Subitizing and Finger Gnosis Predict Calculation Fluency in Adults

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Abstract

Both *subitizing*, the ability to enumerate small sets without counting, and *finger gnosis*, the ability to mentally represent one's fingers, have been found to predict calculation skill in children (Penner-Wilger et al., 2007, 2009). In the current paper, we examined whether these same relations hold for young adults. Consistent with the developmental data, both subitizing and finger gnosis were significantly related to university students' (N = 51) calculation fluency, jointly accounting for 33% of variability in fluency. The findings demonstrate that early precursor skills to mathematics remain similarly related into adulthood.

Introduction

The calculation skills of young adults are falling (LeFevre et al., 2014; Mulhern & Wylie, 2004). LeFevre et al. (2014) report data showing that the arithmetic fluency of university students has declined by 24% across a 12-year span. This decline is problematic, because without basic calculation skills students have insufficient resources to grasp more advanced material (Walcyzk & Griffith-Ross, 2006). Also, strong calculation skills are related to positive employment outcomes including obtaining and retaining a job, higher employment income, and home ownership (Bynner & Parsons, 1997; Parsons & Bynner, 1997; Ritchie & Bates, 2013). Indeed, the links are stronger for numeracy than for literacy skills. Recent research has identified foundational capacities that underlie the development of calculation skills in children (Butterworth, 2010; Penner-Wilger et al. 2007). In the current paper, we examine whether these childhood precursors to calculation skill, subitizing and finger gnosis, remain related to calculation skill into young adulthood. The answer to this research question will address whether the relation in childhood simply reflects the cognitive strategies employed by children while learning arithmetic or a more fundamental and enduring relation.

Subitizing, the ability to enumerate small sets without counting, predicts calculation skill in children (Penner-Wilger et al., 2007; Reeve, Reynolds, Humberstone, & Butterworth, 2012). Children can typically subitize to three items, whereas adults can typically subitize to four items (Svenson & Sjoberg, 1983). Penner-Wilger et al. (2007) examined the relation between subitizing and calculation skill in Grade 1 children. Children completed an enumeration task, where they were shown a set of dots and asked to verbally report how many dots were shown. The response time (RT) slope as a function of set size across 1-3 items was used as the subitizing measure. A slope close to zero indicates subitizing (items enumerated in parallel) and a positive slope indicates counting. Calculation skill was measured using the calculation subtest from the Woodcock-Johnson Psycho-Educational Battery-Revised (Woodcock & Johnson, 1989). Penner-Wilger et al. found that subitizing was significantly related to calculation skill [$r(145) = -.31$] concurrently in Grade 1. Children with better subitizing ability, indicated by flatter slopes, displayed better calculation skills. These findings were extended by Reeve et al. (2012) who found that subitizing ability at six years of age was related to calculation skill both concurrently and also longitudinally at nine and eleven years of age.

Children with developmental dyscalculia, a deficit in numeracy and/or arithmetic, have poorer subitizing abilities than typically developing children (Andersson & Ostergren, 2012; Landerl, 2013; Schleifer & Landerl, 2011). Schleifer and Landerl (2011) found that dyscalculic children in Grades 2, 3 and 4 had response time slopes across 1-3 items that were twice as steep as the slopes of their typically developing peers. Landerl (2013) replicated the finding that dyscalculic children in Grades 2-4 have steeper RT slopes than typically developing children for dot enumeration across 1-3 items. Thus, there is evidence for a relation between subitizing ability and calculation skill both within typically developing samples and when comparing between typically developing and dyscalculic samples.

Why does subitizing predict calculation skill? Butterworth (1999; 2010) proposed that the capacity to abstractly represent *numerosity*, the number of items in a set, is a foundational ability for building calculation skill. The dot enumeration task indexes this capacity to abstractly represent numerosities by requiring the mapping of an abstract symbol (e.g., 3 or "three") onto a set. In a test of

Butterworth's proposed mechanism, Reeve et al. (2012) found that group differences in dot enumeration were found in the subitizing range, but not in the counting range. As such, subitizing may predict calculation skill because it indexes this foundational capacity to abstractly represent numerosities, and it is precisely these abstract representations of number that we use to perform arithmetic calculations. If Butterworth is correct, subitizing should predict not only calculation skill but other numeracy measures as well. Consistent with this view, Penner-Wilger et al. (2007) found that subitizing slope also predicts Grade 1 children's number system knowledge, including recognizing digits, identifying which number comes next, ordering and understanding of place value, [$r(145) = -.30$].

Finger gnosis, the ability to mentally represent one's fingers, also predicts calculation skill in children (Fayol, Barrouillet & Marinthe, 1998; Noël, 2005; Penner-Wilger et al., 2007, 2009). Finger gnosis is commonly measured using variants of a finger localization task (Baron, 2004), where the participant's hand is occluded from view, the researcher touches one or two fingers, and the participant is asked to report the touched finger(s). The number of correct identifications is used as the finger gnosis measure. Fayol, Barrouillet and Marinthe (1998) found that a set of neuropsychological tests, including tests of finger gnosis, predicted Grade 1 children's math scores. Noël (2005) found that children's finger gnosis scores predicted accuracy and fluency on a variety of mathematical tests, both concurrently in Grade 1 and longitudinally one year later. Penner-Wilger et al. (2007) replicated the relation between finger gnosis and calculation skill in Grade 1 children, finding that finger gnosis was significantly related to calculation skill [$r(145) = .19$] concurrently in Grade 1. In sum, there is robust evidence of a relation between finger gnosis and calculation skill in unselected populations across different countries and educational systems including France and Canada.

To pinpoint the relation between finger gnosis and calculation skill, Penner-Wilger et al. (2009) examined finger gnosis and two tests that index the strength of number representations – magnitude comparison and number line estimation. Finger gnosis was measured using the finger localization task. The child's hand was occluded and two fingers on the same hand were touched simultaneously; children were asked to point to the touched fingers. Children with better finger gnosis scores in Grade 1 had smaller distance effects on the magnitude comparison task [$r(99) = -.35$] and more linear estimates on the number line comparison task [$r(99) = .27$] in Grade 2, reflecting a more precise mapping between numerals and their associated magnitude (Dehaene et al., 1998; Siegler & Booth, 2004).

Why does finger gnosis predict calculation skill? Using cross-domain modeling, Penner-Wilger and Anderson (2011) showed that finger and number representations use overlapping neural substrates. They argue that this overlap is the result of neural reuse (Anderson, 2010), wherein a neural circuit originally evolved for one use, such as finger representation, has be redeployed for a later developing use, number representation. This overlapping neural circuit forms part of the functional complex supporting each use – finger and number representation – and, importantly, performs the same low-level working in both uses (Penner-Wilger & Anderson, 2013). Thus, children with more distinct mental representations of their fingers also have more distinct mental representations of number because both representations are supported by the same neural substrate. Alternatively, Butterworth (2010) argues that poor numerosity representations could impair the use of fingers in children's arithmetic development, leading to poor calculation skill. If this were the mechanism linking finger gnosis and calculation skill, however, we would expect subitizing and finger gnosis to be related.

Subitizing and finger gnosis are independent predictors of calculation skill. No significant relation exists between children's subitizing and finger gnosis scores (Penner-Wilger et al., 2007; 2009), contrary to Butterworth's hypothesis. Penner-Wilger et al. (2007) found that each precursor predicted significant unique variance in calculation skill, [$r(99) = -.19$ and $r(99) = .28$ for subitizing and finger gnosis, respectively, controlling for receptive vocabulary and processing speed]. Children with flatter subitizing slopes and more distinct representations of their fingers had stronger calculation skills. Jointly, subitizing and finger gnosis accounted for 28% of variability in calculation skill. Moreover, children's patterns of performance on subitizing and finger gnosis tasks in Grade 1 can be used to identify and differentiate children with math-specific difficulties and children with more general non-verbal learning difficulties longitudinally (Penner-Wilger et al., 2009).

Subitizing and finger gnosis jointly and independently predict calculation skill in children. It remains an open question, however, as to whether the same relations hold in adults. The observed relations may reflect the solution strategies young children use to perform arithmetic calculations, strategies that undergo significant developmental changes (Siegler & Shrager, 1984). As a result, the relations may hold only for children. In support of this hypothesis, the relation is currently reported for kindergarten and early elementary school children, when arithmetic calculation skills are being taught.

Alternatively, given that subitizing and finger gnosis are posited to relate specifically and fundamentally to the representation of number, we might predict that the relations between subitizing, finger gnosis and calculation skill hold for young adults. For example, phonological awareness is a foundational capacity for the development of reading skill (Goswami & Bryant, 1990). Consistent with the foundational nature of this relation, phonological awareness continues to relate to reading ability beyond childhood (Durgunoğlu & Öney, 2002; Wilson & Lesaux, 2001).

In the current paper, the relations among subitizing, finger gnosis and calculation skill in university students were tested. Given that both subitizing and finger gnosis are posited to play a foundational role in the development of number representations, it was hypothesized that the relations previously seen for children between subitizing and calculation skill and finger gnosis and calculation skill would hold for young adults – at level similar to that shown previously in children.

Method

Participants

The participants for the study consisted of 51 students (*M* = 19.8 years ± 1.0 years, Range = 18-23 years), from King's University College and Western University (27 male, 24 female). All participants completed their elementary and secondary education in Canada. Participation for this study was on a voluntary basis.

Materials

Subitizing The subitizing task is based on the number discrimination task from Trick et al. (1996). In this task, participants were presented with a set of dots (ranging from 1 to 9), and were asked to identify the number of dots as fast as they could without making any errors. Participants selected their response by touching one of two alternatives. The following eight discriminations were made twice (once with the smaller number of dots displayed and once with the larger number of dots displayed) in each of five blocks: 1 vs. 2; 2 vs. 3; 3 vs. 4; 4 vs. 5; 5 vs. 6; 6 vs. 7; 7 vs. 8; 8 vs. 9. Stimuli remained on the screen for 7800ms or until the participant made a choice, and the time between trials was 1000ms. Participants performed a total of 80 trials and the order of the problems was randomized. The dependent variable was response time slope as a function of set size across 1 – 4 items. This subitizing range was selected based on previous research showing that adults generally subitize to four items (Svenson & Sjoberg, 1983).

Finger Gnosis The finger gnosis task is based on Noël (2005). During this task, participants were first asked to place either their dominant or non-dominant hand palm down on a desk (starting hand was counterbalanced). The participant's view of their hand was subsequently occluded by a box placed over their hand. For each trial, the experimenter lightly touched the top of two fingers (simultaneously) on the participant's hand (right below the fingernail). The experimenter then removed the box and asked the participant to point to the two fingers that were touched. Participants performed a total of 10 trials on each hand for a total of 20. The dependent variable was number of fingers correctly identified (out of 40).

Calculation Fluency As a measure of calculation fluency, participants completed the addition and subtraction-multiplication subtests of the Kit of Factor-Referenced Cognitive Tests (French, Ekstrom, & Price, 1963). Each subtest of this paper-and-pencil task consisted of two-pages of multi-digit arithmetic problems (two pages of three-digit addition problems, and two pages containing both two-digit subtraction problems and two-digit multiplication problems). Participants were instructed to solve the problems as quickly and accurately as possible and were given two minutes per page. Calculation fluency was measured as the total number of correct solutions on both tests, and reflects an individual's ability to quickly and accurately execute simple arithmetic procedures on multi-digit problems.

Procedure

Participants were seated in a quiet room in front of an iPad. Once comfortable, participants completed the subitizing task on the iPad. Following the subitizing task, participants completed the finger gnosis task and the addition and subtraction-multiplication subtests from the Kit of Factor-Referenced Cognitive Tests. These tasks were completed in one session lasting approximately one hour.

Results

Descriptive statistics for calculation fluency, subitizing slope, and finger gnosis can be found in Table 1.

Table 1: Descriptive statistics for all measures

Measure	*M (SD)*
Calculation fluency (# correct)	53.7 (20.0)
Subitizing slope (ms)	143.0 (54.6)
Finger gnosis (# correct)	38.2 (1.9)

Correlations

A Pearson's bivariate correlation was used to determine the associations among subitizing slope, finger gnosis, and calculation fluency. Correlations are summarized in Table 1. There was a significant correlation between subitizing slope and calculation fluency, r = -.52, p < .001, indicating that flatter subitizing slopes were associated with greater calculation fluency. There was also a significant correlation between calculation fluency and finger gnosis, r = .29, p = .04, indicating that higher scores on the finger gnosis measure were associated with greater calculation fluency. The correlation between subitizing slope and finger gnosis was not statistically significant, r = -.08, *ns*. To determine whether the relation was specific to subitizing, and not enumeration ability more generally, we included the enumeration slope for the entire range and the counting range (5+), but neither was significantly related to calculation fluency and are not discussed further.

Table 2: Correlations among student measures (N = 51)

	1	2	3
1. Calculation Fluency			
2. Subitizing Slope (RT)	-.52**		
3. Finger Gnosis	.29*	-.08	

*p < .05, **p < .01.

Multiple Regression

A multiple regression analysis was used to determine if subitizing slope and finger gnosis significantly predicted participants' calculation fluency. Predictors were entered in a single step. As shown in Figure 1, the two predictors jointly explained 33% of the variability in calculation fluency, $R^2 = .33$, $F(2,48) = 11.58$, $p < .001$. Finger gnosis significantly predicted calculation fluency, $\beta = .25$, $t(48) = 2.07$, $p = .04$, as did subitizing slope, $\beta = -.50$, $t(48) = -4.17$, $p < .001$.

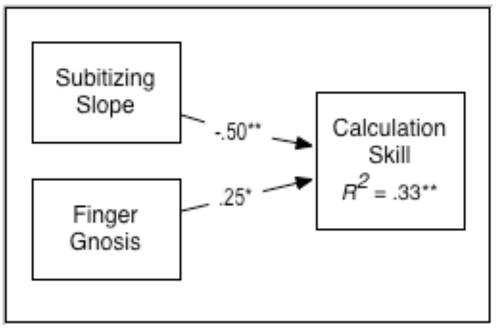

Figure 1: Multiple regression model predicting calculation skill from subitizing slope and finger gnosis.

**p<.001 *p<.05

Conclusion

We found that subitizing and finger gnosis predicted significant unique variance in young adults' calculation fluency, jointly accounting for 33% of the variability. Our findings demonstrate that the relation between the precursor skills (subitizing and finger gnosis) and calculation does not just reflect the early solution strategies used while developing calculation skills, but instead is an enduring relation. Moreover, the strength of the relation was not reduced in young adults. Thus, the findings support the view that subitizing and finger gnosis are foundational capacities for arithmetic calculation.

The finding that subitizing ability is related to adult calculation skill is consistent with the position that the ability to abstractly represent numerosities is foundational for building calculation skills. Contrary to Butterworth's numerosity coding hypothesis (2010), however, this relation is specific to the small numerosity, or subitizing, system. Our findings are consistent with developmental data showing that the relation between enumeration and calculation ability is specific to the subitizing range (Penner-Wilger et al., in prep; Reeve et al., 2012).

The finding that finger gnosis is related to adult calculation skill is consistent with Penner-Wilger and Anderson's (2013) redeployment view. On the redeployment view, finger gnosis and calculation skill are related because finger and number representations share an overlapping neural substrate. Given that the proposed nature of this relation is one of partial identity, it follows that the relation would be observed in both children and adults. Butterworth's (2010) proposed mechanism for the relation between finger gnosis and calculation skill was not supported. Finger gnosis and calculation skill were related in adults but, as in the developmental data, subitizing and finger gnosis were not correlated.

The relations between the precursors and calculation skill may be mediated by the strength of number representations. Both subitizing and finger gnosis contribute unique capacities to the representation of number, as supported by developmental data (Penner-Wilger et al., 2007; 2009). It follows that the relations in adults may also be mediated by the strength of number representations. We are currently addressing this research question by gathering data for two tasks that index the strength of symbolic and non-symbolic number representations: Magnitude comparison and ordinality (Butterworth & Reigosa, 2008; Lyons & Beilock, 2009).

Given the evidence supporting subitizing and finger gnosis as foundational capacities for arithmetic calculation, an obvious next step is to design training studies to see if (1) subitizing and finger gnosis can be improved by training and (2) if this training improves calculation skills. Some preliminary research on these important questions has begun. Gracia-Bafalluy and Noël (2008) report that finger gnosis training improves children's numeracy skills (cf. Fischer, 2010). Further training studies are needed to evaluate the suitability of subitizing and finger gnosis training, both individually and jointly, as effective interventions. Improving these foundational capacities may lead to an increase in numeracy skills in the same way that improving phonological awareness leads to an increase in literacy skills (Goswami & Bryant, 1990).

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