

Spatial Organization and Presentation Mode in the Representation of Complex Data

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Abstract

External representations are more effective when spatial dimensions are used to represent numeric variables. However, this principle may result in suboptimal representations when the number of numeric variables to be represented is large. To test this possibility, participants studied a set of graphs representing a parametrized function under different parameter values. The graphs were displayed either using a grid organization, with parameter values represented by spatial dimensions (horizontal and vertical position of the graphs), or juxtaposed in a single area, with parameter values represented by non-spatial dimensions (color and texture). Juxtaposed organization led to better learning. However, this advantage was eliminated when the graphs were presented successively rather than simultaneously. The results suggest that juxtaposed organization can improve comprehension of complex data by facilitating comparison between parts of the data. Such organization may be preferable even if it precludes use of spatial dimensions for some numeric variables.

Keywords: external representations; graphs; human factors; mathematics; education.

Background

External representations of information, such as pictures, diagrams, and graphs, play an important role in human thought, serving as tools to facilitate learning, problem solving, and communication. The effectiveness of external representations depends in large part on the degree to which they respect cognitive constraints on their design. A well-studied class of constraints known relates to the selection of representational dimensions – that is, the choice of which features or dimensions of the external representation will represent which features or dimensions of the information to be represented. For example, in a network diagram, degree of connectivity between nodes could be represented by either spatial proximity or thickness of the lines connecting them (Tversky, Corts, Yu, Mason, & Nickerson, 2012), while in a graph of experimental results, different data points could be represented by either bars or points on lines, and so on.

One general cognitive constraint on the selection of representational dimensions is congruence: external representations are more effective when dimensions of the information are represented by similarly structured dimensions of the representation (Hegarty, 2011; Tversky, Morrison, & Betrancourt, 2002; Zhang, 1996). For example, in diagrams, categorical variables are best represented by categorical visual dimensions such as inclusion in containing shapes, while continuous variables are best represented by continuous visual dimensions such as proximity (Tversky et al., 2012). Ratio scale variables, including most numeric variables, are best represented by visual dimensions that also possess a ratio

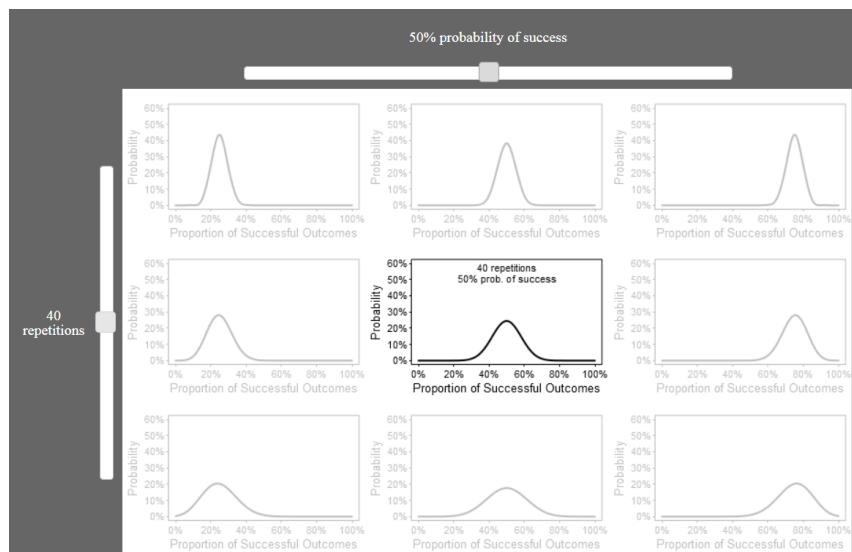
scale (Zhang, 1996). Spatial dimensions, e.g. horizontal or vertical position, fall into this category, while many non-spatial dimensions, e.g. line color and line texture, do not. As an example, it is preferable to represent numeric variables by position on the x -axis of a graph, a spatial dimension, than by the graph legend, typically corresponding to a non-spatial dimension such as line color or texture (Shah & Carpenter, 1995).

The constraints just discussed apply primarily to the selection of representational dimensions for individual dimensions of the represented information. If the information to be represented involves multiple dimensions, additional constraints may arise when these dimensions are considered together. Consequently, a selection of representational dimensions might be optimal from the perspective of each dimension considered separately, but suboptimal when considered as a whole. Thus, a complete understanding of cognitive constraints on the selection of representational dimensions must include an understanding of constraints that apply specifically to the selection of multiple dimensions. This type of constraint has received comparatively little attention in research to date.

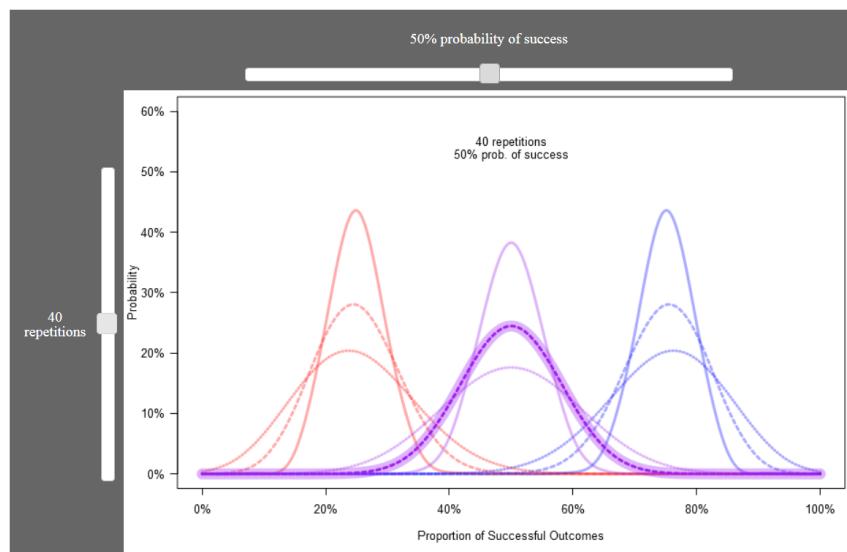
To make this issue more concrete, consider Figures 1A and 1B. These figures present a number of graph lines illustrative of the binomial distribution. The binomial distribution gives the probabilities of the different possible outcomes of a sequence of independent binomial trials, here termed “repetitions,” where an outcome is defined as a certain proportion of repetitions resulting in “successes.” The probability mass function over possible outcomes is determined by two parameters, the number of repetitions n and the probability of success on a single repetition p . Figures 1A and 1B each show 9 graph lines for the probability mass functions corresponding to 9 different combinations of values of n and p . In both figures, the different possible outcomes are represented by the x -axis and the corresponding probabilities by the y -axis. However, the figures differ in their selection of visual dimensions to represent the parameters n and p . In Figure 1A, the graph lines are contained in separate graphs, which are arranged in a grid, and the parameters are represented by spatial dimensions: each vertical position in the grid represents a different number of repetitions n , and each horizontal position a different probability of success p . In Figure 1B, the graph lines are contained in a common plot area, and the parameters are represented by non-spatial dimensions: each line texture represents a different number of repetitions n , and each line color a different number of repetitions p .

The graphs shown in Figures 1A and 1B all involve the same four variables: the proportions of repetitions resulting in success, the probabilities of each such proportion, and the

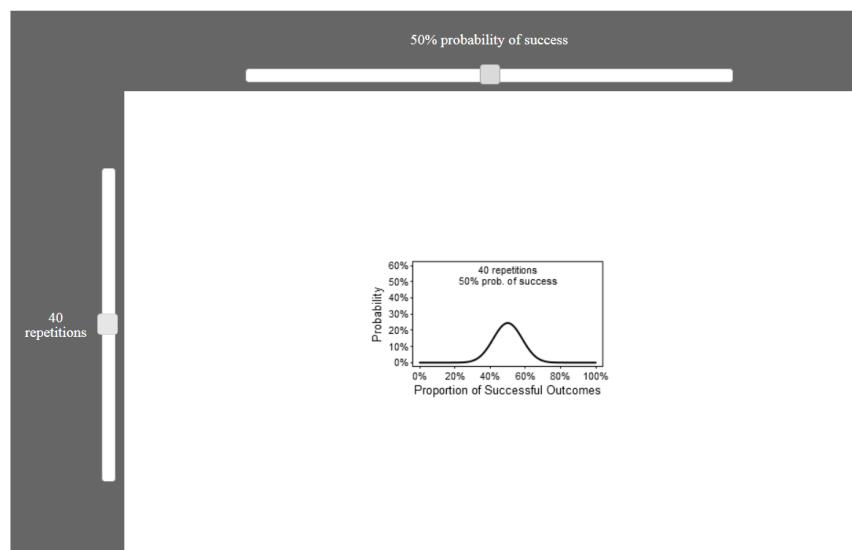
(A)



(B)



(C)



(D)

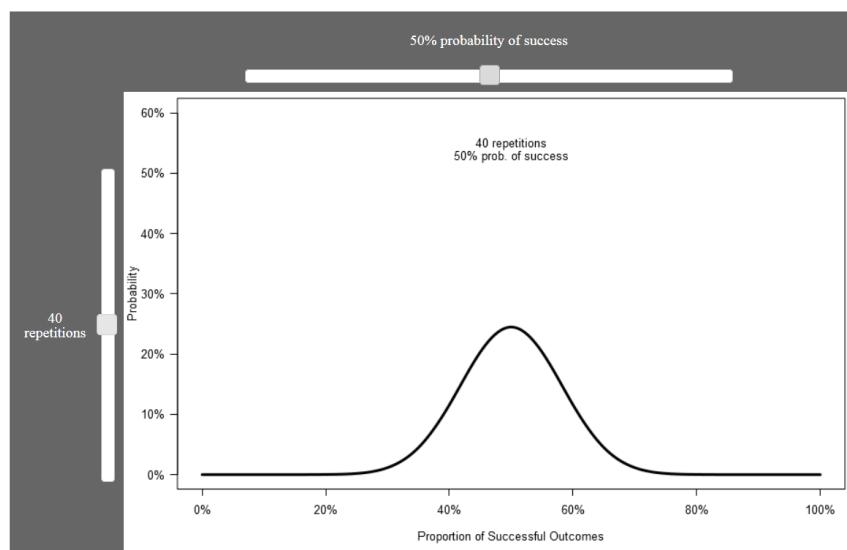


Figure 1. Training interface. (A) Simultaneous/grid version, (B) Simultaneous/juxtaposed version, (C) Successive/grid version, (D) Successive/juxtaposed version.

parameters n and p . Because all of these variables are numeric, the general constraints discussed above imply that they are most suitably represented by spatial dimensions. Thus, if the selection of representational dimensions is considered separately for each variable, the approach employed in Figure 1A, where the parameters are represented by spatial dimensions, should be superior to that of Figure 1B, where the parameters are represented by non-spatial dimensions.

However, consideration of the selections of representational dimensions for all the variables taken together suggests a possible drawback of the approach in Figure 1A, namely, each spatial dimension represents more than one variable. In particular, vertical position represents both the number of repetitions n and the probabilities associated with the different proportions of successes, while horizontal position represents both the probability of success p and the proportion of repetitions resulting in success. “Overloading” a single dimension by using it to represent multiple variables is potentially confusing even if that dimension is a good choice to represent each variable individually.

At this point, it is important to highlight that while the above example may appear highly specific, the issue just mentioned is not idiosyncratic to this example, but instead quite general. Whenever data involving k numeric variables are represented using fewer than k spatial dimensions – for example, when data involving at least 3 numeric variables are represented using 2-dimensional graphs – using spatial dimensions to represent all of the numeric variables will necessarily result in overloading at least one spatial dimension. In such cases, the danger of overloading a single spatial dimension with multiple variables can be avoided by using non-spatial dimensions to represent some of the variables. This approach is illustrated in Figure 1B, in which different values of the parameters of the binomial distribution are represented by different line colors and textures, rather than by different horizontal and vertical positions.

Besides avoiding confusion due to overloading spatial dimensions, the approach illustrated in Figure 1B has an additional potential advantage: By juxtaposing all of the graph lines in a common plot area, it facilitates comparison between them. Comparison has been proposed as a basic cognitive mechanism capable of increasing the salience of both similarities and differences between the things compared (Gentner, 2010; Kurtz, Boukrina, & Gentner, 2013). Drawing comparisons between examples can improve learning of categories and concepts by drawing attention to the critical dimensions on which the examples vary. Thus, even if overloading of spatial dimensions is not an issue, using non-spatial dimensions to represent some variables might still be advantageous by virtue of allowing more data to be juxtaposed in a common space, and thereby facilitating comparisons between different parts of the data.

So far, we have only considered static representations in which all the information available is presented simultaneously, as in Figures 1A and 1B. However, it is not obvious

that simultaneously presenting all available information is the best approach. If the information is complex, as is often the case for datasets involving large numbers of variables, simultaneous presentation could create excessive cognitive load (Mayer & Moreno, 2003), making it difficult to process the information. Presenting parts of the information successively could alleviate this difficulty by encouraging users of the representation to focus on one part at a time. This approach is illustrated in Figures 1C and 1D. The information in these Figures is spatially organized in the same way as in Figures 1A and 1B, respectively. However, only one of the 9 graph lines is available at any given moment.

The effects of simultaneous or successive presentation mode might interact with those of different approaches to spatial organization. Specifically, if juxtaposed spatial organizations, such as the one employed in Figure 1B, are advantageous by virtue of facilitating comparisons, this advantage should be reduced or eliminated if different parts of the represented information are presented successively rather than simultaneously, as in Figure 1D. The reason is that successive presentation interferes with comparison, while simultaneous presentation facilitates it (Loess & Duncan, 1952). On the other hand, if juxtaposed organizations (Figure 1B) are advantageous by virtue of avoiding confusion due to overloading spatial dimensions with multiple variables, then this advantage should apply regardless of whether simultaneous or successive presentation is employed.

The experiment described below was designed to address three related questions resulting from the above discussion. (1) If the number of numeric variables to be represented exceeds the number of spatial dimensions available to represent them, is it preferable to represent all of the variables by spatial dimensions, or instead to represent some of the variables by non-spatial dimensions? (2) If the latter approach is more effective, is the reason due to facilitating comparison or to avoiding confusion caused by overloading spatial dimensions with multiple variables? (3) Is either simultaneous or successive presentation preferable to the other?

Method

Participants learned about how properties of the binomial distribution depend on the values of its parameters by using an interactive tutorial, which presented example graphs of the binomial distribution for different combinations of parameter values. The examples were presented either simultaneously or successively, and were either organized in a grid or juxtaposed in a common space, as described in the Introduction. After completing the tutorial, participants were tested on their understanding by answering recall and comprehension questions without being able to refer to the tutorial examples. Accuracy on the test served as a measure of the effectiveness of the representation used during the tutorial¹.

¹ The experiment can be accessed at https://perceptsconcepts.psych.indiana.edu/experiments/dwb/STOE_01/demo.html.

Participants

Participants were N=164 Indiana University undergraduate students who participated in partial fulfillment of a course requirement. Participants were assigned randomly to one of the four factorial combinations of presentation mode – simultaneous or successive – and spatial organization – grid or juxtaposed. 42 participants were assigned to the simultaneous/grid condition, 40 to the simultaneous/juxtaposed condition, 40 to the successive/grid condition, and 42 to the successive/juxtaposed condition.

Materials

Example graphs representing the probability mass function for the binomial distribution under different values of its parameters were created for use in the tutorial. The values used for the number of repetitions n were 20, 40, and 100, while those used for the probability of success p were 25%, 50%, and 75%. For each combination of parameter values, a graph line for the probability mass function was created in the following manner. First, for each number k in 0, 1, ..., n , the probability that exactly k out of n binomial trials would result in successes was calculated assuming probability of success p . Second, the numbers of successes k were converted into proportions of successes k/n by dividing by n . Third, the values k/n were divided into a fixed number of bins (21) and the total probability mass within each bin was calculated. Finally, the binned data were plotted by interpolating a smooth graph line using the *splines* function in R.

For the simultaneous/grid condition, nine separate example graphs were created, i.e. one graph for each of the nine graph lines described above. The nine graphs were laid out in a grid as illustrated in Figure 1A. The rows and columns of the grid corresponded to different values of the parameters n and p respectively, with positions higher and to the right in the grid corresponding to larger values of the parameters. For the successive/grid condition, the nine graphs were positioned in a grid in the same manner as in the simultaneous/grid condition, but only one graph was visible at a time, while the others were hidden (Figure 1C).

For the simultaneous/juxtaposed condition, a single example graph was created, containing all nine of the graph lines created as described above (Figure 1B). The size of this single graph was approximately equal to the size of the grid containing all nine graphs in the grid conditions. The graph lines corresponding to different values of the parameter n were differentiated by line texture (arbitrarily chosen degree of dash spacing), while the graph lines corresponding to different values of the parameter p were differentiated by color. For the successive/juxtaposed condition, all nine graph lines were displayed in the same graph space just as in the simultaneous/juxtaposed condition, but only one graph line was visible at a time, while the others were hidden (Figure 1D). Because the graph lines were not shown simultaneously, they were all shown with the same texture and color (solid black).

In all four conditions, the example graphs were embedded in a tutorial interface consisting of the example graphs and two sliders (Figure 1). One of the sliders was used to select

a value for the number of repetitions parameter n , while the other was used to select a value for the probability of success parameter p . In the simultaneous/grid condition (Figure 1A), selecting values for the two parameters using the sliders caused the example graph corresponding to the selected values to be highlighted, and the other 8 example graphs to be faded. Similarly, in the simultaneous/juxtaposed condition (Figure 1B), the sliders caused the graph line corresponding to the selected parameter values to be highlighted and the other graph lines to be faded. In the two successive conditions, the sliders caused the example graph or graph line corresponding to the selected values to be displayed, and the other example graphs or graph lines hidden.

Procedure

The experiment was administered by computer using a web-based interface. Participants first read a passage explaining the binomial distribution and its parameters in the context of an example involving coin flips. They were then presented with the tutorial interface corresponding to their experimental condition, as described under Materials. Participants were given a series of tasks intended to lead them to explore the entire space of example graphs/graph lines. Each task required the sliders in the tutorial interface to be set in such a way that the selected graph (or graph line) satisfied some requirement, such as maximizing or minimizing the height of the peak. The tasks were designed so that if all of them were performed correctly, each of the 9 example graphs would be selected at least once (and in most cases, twice) in the course of performing the tasks. Participants were free to manipulate the sliders as long as they wished before submitting a response for each task. The tasks were performed one at a time in a fixed order. If an incorrect response was submitted, participants were given feedback and allowed two chances to correct the response, after which the correct response was shown.

After the tutorial was complete, a series of 16 multiple choice test questions was shown. There were two types of test questions: recall and comprehension, each accounting for half of the questions. The recall questions required participants to identify the parameter values that would produce a given graph, or to select the graph that would result from given parameter values. The comprehension questions tested whether participants were aware of qualitative relations between the values of the parameters n and p and characteristics of the probability mass function, such as the fact that the value of n does not affect the horizontal location of the peak of the graph line, but does affect how flat or sharp the peak is. Each test question had 4 possible responses, of which only one was correct. The questions were presented one at a time, in a (different) random order for each participant, and no feedback was given. Participants could not refer to the tutorial interface during the test.

Results

Tutorial Behavior

Although we were primarily interested in the effects of experimental condition on test performance, several measures of participants' behavior during the tutorial were also analyzed for possible effects of condition. In particular, for each participant, the following measures were calculated: tutorial accuracy, defined as the percent of tutorial tasks answered correctly on the first try, completion time, defined as the average time from beginning to end of one tutorial task, and positions visited, defined as the average number of slider settings viewed in the course of performing one tutorial trial (a given slider setting could be counted more than once if it was revisited after the sliders were changed to a different setting).

Average tutorial accuracy was 71.3%. Tutorial accuracy showed no effect of presentation mode (simultaneous or successive), spatial organization (grid or juxtaposed), or their interaction, $p > .25$. Average completion time was 27.8 seconds. Completion time showed no effect of either experimental factor or their interaction, $p > .50$. Average number of positions visited was 4.63. In contrast to the other two measures, positions visited showed effects of both presentation mode, $F(1,160) = 27.92$, $p \approx .000$, and spatial organization, $F(1,160) = 8.57$, $p = .004$, though not of their interaction, $p = .115$. Participants visited more positions per task in the successive condition (5.14) than in the simultaneous condition (4.12), and more in the juxtaposed condition (4.92) than in the grid condition (4.34). However, number of positions visited was uncorrelated with accuracy on the test, $r(162) = .080$, $p = .310$, suggesting that effects of experimental condition on positions visited are unlikely to explain any effects of experimental condition on test accuracy.

Test Performance

On average, participants answered 61.2% of the test questions correctly, with 25.0% representing chance performance. Test accuracy scores were submitted to a mixed ANOVA with presentation mode (simultaneous or successive) and spatial organization (grid or juxtaposed) as between-subjects factors and question type (recall or comprehension) as a within-subjects factor. The effect of question type was highly significant, $F(1,160) = 549.09$, $p \approx .000$, indicating that accuracy was higher for recall questions (85.5%) than for comprehension questions (36.9%). However, accuracy was significantly higher than chance for both recall and comprehension questions, $t(163) = 31.8$, $p \approx .000$ for recall and $t(163) = 8.39$, $p \approx .000$ for comprehension. Question type did not interact with any other factor, $p > .35$.

The main effect of spatial organization was also significant, $F(1,160) = 5.61$, $p = .019$, indicating that test accuracy was higher in the juxtaposed condition (64.3%) than in the grid condition (58.2%). While the main effect of presentation mode was not significant, $F(1,160) = 0.51$, $p = .474$, presentation mode did show a significant interaction with spatial organization, $F(1,160) = 8.76$, $p = .004$. Average test accuracies

by presentation mode and spatial organization are displayed in Figure 2. As shown in the figure, accuracy was highest in the simultaneous/juxtaposed condition (67.2%) and lowest in the simultaneous/grid condition (53.6%), with the other two successive conditions intermediate (successive/juxtaposed: 61.5%, successive/grid: 63.0%). Post-hoc t -tests indicated that when the presentation mode was simultaneous, the two spatial organization conditions (juxtaposed and grid) differed significantly, $t(71.6) = 3.57$, $p = .001$, but when the presentation mode was successive, the two spatial organization conditions did not differ $t(79.7) = .449$, $p = .655$.

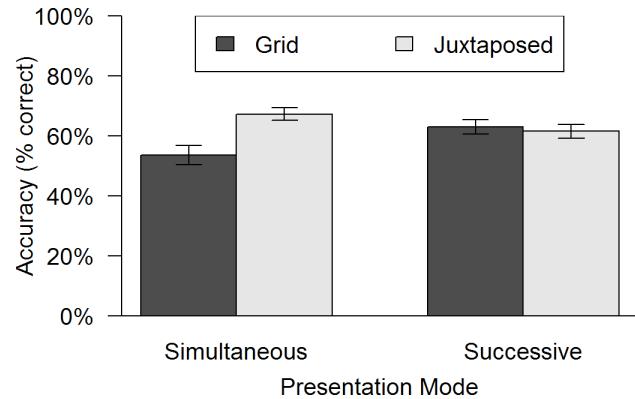


Figure 2. Test accuracy by presentation mode and spatial organization. Error bars indicate standard errors.

Discussion

Studying function graphs juxtaposed in a common space, with different values of function parameters represented by different line colors and textures, led to better learning of the properties of the function than studying the same graphs laid out in a grid, with different parameter values represented by different horizontal and vertical positions in the grid. This result is surprising from the point of view of cognitive constraints on the selection of representational dimensions, which would suggest that numeric variables such as the function parameters are more effectively represented by spatial dimensions, as in the grid organization, than by non-spatial dimensions, as in the juxtaposed organization. This result supports the general point that representations of multivariate data may be well designed with respect to each individual variable in the data, but still poorly designed when considered as a whole. Thus, it is important to understand not only how to select representational dimensions for individual variables, but also how multiple selections of representational dimensions may interact with each other.

A possible drawback of the grid organization, mentioned in the Introduction, is that its representation of multiple variables by each spatial dimension could cause confusion. However, if this drawback was the reason for the advantage of juxtaposed over grid organization, then that advantage should have been observed in both the successive and simultaneous presentation modes. To the contrary, the advantage was observed only in the simultaneous presentation mode. This

finding argues in favor of a different explanation, also mentioned in the Introduction, for the advantage of the juxtaposed organization – namely that that juxtaposing graph lines in a single graph facilitates making comparisons among them, and these comparisons in turn improve learning. We hypothesize that in general, representations will be more effective when representational dimensions for multiple variables are selected in such a way as to facilitate comparisons, and that the facilitative effects of such selection may outweigh the benefits of choosing the best representational dimension for each variable individually, as was the case in the present study. However, further research is needed to test the validity of this hypothesized principle over a wider range of cases.

Our analysis of tutorial behavior permits the elimination of several alternative explanations for the observed advantage of juxtaposed over grid organization. In particular, this advantage is unlikely to result from juxtaposed organization facilitating performance of the tutorial tasks, nor, conversely, from its creating “desirable difficulties” (Bjork, 1994) by inhibiting performance of those tasks, because no effect of condition on tutorial accuracy was found. Nor does the advantage result from participants spending more time studying in the juxtaposed condition, because condition had no effect on time to complete the tutorial tasks. Participants in the juxtaposed condition apparently engaged in more exploration of the parameter space during the tutorial, as measured by number of positions visited per tutorial task, but this difference is also unlikely to explain the advantage of the juxtaposed condition, as number of positions visited actually showed a (non-significant) negative correlation with test performance.

If facilitating comparison improves learning, then one might expect an advantage for simultaneous over successive presentation, on the grounds that the former facilitates comparison (Loess & Duncan, 1952). The absence of such an advantage implies that – if our account for the advantage of juxtaposed organization is correct – successive presentation may offer some compensatory advantage. One possibility is that successive presentation reduces cognitive load by reducing the total amount of information presented at any given moment. If this explanation is correct, successive presentation might be positively beneficial for data more complex, but inferior to simultaneous presentation for data less complex, than the data used in the present study. This prediction could be tested in future research.

However, in the context of the present finding showing no net advantage for either simultaneous or successive presentation, it is worth noting that simultaneous presentation is a less technologically demanding approach than successive presentation. Simultaneous presentation can easily be implemented through static media, such as print, while successive presentation requires some form of dynamic media, such as the interactive interface employed in the present study. If, as this consideration suggests, simultaneous presentation requires less effort to implement than successive presentation, and neither presentation mode is more effective than the other, then simultaneous presentation may be preferable for efficiency reasons. This conclusion dovetails with the findings

of studies comparing the instructional effectiveness of animations and static diagrams, which have often found no advantage of the former or even an advantage of the latter (Mayer, Hegarty, Mayer, & Campbell, 2005; Tversky et al., 2002; but see Höffler & Leutner, 2007).

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