Numerical Estimation Under Supervision

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Abstract

Children's number-line estimation has produced a lively debate about representational change, supported by apparently incompatible data regarding the descriptive adequacy of logarithmic (Opfer et al., 2011) and power models (Slusser et al., 2013). To test whether methodological differences might explain discrepant findings, we created a fully crossed 2x2 design and assigned 96 children to one of four cells. In the design, we crossed sampling (over-, even-) and supervision (with feedback, without feedback), which were candidate factors to explain discrepant findings. In three conditions (over-sampling/unsupervised-83%, even-sampling/unsupervised-67%, and over-sampling/supervised-58%), the majority of children provided estimates better fit by the logarithmic than by the power function. In the last condition (even-sampling/supervised-30%), the reverse was found. Overall, a reliable association ($p < .0001$) was found between proportion best fit by the power function and supervision. Results suggest that the fit of the power function to children's estimates is likely an artifact of supervision.

Keywords: numerical cognition; estimation; learning; cognitive development

Introduction

In this paper, we attempt to reconcile seemingly incompatible data (Barth & Paladino, 2011; Opfer & Siegler, 2007; Opfer, Siegler, & Young, 2011; Slusser, Santiago, & Barth, 2012) regarding the psychophysical functions that link numbers to children's estimates of numerical magnitude.

The psychophysical functions that link numbers to subjects' estimates of numerical magnitude are both theoretically and practically important. Of theoretical interest, the functions generating young children's numerical magnitude estimates have been observed in the non-symbolic number discrimination of a wide range of species (for review, see Nieder and Dehaene, 2009), to change abruptly with limited experience (Opfer & Siegler, 2007; Izard & Dehaene, 2008), and to closely track abilities to deal with numbers in other contexts (Booth & Siegler, 2006; Thompson & Siegler, 2010). Thus, just as an animal can better discriminate 1 and 10 objects than 101 and 110 objects, so too do children estimate the magnitudes of the symbols 1 and 10 to differ more than 101 and 110. These results suggest that (1) over the course of development, numerical symbols are linked to an innate "mental number line" that allows infants and other animals to discriminate numbers and match them across modalities (see Fig. 1) and (2) the linking between symbolic numbers and mental magnitudes is plastic and undergoes qualitative change (Opfer & Siegler, 2012).

The psychophysical functions linking numbers and estimates of numerical value have emerged as practically important as well. Specifically, the functions generating children's numerical estimates correlate highly with real-world behavior, including children's memory for numbers, their ability to learn arithmetic facts, their math grades in school, and their math achievement scores (Booth & Siegler, 2006, 2008; Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Thompson, 2014; Siegler, Thompson, & Schneider, 2011). These findings suggest that children's representations of numerical magnitude play an important role in development of mathematical ability and should be a target for educational interventions.

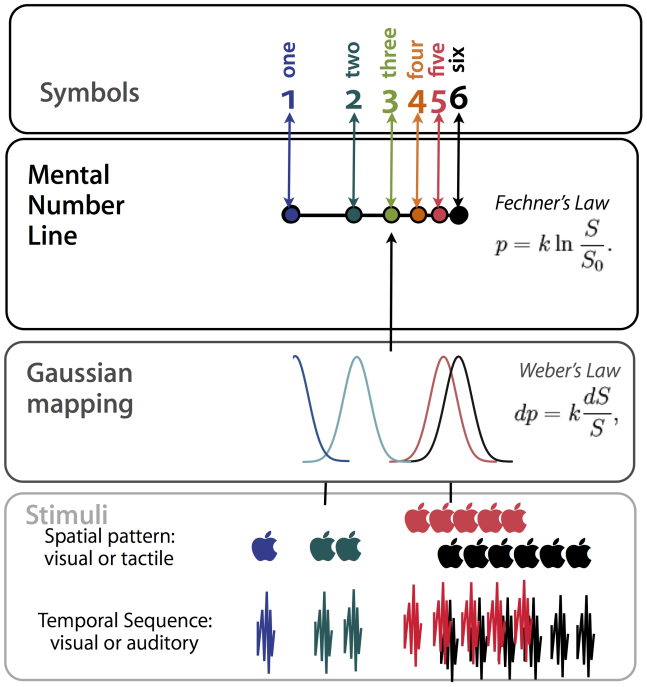

Figure 1. Model of early numerical magnitude representations (from Opfer & Siegler, 2012).

What psychophysical functions then are the most likely ones to generate estimates of numerical value? Across a

wide range of tasks and age groups (for review, see Opfer & Siegler, 2012), we have observed two functions as being the most likely contenders: the logarithmic function given by Fechner's Law ($y = k \ln x + b$) and a standard linear function ($y = mx + b$). For example, on number line estimation tasks, children are shown a blank line flanked by two numbers (e.g., 0 and 1000) and asked to estimate the position of a third number on the line. Because line length itself is not psychophysically compressive or expansive (Lu & Dosher, 2013), the task provides a relatively straightforward method for assessing compression in numerical magnitude representations.

In many studies using the number-line estimation task, a logarithmic-to-linear shift has been observed. For example, on a 0-1000 task, second graders' median estimates were best fit by a logarithmic function, whereas sixth graders' and adults median estimates were best fit by the linear function; similarly, over 90% of individual second graders' estimates were better fit by the logarithmic than linear function, whereas the reverse was true of sixth graders and adults (Siegler & Opfer, 2003). This developmental sequence has been observed to occur at different ages with different numerical ranges. It occurs between preschool and kindergarten for the 0-10 range, between kindergarten and second grade for the 0–100 range, between second and fourth grade for the 0–1,000 range, and between third and sixth grade for the 0–100,000 range (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Opfer & Siegler, 2007; Siegler & Booth, 2004; Thompson & Opfer, 2010). The same transition occurs roughly a year later for children with mathematical learning difficulties (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). The timing of the changes corresponds to periods when children are gaining extensive exposure to the numerical ranges: through counting during preschool for numbers up to 10, through addition and subtraction between kindergarten and second grade for numbers through 100, and through all four arithmetic operations in the remainder of elementary school.

Against the idea of a logarithmic-to-linear shift, however, Barth and colleagues have recently presented evidence that estimates of numerical value may follow cyclical power functions rather than being truly Fechnerian logarithmic functions or arithmetically correct linear functions. For example, on a 0-100 number line task, estimates of 7-year-olds were found to follow a 2-cycle power function originally described by Hollands & Dyre (2000). Indeed, the fit of the 2-cycle power function was strongest for 7-year-olds' ($R^2 = .968$) and 8-year-olds' ($R^2 = .995$) estimates on the 0-1000 number line task, which we examine in our present study. Further, rather than observing an abrupt, single-trial increase in linearity (as Opfer & Siegler, 2007, reported), Barth and colleagues observed a gradual, age-related increase in the value of the exponent of the power function. If true, these quantitative findings are theoretically important. First, they suggest that the commonalities between estimates of symbolic and non-symbolic magnitude are mostly illusory, with estimates of symbolic magnitude being affected by children's prior knowledge of proportions (e.g., that 500 is half of 1000). Second, they suggest that changes in numerical magnitude estimates are *quantitative* (in the sense that one parameter in the same function changes over time) rather than *qualitative* (in the sense that different functions are needed to describe younger versus older children's estimates).

Why Different Functions? Sampling versus Supervision. To illustrate the difference between the data cited in support of the logarithmic-to-linear shift account and the proportion-judgment account, it is useful to compare 7- and 8-year-olds' number line estimates on the 0-1000 task (Fig. 2), where Slusser et al. (2012) found a better fit for the 2-cycle power function over the logarithmic, despite the logarithmic function providing a better fit in the data collected by Opfer and Siegler (2007). Given that the ages of the children and the numeric ranges were the same, something must explain these discrepant findings.

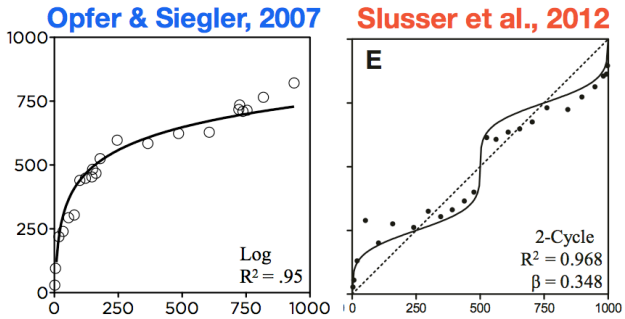

Figure 2. Discrepant results in 0-1000 number line estimation: Left, Pretest number line estimates from Opfer & Siegler, 2007; Right, number line estimates from Slusser et al., 2012.

One potential cause of the discrepancy is methodological differences in *sampling* (Barth, Slusser, Cohen, & Paladino, 2011; Slusser et al., 2012), with the fit of the logarithmic function being an artifact of sparsely sampling at the upper ranges (e.g., obtaining few estimates for numbers 750-1000) and heavily sampling at the lower ranges (e.g., obtaining many estimates for numbers 0-250). As Slusser et al. (2012) write, "there is a resounding tendency for researchers to sample heavily from the lower end of the number line and scarcely from the upper end. This is because most studies aim specifically to distinguish between logarithmic and linear fits in the context of the representational-shift hypothesis…. This practice focuses on participants' propensity to overestimate small numbers, but yields little data to reveal the details of underestimation patterns for larger numbers" (p. 4). This observation has potential force. As can be seen in Fig. 2, Opfer and Siegler (2007) collected estimates for 13 numbers in the 0-250 range and 3 numbers in the 750-1000 range, whereas Slusser et al. (2012) collected estimates for 7 numbers in each of the two ranges.

Another potential cause of the discrepancy is methodological differences in *supervision* (Opfer, Siegler,

& Young, 2011), with the fit of the 2-cycle power function being an artifact of experimenters supervising children's estimate of 500. In the typical number line task (Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2006; Laski & Siegler, 2007; Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008; Opfer & Martens, 2012), children are given no supervision in any of their number line placements. In contrast, in all studies finding a superior fit of the 2-cycle power function, children's estimate of the halfway point is supervised. For example, in Slusser et al. (2012), children were told, "Because 500 is half of 1000, it goes right in the middle between 0 and 1000. So 500 goes right there, but it's the only number that goes right there." Given previous training studies (Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008), such supervision seems highly likely to affect children's estimates.

While these two potential causes of the discrepancy in findings are not mutually exclusive, each cause has different theoretical implications. From the perspective of the logarithmic-to-linear-shift account, differences in sampling are predicted to be minor because oversampling has only a small impact on the absolute fit of the logarithmic and linear regression models. In contrast, from the perspective of the proportion-judgment account, differences in supervision are predicted to be minor because if the child already represents numbers as proportions (e.g., 500 as half of 1000), supervision tells the child nothing new. Thus, in addition to helping to explain the discrepancy in previous results, an effect of either sampling or supervision is meaningful.

The Current Study. To test the impact of supervision and sampling on the fit of the logarithmic and 2-cycle power functions to children's number line estimates, we created a fully crossed 2x2 design and assigned 96 children to one of four conditions.

In the design, we crossed sampling (over-sampling, even-sampling) and supervision (supervised, unsupervised). Two conditions were direct replication attempts of previous findings: over-sampling/unsupervised was a direct replication of Opfer and Siegler (2007), and even-sampling/supervised was a direct replication of Slusser et al. (2012). In these two cells, we expected to replicate previous findings (i.e., best fit by the logarithmic function for the over-sampling/unsupervised condition and best fit by the 2-cycle power function for the even-sampling/supervised condition as in Fig. 2). The remaining two conditions have not been tested previously. Both the logarithmic-to-linear-shift and proportion-judgment account expect slightly worse fits of their preferred models in the over-sampling/supervised cell (though for different reasons). Thus, the most interesting condition is the even-sampling/unsupervised cell. If the fit of the logarithmic function is simply an artifact of over-sampling, estimates in this condition are expected to be best fit by a 2-cycle power function. In contrast, if the fit of the power function is an artifact of supervision, estimates are expected to be best fit by a logarithmic function.

Method

Participants

Participants were 96 first and second grade students (M = 7.62 years, SD = 0.59 years; 55% females; 74% Caucasian, 8% Biracial, 6% Asian, 5% African American, 4% Native American, and 2% Hispanic) who attended one of five public elementary schools in Norman, OK. On average, 45% of students at these five schools were eligible for free or reduced-price lunches; Oklahoma's state average is 61%. Two female research assistants presented the procedure.

Procedure and Design

All children completed the estimation task one-on-one with a trained experimenter in a quiet room in their school. For each problem, children were shown a line 21.8-cm long, with the left endpoint labeled 0 and the right endpoint labeled 1000. The child's task was to estimate the position of a third number by making a hatch mark on the number line corresponding to its location.

Children differed in the numbers that they estimated and the instructions they received. Specifically, children were randomly assigned to one of four fully-crossed experimental conditions that differed with respect to the numbers that were estimated on number lines (over-sampling/even-sampling conditions) and whether corrective feedback was given about the location of 500, the midpoint of the 0-1000 number line (supervised/unsupervised conditions).

In the oversampling conditions, children were asked to estimate the positions of 2, 5, 18, 34, 56, 78, 100, 122, 147, 150, 163, 179, 246, 366, 486, 606, 722, 725, 738, 754, 818, and 938. These numbers had been used in Opfer and Siegler (2007). In the even sampling conditions, children were asked to estimate the positions of 3, 7, 19, 52, 103, 158, 240, 297, 346, 391, 438, 475, 525, 562, 609, 654, 703, 760, 842, 897, 948, 981, 993, and 997. These numbers had been used in Slusser et al. (2012).

In the supervised conditions, children received the following instructions (adapted from Slusser, Santiago, & Barth, 2012): "*This task is with number lines. There will be a number up here.* [Researcher pointed to the top left corner of the blank number line data sheet where the to-be-estimated number was located.] *Your job is to show me where that number goes on a number line like this one. Each number line will have a 0 at this end* [Researcher pointed to 0.] *and 1000 at the other end* [Researcher pointed to 1000.]. *When you decide where the right place for the number is, I want you to make a mark through the line like this.* [Researcher made a vertical hatch mark in the air in front of the child.] *Can you show me where 0 goes? Great! Now, can you show me where 1000 goes?* [Researcher provided corrective feedback if the participant did not mark the right location for these two numbers.] *So if this is 0, and this is 1000, where would you put 500?* [Researcher provided corrective feedback on the location of 500.] *Because 500 is half of 1000, it goes right in the middle*

between 0 and 1000. So 500 goes right there [Researcher pointed to vertical hatch mark that indicated the correct location of 500]*, but it's the only number that goes there. I am going to show you a lot of numbers, so just mark where you think each one should go. Don't spend too long thinking about each one. I will read you the number above the line, and then you should decide where that number goes. Are you ready to give it a try?"*

Children assigned to the no supervision conditions received the same instructions except these children were not asked to estimate 500, and thus were not given feedback about the correct location of the midpoint of the 0-1000 number line.

Results

To ensure that our random assignment to condition resulted in equivalent groups, we first confirmed that age did not differ significantly by experimental condition, $F(1, 92) = .41$, $p > .05$.

Over-sampling/Unsupervised. As in all analyses, we regressed children's median estimates against number using both the logarithmic and 2-cycle power function (Fig. 3). While the logarithmic function accounted for 95% of variation in children's estimates, the 2-cycle regression function accounted for only 4%. To interpret this difference in R^2 values, we calculated differences in AICc scores. Although this model selection tool is somewhat biased for models with parameters having greater flexibility of form (e.g., favoring power models over logarithmic ones; Pitt, Myung, & Zhang, 2002), AICc scores were used by Slusser et al. (2012) and so we used them here as well. In this case, we observed an AICc difference of 62.89, meaning that there was a greater than 99.99% probability that the data-generating function was logarithmic rather than a 2-cycle power function. To ensure that these results were not artifacts of averaging data over subjects, we also fit the logarithmic and 2-cycle power function to each individual child's estimates, with the result that 84% of children were better fit by the logarithmic regression function than by the 2-cycle power function. Thus, as expected, we replicated results from Opfer and Siegler (2007) when using their combination of numbers and no supervision.

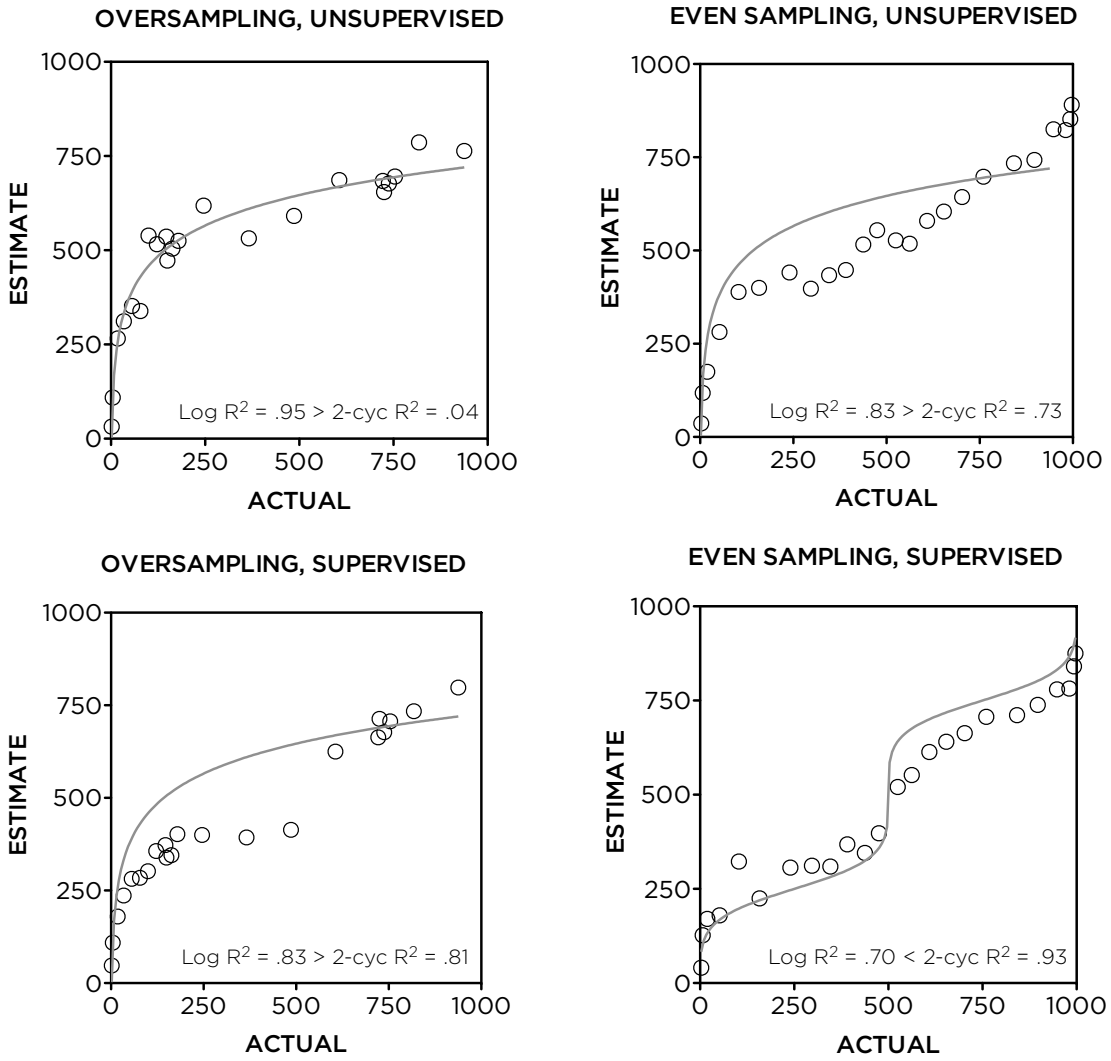

Figure 3. Estimates by experimental condition.

Even-sampling/Supervised. We next analyzed results from the condition using the numbers and supervision provided by Slusser et al. (2012). In this condition, we found that the fit of the 2-cycle power model (R^2 = .93) was greater than the fit of the logarithmic model (R^2 = .70). To interpret this difference, we calculated differences in AICc scores, and we found a difference of 35.84, meaning that the probability that the 2-cycle power function is the data-generating model is >99.99%. We also fit the logarithmic and 2-cycle power function to each individual child's estimates, with the result that 70% of children were better fit by the logarithmic regression function than by the 2-cycle power function. Thus, as expected, we replicated results from Slusser et al. (2012) when using their combination of numbers and supervision.

Over-sampling/Supervised. Results from the condition using the numbers tested by Opfer & Siegler (2007) and the supervision provided by Slusser et al. (2012) were examined next. In this condition, we found that the fit of the logarithmic model (R^2 = .83) was greater than the fit of the 2-cycle power model (R^2 = .81). To interpret this difference, we calculated differences in AICc scores, and we found a difference of .31, meaning that the probability that the logarithmic function is the data-generating model is 46.1%. We also fit the logarithmic and 2-cycle power function to each individual child's estimates, with the result that 58% of children were better fit by the logarithmic regression function than by the 2-cycle power function. Thus, whether because over-sampling penalized the power function (as suggested by Slusser et al., 2012) or because supervision increases the linearity of estimates (Opfer & Thompson, 2008; Thompson & Opfer, 2008), the absolute fit of both models was less than had been previously observed and the relative differences between the models were less as well.

Even-sampling/Unsupervised. Finally, we analyzed results from the condition using the numbers tested by Slusser et al. (2012) but with no supervision (as in Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2006; Laski & Siegler, 2007; Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008; Opfer & Martens, 2012). In this condition, we found that the fit of the logarithmic model (R^2 = .83) was greater than the fit of the 2-cycle power model (R^2 = .73). To interpret this difference, we calculated differences in AICc scores, and we found a difference of 8.73, meaning that the probability that the logarithmic function is the data-generating model is 98.74%. We also fit the logarithmic and 2-cycle power function to each individual child's estimates, with the result that 67% of children were better fit by the logarithmic regression function than by the 2-cycle power function. Thus, rather than the fit of the logarithmic function being an artifact of the numbers tested, numerical magnitude estimates -- when unsupervised -- appear to follow a logarithmic function.

Why might supervision raise the relative fit of the 2-cycle power function over the logarithmic function? One possibility is that supervision simply causes children to improve their estimates, thereby increasing the fit of the linear function. To test this idea, we re-analyzed results to determine whether they might be better explained by a linear rather than 2-cycle function. For median estimates, the linear function provided a better fit than the 2-cycle function across all four conditions (AICc's > 12; prob. of lin > 99%). However, the proportion of individual children best fit by logarithmic versus linear functions varied with age and supervision. Among the 24 7-year-olds, 75% of children in the unsupervised conditions were best fit by log, whereas only 43% were best fit by lin, χ^2 = 5.49, p = .019. Among the 24 8-year-olds, 33% of children in the unsupervised conditions and 30% of children in the supervised condition were best fit by log. Thus, as in Opfer and Siegler (2007), both supervision and age increased the proportion of children generating estimates best fit by the linear function.

Discussion

In this paper, we sought to reconcile seemingly incompatible data (Barth & Paladino, 2011; Opfer & Siegler, 2007; Opfer, Siegler, & Young, 2011; Slusser, Santiago, & Barth, 2012) regarding the psychophysical functions that link numbers to children's estimates of numerical magnitude. Specifically, we sought to identify the influence of sampling and supervision on the relative fits of the 2-cycle and logarithmic functions.

The results of our study indicate that young children's unsupervised estimates of numerical magnitude tend to increase logarithmically with the actual value of the numbers estimated. This finding held regardless of whether the numbers that were presented to children oversampled the low end of the range or sampled all numbers equally. This result is not consistent with the speculation of Barth et al. (2011) and Slusser et al. (2012) that the superiority of the fit of the logarithmic function to the power function is an artifact of sampling. This was an important issue to test because the only previous study examining the relative fits of the two models to unsupervised estimates (Opfer, Siegler, & Young, 2011) had relied on data that used over-sampling and found that over 90% of individual children's estimates were best fit by the linear and logarithmic functions.

Results also suggest that it is not very likely that young children spontaneously make use of numerical proportions when estimating the positions of numbers on number lines. This is a key claim of the proportion-judgment account, and it guides the choice of models for testing. Against this view, however, few second graders know that the number that is half of 1000 is 500. Thus, telling them this fact in the context of number line estimation is likely to have a large effect on their estimates. Consistent with this idea, 67% of children making unsupervised estimates (with even sampling) were best fit by the logarithmic function, whereas only 30% of children making supervised estimates (with even sampling) were best fit by the logarithmic function. This result would not be expected if children already knew the proportions being given by Slusser et al. (2012) in their instructions to children.

Most generally, however, the results suggest that supervision has a powerful effect on children's number line estimates. We found that the largest impact of supervision was to increase the linearity of estimates (regardless of sampling), not to cause children's estimates to follow a 2-cycle power function. This result is consistent with a number of training studies of children's number-line estimates (Opfer & Siegler, 2007; Opfer & Thompson, 2008; Thompson & Opfer, 2008). An important conclusion from these and the present study is that representations of symbolic numeric magnitude are plastic and modifiable by experience. Given that linearity of children's numerical estimates correlate highly with real-world behavior, including children's memory for numbers, their ability to learn arithmetic facts, their math grades in school, and their math achievement scores (Booth & Siegler, 2006, 2008; Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Thompson, 2014; Siegler, Thompson, & Schneider, 2011), the present results suggest that supervision of numerical magnitude judgments could have an important effect on children's general math proficiency as well.

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