher ratings overall for Hong Kong children.

The Hong Kong sample had far fewer boys than girls, so the culture differences above could be sex differences in disguise. However, if sex is included in the analysis, all effects remain, and the only additional effect is the sex x culture x solid interaction, $F(1.71, 326.02) = 7.02$, due to a slightly smaller solid effect in Hong Kong boys than UK boys or girls of either culture. If the sample was gender-balanced, therefore, cultural differences in solid effect would likely increase, not decrease, while there were no sex differences in liquid effect or interaction in the first place. Sex differences therefore do not provide an alternative account of the cultural differences found here.

When the two cultures were analysed separately, both showed main effects of liquid, $F(1.70, 142.70) > 8.11$, of solid, $F(1.63, 184) > 308.89$, and the interaction, $F(4, 336) > 5.74$, with bilinear components, $F(1,84) > 4.61$. Only the solid effect differed by age for the Hong Kong children, $F(3.26, 184) = 2.64$. For UK children, the solid effect and the solid x liquid interaction differed by age, $F(3.18, 133.39) = 2.92$ and $F(8,336 = 2.57)$, the liquid x age interaction missed significance $F(3.40, 142.70) = 2.13$, $p = .091$.

Despite these age differences, the solid effect appeared at each age in each culture, $F(1.47, 51.37) > 51.55$, confirming the visual impression. The liquid effect was significant in all three Hong Kong groups, $F(1.32, 59.90) > 9.35$, but only in the oldest UK children, $F(1.59, 41.24) = 14.25$. The liquid x solid interaction was significant for the youngest and oldest Hong Kong children $F(2.67, 93.60) > 3.25$, with bilinear trend, $F(1, 35) > 4.27$ and the two older UK groups, $F(4, 124) > 3.72$, with bilinear trend close to significance $F(1, 26) = 3.60$, $p =.069$ and $F(1, 31) = 3.90$, $p = .057$, as expected under the proportion model.

Overall, the results fit with the often-reported advantage in maths/science for East Asian children (Siegler & Mu, 2008; Stevenson et al., 1990). Hong Kong children of all ages understood that sweetness increases with amount of solid, but decreases with amount of liquid, with support for the normative concentration model from 4 and 5 years of age. UK children's judgments were less affected by liquid amount at all ages and support for the concentration model appeared only from 6 or 7 years.

Individual Subject Analysis

In developmental work, it is important to check whether group data are representative of individuals, so we also considered the data at the single subject level. We worried, in particular, that small liquid effects at younger ages or in UK children may reflect averaging of children with dilution effect, and children with incorrect, direct liquid effect, as noted in previous literature. Seventeen children here (of 201 for which we had data on this, all but one at the younger ages), made direct errors when initially asked about the effect of adding water. To assess if such errors also occurred on experimental trials, we classified individuals into 3 groups, with direct, inverse or no main effect of solid/liquid. As single subject ANOVAs have little power with children, who will not sit through many replications, we used a combined means-based and statistical criterion, attributing an effect at individual level by either significance or size of effect. In previous applications this approach amplified statistical trends without changing the patterns (Schlottmann, 2001) and the same appeared here.

Children were thus taken to show a direct (or inverse) effect, if mean judgments for the largest volume were higher (lower) than for the smallest volume, with this difference either 2 points or larger, or significant in a single subject ANOVA with 9 df for error. No effect was diagnosed if this difference was less than 2 points and not significant. The percentage of UK and Hong Kong children falling into each of the 3 groups are in Table 1.

Table 1: Proportion (%) of Hong Kong (top) and UK children (bottom) with solid and liquid main effects (correct pattern in bold)

Group	Effect	Inverse	Direct	None	N
HK 4&5		6	**72**	22	36
HK 6&7	Solid	2	**91**	7	44
HK 8&9		0	**89**	11	36
HK 4&5		**50**	0	50	36
HK 6&7	Liquid	**48**	16	36	44
HK 8&9		**72**	3	25	36
UK 4&5		0	**86**	14	28
UK 6&7	Solid	0	**100**	0	32
UK 8&9		0	**100**	0	27
UK 4&5		**21**	11	68	28
UK 6&7	Liquid	**31**	3	66	32
UK 8&9		**48**	4	48	27

In the top and third block, most children, at all ages, in both cultures show direct effects of solid, confirming the impression of uniformly good understanding in the group data. In both cultures, a small developmental increase, less than 20% across all three ages, is also evident.

In the bottom and second block, fewer children of all ages and in both cultures show appropriate inverse liquid effects, also confirming the group impression. Nevertheless, in Hong Kong, about half at even the youngest age display this inverse effect, while this appears for only about 20% in the UK. The developmental increase for liquid is well over 20% across the three ages.

The individual data confirm age and culture differences apparent in the group data. Moreover, they show that inappropriate direct water effects (Stavy et al., 1982) are rare in our sample. Thus the impression of a weaker effect for amount of liquid is not due to averaging of children with direct and inverse patterns. Rather, many children showed no effect for amount of liquid at all, judging sweetness of a solution largely based on the amount of solid in it, ignoring the amount of liquid.

Qualitative Data

After children made their first mixture, before intensity was ever mentioned, we asked, "What would happen if we added more sugar/water to this?" to determine what aspect of the event would be children's natural focus. (These questions were asked only in one Hong Kong sample and in the UK). Responses in Figure 3 fell into four categories: Children talked about (1) intensity (i.e., sweetness or more/less sugary; directionless, correct and incorrect direction effects fell in this category), (2) other physical features (i.e., colour, fizzy, floating/sinking, volume), (3) they re-described once more that the sugar would dissolve/melt or merely stated that the flask would look the same as before, or (4) they gave unclear/don't know responses.

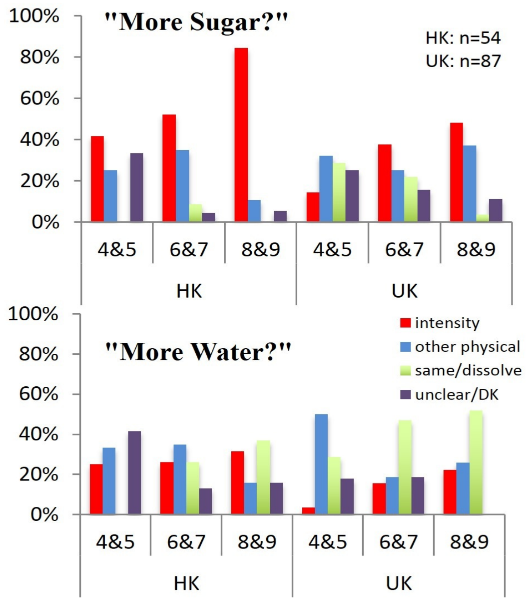

Figure 3: Proportion of verbal responses to questions on what would happen if we added more solid (top panel) or more liquid (bottom panel) to the mixture

The red bars in Figure 3 show that Hong Kong children more often focus naturally on intensity/sweetness than UK children, $\chi 2(1)=9.34$. This focus increases with age in both cultures. The latter is clear in the top panels for responses when asked about effects of adding more sugar, $\chi 2(2)>6.94$, but non-significant age trends appear as well (bottom panel) for adding liquid. Children generally talked less about sweetness in the latter case, suggesting more natural focus on intensity for solid than liquid. Alternatively, this may reflect simply that the question about adding water came second, and children did not want to repeat previous answers about sweetness. However, that children did not mind repeating event descriptions (green bars) speaks against the latter option.

When taste intensity answers are compared to other types of answer, it seems that Hong Kong children from the earliest age talked more often about taste than about any other physical feature of the solution (colour, fizz, floating or volume, summed together in the blue bars) and rarely merely re-described the dissolving event they had already heard about (green). For UK children, in contrast, taste was no more salient than other physical feature (blue) and they often just repeated the previous description (green). These qualitative data fit well with our quantitative results, highlighting similar age and cultural differences in children's natural focus on taste intensity of these mixtures as we found in their judgments.

Discussion

The present study used a novel judgment method and cross-cultural comparison to study children's understanding of an intensive property of solid/liquid mixtures. The results both confirm and disconfirm that children have deep-seated difficulty with intensity judgment. For UK children, results largely confirm this impression. For Hong Kong children, however, our method produced substantially better performance, from the youngest age. This contrast in the data suggests that what previous views have typically taken to reflect children's cognitive limitations substantially depends on children's cultural experience.

Had we tested only UK children, our results would have cross-validated previous findings with a novel judgment method, one more sensitive than standard choice paradigms to early intuitive knowledge of comparable complexity in other domains. Probability, for instance, like intensity, involves a direct and an inverse variable. When young children chose which of two options is more probable/intense, they often ignore the inverse variable in both cases. When children judge how probable each option is, in contrast, they show strong effects of the inverse variable by age 4. Children's intensity judgment here improved as well – weak effects of the inverse variable appeared from age 7, plus we found few direct errors – but not as dramatically as in other areas. This would seem to confirm that children have problems with intensive concepts that go beyond difficulties with inverse variables (Nunes et al., 2003) and that appear even in intuitive judgments.

One might object that the difficulty lies with the specific materials used, because everyday experience provides no continuously visible cue to taste intensity. This may delay children's learning about such concepts. The same point could be made about many other intensive concepts that trouble children. Taste, heat and density can all be felt, of course, but this typically involves short experiences one has to focus on, while visual information can be taken in more continuously and incidentally. This plausible view does not explain the present data, because in other work, children perform similarly for sugar-water and colour-water solutions (Schlottmann, Moss, Hill, Ellefson, & Taber, 2013). More strikingly, Jäger and Wilkening (2001) who first proposed this, found, contrary to expectation, that children believe a mixture of dark and light liquids is darker than the darker of the constituents. Availability of visual intensity cues thus does not seem to help children reason about intensive concepts. Jäger and Wilkening also concluded that children had genuine difficulties with intensive concepts.

Our findings with Hong Kong children, however, rule out this view. From ages 4 or 5, these children gave sophisticated intensity judgments, with strong liquid effects and a data pattern as predicted by the normative proportion model. Children's good understanding appeared with our Information Integration judgement method, and it remains to be seen whether the Hong Kong advantage is strong enough to show up also with less sensitive, traditional choice measures. Some doubt remains, in any event, about whether Hong Kong children's judgment conforms to the proportion model from the youngest age, because the 6-year-olds' data did not diverge. Nevertheless, that children this young are able to cope with the inverse variable is clear: it showed up at all ages at group and individual level. Future studies should address whether the inverse relation precedes the proportion model, whether both appear at the same age, or whether development is, as on face value here, U-shaped, with younger and older children doing better than children around school entry. Stavy (1982) already reported U-shaped trends in intensity choices, suggesting this reflected a transition from intuitive to analytic thought. In Stavy's view, echoing Piaget's, however, intuition is a global, undifferentiated, immature form of thought, while children here displayed highly structured, adaptive intuitions from the youngest age. Our Hong Kong data show that by 4 or 5 years, children can have a functional intensity concept.

Further research should investigate what elements of Hong Kong, but not UK pre-schoolers' experience support this advantage. Work on cross-cultural differences in mathematics cites better socio-cultural and academic support and attitudes, differences in curriculum and teaching style from the early years, as well as differences in number language, all supporting Asian children's better, earlier achievements (e.g., Aunio, Aubrey, Godfrey, Pan, & Liu, 2008). There may be similar positive attitudes towards, and more/better practices of, science from a young age, e.g., more focused, teacher-centric teaching may lead to more focused science observation which might help with noting variables.

Language might also support or amplify such effects. English is a count/mass language, while Chinese is a classifier language. The latter may help to make substances salient, with effects noted from the ages considered here (Li, Dunham & Carey, 2009). In addition, Chinese allows dropping much pragmatically inferable information from a sentence, which might also support focus on situational essentials. Indeed, Hong Kong children's verbal answers here were not just more focused on intensity than same-aged UK children's, but also had only half the number of words. Howe, Nunes, and Bryant (2010) discuss, for English, how intensive quantity reasoning depends greatly on how language makes some variables salient, e.g., talk of sweetness highlights the direct variable, when talk of wateriness might highlight the indirect variable. More subtle effects, e.g., depending on which variable is unitized, are also described. Cross-linguistic differences in how language affects variable salience seem likely and require further investigation.

Finally, we need to point to the relevance of intensive quantity reasoning to children's learning about substances/materials, i.e., early chemistry. There is little work on young children's chemistry and much of what there is concludes that children have little understanding; an impression largely derived from interview studies (e.g., Liu & Lesniak, 2006). These are not sensitive to young children's abilities, but clearly problems remain even in less verbal experimental paradigms as considered here. How substances mix is thought to be one of the earliest aspects of chemistry accessible to children, and even in this area errors abound. The present study and a few others (Quinn, Ellefson, Schlottmann, & Taber, 2013) suggest that even pre-schoolers are capable of using appropriate reasoning in this area. While the conditions under which children do so are not well understood at present, this study provides an important new perspective both on children's understanding of intensive quantities and on their understanding of substances/materials.

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