Pierced by the number line: Integers are associated with back-to-front sagittal space

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Abstract

Mathematical cognition, paragon of abstraction, is marked by systematic associations between number and space, often described as "mental number lines." In three experiments, we demonstrate a novel effect: a sagittal (back-to-front) number line for negative and positive integers. In a paradigm requiring full-body movements, participants judged numerical magnitude (Exp. 1-2) and parity (Exp. 3). Across all three experiments, participants associated numerical magnitude with locations in front of and behind the body. Responses to negative integers were faster when moving backward than forward; responses to positive integers, faster forward than backward. This sagittal number line appears to require the involvement of negative numbers (Exp. 1-2) and is most pronounced when judging magnitude (Exp. 3). In sum, reasoning about integers induces systematic dispositions to act along the sagittal axis. Such dispositions may reflect our mathematical *habitus*, habits of action and thought that reflect and enact our conceptual systems.

Keywords: numerical cognition; space; SNARC; embodiment; integers; sagittal number line

Introduction

"[Space] provides a location for all things that come into being. […] [E]verything that exists must of necessity be somewhere, in some place and occupying some space, and that which doesn't exist somewhere, whether on earth or in heaven, doesn't exist at all." – Plato, Timaeus

Mathematics is paradigmatically abstract, and its objects are beyond the reach of perception and interaction. And yet we still manage to engage in mathematical reasoning—for some, with more than a modicum of success and confidence. This may depend, in part, on weaving mathematics into concrete practices, habits, and concepts (Kitcher, 1984; Lakoff & Núñez, 2000). Indeed, numbers are tightly linked to space in both action and thought (Hubbard et al, 2005). A now-classic finding is the "SNARC" effect: among literate individuals from cultures that read left-to-right, smaller numbers induce dispositions to act in left space, and larger numbers, in right space (Dehaene et al, 1993). Negative integers, too, induce spatial dispositions, although task-demands influence whether they are located to the "left" of zero, in line with their relative numerical magnitude, or mixed in with positive integers on the basis of their absolute value (e.g. Fischer, 2003; Ganor-Stern & Tzelgov, 2008). This horizontal "mental number line" reflects a lifetime of enculturation into spatial practices; indeed, its orientation switches in cultures where numbers and words are read right-to-left (Shaki, Fischer, & Petrusic, 2009).

The horizontal number line is but one in a system of spatial-numerical associations, including a bottom-to-top association with vertical space (e.g. Hartmann, Grabherr, & Mass, 2012). Spatial dispositions may even play a role in more complex tasks: Mental arithmetic, for instance, induces systematic dispositions to respond spatially, with addition biasing responses rightward and subtraction biasing responses leftward (Knops et al, 2009), even during exact, symbolic calculation (Marghetis et al, 2014).

A back-to-front sagittal number line?

These spatial dispositions seem to reflect asymmetries in spatial experience. The vertical axis is oriented in virtue of the asymmetry of our bodies, as well as by experiential correlations between *more* and *up* (Lakoff & Núñez, 2000). And while our bodies are bilaterally symmetric, the horizontal axis is oriented by cultural practices such as reading and counting (e.g., Shaki, Fischer, & Petrusic, 2009). It is surprising, therefore, that little attention has been given to dispositions along an especially prominent axis: the sagittal axis, running from behind the body to the front.

The sagittal axis, after all, is striking in its asymmetry. Things in front can be seen, heard, touched; things behind are far more difficult to access. The sagittal axis, moreover, *is* associated with another abstract domain: Time (see Núñez & Cooperrider, 2013, for review). In English, language typically reserved for sagittal space is used to talk about time (e.g. look *forward* to the future); co-speech gestures also reflect this pattern; and temporal reasoning induces dispositions to move forward or backward (*ibid*). Language, gesture, and thought all enact a sagittal timeline.

What about number? In English, decreasing or increasing counting is most often described spatially as counting *down* or *up*, but also sometimes as counting *backward* or *forward*. But there is little evidence that we actually conceptualize number using the sagittal axis, using a back-to-front sagittal number line (SNL). Some suggestive evidence comes from the so-called "vertical" SNARC, perhaps more accurately described as a *radial* effect: Responses to smaller numbers are faster in near space, and to bigger numbers, in far space (Ito & Hatta, 2004). However, this radial SNARC effect has always been tested with response buttons placed directly in *front* of the body, thus confounding location along the sagittal axis with *distance from the body*; larger numbers may just prime responses that are *farther away.* Hartmann et al (2012) tried to find evidence for an unambiguously back-to-front representation of number. Their Swiss participants

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generated significantly higher random numbers (1-30) while experiencing upward (compared to downward) or rightward (compared to leftward) motion—but *not* while experiencing forward (compared to backward) motion. In contrast, Seno et al (2011) reported that Japanese and Chinese participants generated significantly higher numbers while experiencing a visual illusion of *backward* movement, compared to an illusion of *forward* motion—a reversal of the way numbers are described in English. It is unknown whether this effect extends to Western populations.

These results suggest that a sagittal representation of the natural numbers, if it exists, may be fragile or task-dependent. Upon reflection, the sagittal axis is an unlikely model of the natural numbers: the body divides space categorically; natural numbers are undivided. The *integers*, on the other hand, share many structural properties with the sagittal axis: positive and negative are separated by zero; front and back, by one's body. Might we use the sagittal axis to conceptualize the integers?

The current study

Three experiments investigated the possible existence of a back-to-front sagittal number line (SNL). Participants judged the magnitude (Exp. 1 and 2) or parity (Exp. 3) of positive and negative integers, and responded by stepping forward or backward in space. We foresaw a number of possible outcomes. For one, there could be no systematic association between numbers and sagittal space; the spatialization of number might be exhausted by known horizontal, vertical, and radial dispositions. Second, both natural numbers and integers alike might prime forward and backward motion, perhaps due to so-called *Polarity Correspondence* (Proctor & Cho, 2006): Numbers and the sagittal axis both have a clear orientation, so responses might be facilitated when their "polarity" is aligned. Finally, if there is a specifically *numerical* association between integers and sagittal space, then we may find evidence of selective, systematic spatial dispositions: forward for positive numbers, backward for negative numbers, with zero mapped to the natural origin of the body.

Experiment 1

Participants

Undergraduate students (n=32, mean age = 21, 22 women) participated in exchange for partial course credit.

Procedure

In a fully within-subjects design, each participant completed two tasks—the Integer and Whole Number tasks—with task order counterbalanced between participants. In both tasks, participants had to judge the relative magnitude of visually-presented single-digit numbers. They responded by stepping forward or backward onto rectangular, colored targets on the floor (Fig. 1, left panel). Targets were approximately two feet in front of (yellow) or behind (red) a central foot-pedal and were described by their color, not their location.

In the ***Integer task***, participants judged whether integers from -9 to 9 (not including 0) were greater or less than 0. In the ***Whole Number*** **task**, they judged whether positive integers from 1 to 9 (not including 5) were greater or less than 5. The Whole Number task was thus similar to the classic SNARC paradigm (Dehaene et al, 1993).

Each task consisted of two blocks. In one block, participants moved forward for *greater* numbers (i.e. greater than 0 or 5), and backward for *lesser* numbers; response assignment reversed in the other block; block order was randomly assigned.

Each trial begin when an image of a shoe appeared on a computer monitor, approximately four feet ahead of the participant, which prompted the participant to depress a foot-pedal with their right foot. Once the foot-pedal was depressed, a central fixation cross appeared for 500ms, followed by single-digit numeral. The numeral disappeared when participants lifted their foot to begin their response, or after 5000ms. Reaction time was measured, via the foot-pedal, from stimulus onset to response initiation (Fig. 1). Response direction was recorded online by an experimenter in the room. Participants were instructed to begin moving only once they had made their decision; if participants changed direction after initiating their response, the trial was discarded. In each block, each number was presented 10 times, in random order. Each block began with 8 practice trials, for a total of 176 trials in the *Whole Number* task and 376 trials in the *Integer* task. Participants were allowed to rest between tasks. The entire experiment took approximately 30 minutes.

Results

Four participants did not complete both tasks and were replaced before analysis. Overall accuracy was quite high (M = .97). A 2x2x2x2 mixed-design ANOVA, with Magnitude (greater or less than reference), Direction (forward or backward), and Task as within-subjects factors, and Task Order as a between-subjects factor, revealed only one significant effect, a two-way interaction between Magnitude and Direction, $F_{(1,29)} = 5.3$, $p < .029$, $\eta_p^2 = .15$. Participants responded correctly to larger numbers more often when moving forward than backward (M = .982 *vs.* .971), and to smaller numbers more often when moving backward than forward (M = .973 *vs.* .969).

Before analyzing response times, incorrect trials were removed, followed by trials with reaction times that were

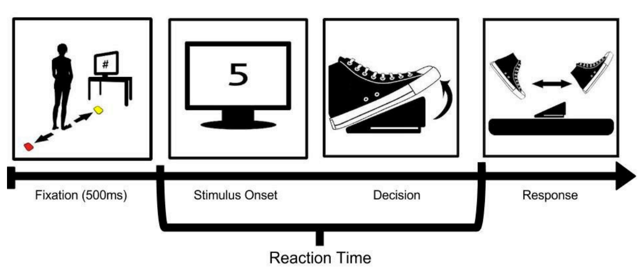

Figure 1: Procedure for all three experiments. Reaction time was recorded via foot-pedal.

slower than three standard deviations above each participant's mean response time in each condition (3% of trials). In addition, one participant was removed due to exceptionally low accuracy, answering only 73% of the trials correctly (accuracy > .92 for all other participants).

We again conducted a 2 (Magnitude) x 2 (Direction) x 2 (Task) x 2 (Task Order) mixed-design ANOVA. Participants were faster overall during the Integer than the Whole Number task (*M* = 441ms *vs* *M* = 466ms), $F_{(1,29)}$ = 4.2, *p* = .05, η_p^2 = .13. They were also faster to respond to smaller than to larger numbers (*M* = 448ms *vs* *M* = 458ms), $F_{(1,29)}$ = 12.0, *p* = .002, η_p^2 = .29. And there was a marginally significant effect of direction, with faster responses backward than forward (*M* = 448ms *vs* *M* = 458ms), $F_{(1,29)}$ = 4.1, *p* = .053, η_p^2 = .12.

Only two other effects approached significance. There was a two-way interaction between Magnitude and Direction, $F_{(1,29)}$ = 4.4, *p* = .046, η_p^2 = .13. Backward responses were faster for smaller than for larger numbers (*M* = 433ms *vs.* 462ms), while forward responses were faster for larger than for smaller numbers (*M* = 453ms *vs.* 463ms).

This interaction, however, was complicated by a 4-way interaction between all four factors. It is easy to see why: the two-way interaction between Magnitude and Direction was driven almost entirely by the Integer task, and only when the Integer task was completed first (Fig. 2). This was confirmed by four follow-up 2 (Magnitude) x 2 (Direction) ANOVAs, performed for each Task and Order. When the Integer Task was performed first, the two-way interaction between Magnitude and Direction was highly significant ($F_{(1,15)}$ = 9.9, *p* < 0.007, η_p^2 = .40), with responses to negative numbers an average of 49ms faster when moving backward than forward ($t_{(15)}$ = -3.0, *p* = 0.01) but responses

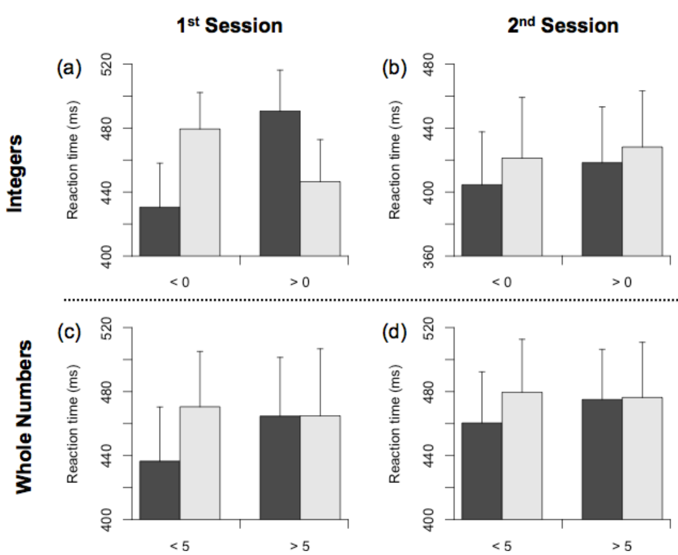

Figure 2: In *Experiment 1*, the relation between numerical magnitude and response direction differed by task (rows) and order (columns). (a) When participants completed the Integer task first, there was a highly significant interaction. (b-d) In no other case was there an interaction.

to positive numbers an average of 44ms faster when moving forward ($t_{(15)}$ = -2.5, *p* = 0.02; see Fig. 1a). By contrast, the interaction did not even approach significance when the Integer task was performed second, or for the Whole Number task ever (all *F*s < 1.1, *p*s > .3, η_p^2s < .071).

To further characterize this selective association between Magnitude and Direction, we performed a regression analysis of reaction times on the Integer Task, when it was performed first, adapting an approach used to analyze the classic horizontal SNARC (Fias et al, 1996). For each subject and number, we calculated the difference between backward and forward responses (dRT). If dRT is positive, backward responses are faster than forward responses. Then, for each subject, we regressed dRT onto numerical magnitude. The slopes of these linear regression lines index the orientation and intensity of the association: negative slopes indicate that smaller numbers are associated with the back, larger numbers with the front. As predicted, the slopes of these regressions were negative for nearly everybody (13/16, *p* = 0.02, binomial test), and overall the slopes were significantly different from zero (M_β = -7.7, $t_{(15)}$ = -3.4, *p* = .004). There was evidence, therefore, that nearly every participant associated negative numbers with the space behind them, positive numbers with the space ahead.

Discussion

Experiment 1 demonstrated a novel effect: a *sagittal number line*. Negative numbers were associated with the space behind the body, and positive numbers with the space in front. This effect, moreover, was restricted to the Integer task; there was no evidence of systematic spatial dispositions during the Whole Number task, which did not involve negative numbers. In fact, note that the stimuli in the Whole Number task (1 to 9) were identical to the *greater* half of the stimuli on the Integer task (i.e. integers greater than 0). The exact same numbers, therefore, were associated with front space when they were processed in the context of negative numbers (Integer task; Fig. 2a, right bars) but had no associations—or even a slight association with the rear—when processed on their own during the Whole Number task (Fig. 2c, d). The SNL, therefore, may be limited to instances where both positive and negative integers are considered together as part of the task.

However, the two tasks differed in more ways than the mere presence of negative integers. First, stimuli in the Whole Number task ranged only from 1 to 9 (range = 8) while those in the Integer task ranged from -9 to 9 (range = 18). Second, participants could succeed at the Integer task simply by checking for the presence of a minus sign (e.g. -4 vs 4), while the Natural Number task required access to the magnitude represented by the numeral. This categorical strategy for the Integer task may have induced associations between space and magntiude based entirely on the dimensions' "polarity" (cf. Proctor & Cho, 2006).

To remove these two confounds, the Whole Number task in Experiment 2 used numbers from 11 to 29, judged relative to 20. These numbers have the same range as those in the original Integer task, and their relative magnitude (i.e. greater or less than 20) can be determined from the first digit alone (e.g. 11 *vs*. 21). If the interaction in Experiment 1 was an artifact of a categorical strategy rather than evidence of sagittal representation of the integers (i.e.. an SNL), then we should find an analogous interaction for this modified Whole Number task. If, on the other hand, the interaction reflected selective spatial dispositions for positive and negative integers, then we should find the effect only when the task involves negative integers.

Experiment 2

Participants

Undergraduate students (n = 32, mean age = 21, 22 women), who had not participated in the first experiment, participated in exchange for partial course credit.

Procedure

In a between-subjects design, participants completed either an *Integer* task or a modified *Whole Number* task. The procedure for the Integer task was identical to Experiment 1. For the Whole Number task, the procedure was modified in two ways: the stimuli ranged from 11 to 29 (instead of 1 to 9) and numbers were compared to 20 (instead of 5). The tasks were therefore matched in two ways. First, participants could complete the task by attending only to the most rightward symbol ("1" or "2"; presence or absence of a negative sign). Second, the stimuli had an identical range.

Results

Overall accuracy was quite high (M = .96), and did not differ by task, $t_{(30)}$ = 0.66, p = .51. A 2x2x2 mixed-design ANOVA, with Task as a between-subjects factor, and Magnitude (greater or less than) and Direction (forward or backward) as within-subjects factors, revealed no significant main effects or two-way interactions (all Fs > 2.2, ps > .15).

There was, however, a marginally significant three-way interaction between Task, Direction, and Magnitude, $F_{(1,30)}$ = 3.9, p = .057. To unpack this three-way interaction, we conducted follow-up 2 (Magnitude) x 2 (Direction) within-subjects ANOVAs for each task separately. For the Integer task, there was a significant interaction between Magnitude and Direction, $F_{(1,15)}$ = 7.28, p = .017. Accuracy was higher for negative numbers when moving *backwards* than forward (M = .97 *vs.* .93); accuracy was higher for positive numbers when moving *forward* than backwards (M= .98 *vs.* .94). For the Whole Number task, by contrast, there was no such interaction, $F_{(1,15)}$ = 0.47, p = .51. Thus, there was a task-specific interaction between space and numerical magnitude, with better accuracy when negative numbers were mapped to the space behind the participants, and positive numbers mapped to the space in front.

We next analyzed reaction times. As in Experiment 1, before analyzing response times, incorrect trials were removed, followed by trials with reaction times that were slower than three standard deviations above each participant's mean response time in each condition (5% of trials).

Reaction times were analyzed with a 2 (Magnitude) x 2 (Direction) x 2 (Task) mixed-design ANOVA. Once again, there was a main effect of Direction: backward responses (M = 441ms) were faster than forward responses (M = 458ms), $F_{(1,30)}$ = 21.3, p < .001, η_p^2 = .42. The only other significant effect was a three-way interaction between Task, Magnitude, and Direction, $F_{(1,30)}$ = 9.72, p = 0.004, η_p^2 = .24. Follow-up analysis of each individual task, using 2 (Magnitude) x 2 (Direction) repeated-measures ANOVAs, revealed that this was driven by a significant two-way interaction for the Integer task, $F_{(1,15)}$ = 5.82, p = .029, η_p^2 = .28, which approached but did not reach significance for the Whole Number task, $F_{(1,15)}$ = 4.45, p = .052. When moving backwards during the Integers task, participants were significantly faster to respond to negative than to positive integers (M = 442ms *vs.* 404ms), $t_{(15)}$ = -2.357 p = 0.02; by contrast, when moving forwards they were faster to respond to positive than to negative integers (M = 422ms *vs.* 458ms), $t_{(15)}$ = 2.19, p = .04 (Fig. 3).

As in Experiment 1, we performed a regression analysis to further character the association between numerical magnitude and sagittal space. For each subject and number, we calculated the difference between backward and forward responses (dRT) and regressed dRT onto numerical magnitude for each subject. The slopes of these linear regression lines index the direction of the association between number and space: negative slopes indicate that smaller numbers are associated with the space behind the body, larger numbers with the space in front. For the Integer task, as predicted, the mean slope across participants was significantly less than zero, β = -5.8, $t_{(15)}$ = -2.45, p = .027. Approximately 70% of participants (11/16) had a negative slope, comparable to what is typically found for the classic horizontal SNARC (e.g., Cipora & Nuerk, 2013). For the Whole Number task, by contrast, slopes were not significantly different from zero, β = 2.7, $t_{(15)}$ = 1.9, p = .08.

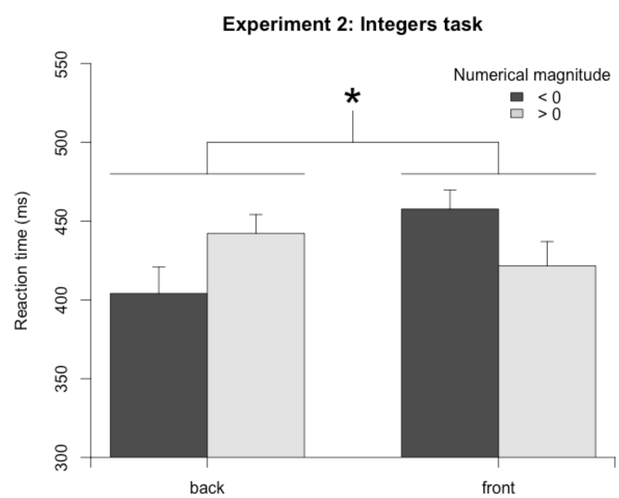

Figure 3: In Experiment 2, negative integers were associated with the space behind the participant, and positive integers with the space in front.

Discussion

Experiment 2 replicated the main finding of Experiment 1—a back-to-front SNL—and ruled out a number of deflationary accounts of the difference between the first Whole Number and Integer tasks. Furthermore, the lack of a reliable interaction on the Whole Number task in Experiment 2 reinforces the genuinely *numerical* quality of the SNL. Since stimuli in both tasks had an identical range (-9 to 9 and 11 to 29), it is unlikely that they differ solely in depth of processing (e.g. due to the Distance Effect). And since participants could complete both tasks by attending only to the leading symbol, it is unlikely that the effect in the Integer task was due solely to a categorical strategy or Polarity Correspondence (Proctor & Cho, 2006). Integers are spontaneously, systematically associated with sagittal space, but only when negative integers are involved.

Experiment 3

The classic SNARC effect is thought to reflect *automatic* activation of spatial information during number processing. Associations between number and horizontal space appear even if the task does not require participants to process numerical magnitude—for instance, deciding if numbers are even or odd (Dehaene et al, 1993). Is the SNL similarly automatic, or does it require explicit attention to magnitude?

To answer this question, Experiment 3 introduced a Parity task (i.e. even *vs.* odd) for negative and positive integers. If the SNL is automatic—activated autonomously when processing integers—then numerical magnitude should interact with spatial location during both Magnitude (greater or less than 0) and Parity (even *vs.* odd) tasks.

Participants

Undergraduate students (n = 32, mean age = 20, 26 women), who had not participated in the first two experiments, participated in exchange for partial course credit.

Procedure

In a between-subjects design, participants participated in one of two tasks. The *Integer* task was unchanged from Experiments 1 and 2. In the new *Parity* task, participants had to determine the *parity* (even *vs.* odd) of integers ranging from -9 to 9, not including zero. All other details of the design (number of trials, timing, etc.) were identical to the Integer task. Task assignment was counterbalanced.

Results

Accuracy was high and did not differ between tasks (both Ms = .96, $t_{(15)} = 0.07$, $p = .9$). A mixed-design ANOVA with Magnitude and Direction as within-subjects factors, and Task as a between-subjects factor, revealed only a main effect of direction, $F_{(1,30)} = 6.25$, $p = 0.02$, $\eta_p^2 = .17$.

Once again, incorrect trials were removed, followed by trials with reaction times more than three standard deviations from the participant's mean response time in each condition (5% of trials). To analyze reaction times, we performed a 2 (Magnitude) x 2 (Direction) x 2 (Task: Magnitude *vs.* Parity) mixed-design ANOVA. There was a significant main effect of Direction, $F_{(1,30)} = 10.2$, $p = .003$, $\eta_p^2 = .25$. The only other significant effect was the predicted two-way interaction between Magnitude and Direction, $F_{(1,30)} = 4.66$, $p = .04$, $\eta_p^2 = .13$. Backward responses were faster for negative integers (M = 475ms *vs.* 489ms), while forward responses were faster for positive integers (M = 504ms *vs.* M = 489ms). Crucially, there was no three-way interaction with Task. Indeed, separate regression analyses for each task found that the mean slope was less than zero in both tasks (Magnitude: $\beta = -3.5$; Parity: $\beta = -1.0$), that is, increasing integers were associated with back-to-front sagittal space regardless of the task. Though the slope was more negative for the Magnitude task, suggesting a more pronounced association, this difference was not significant ($t_{(30)} = -1.16$, $p = .25$). Moreover, there was no difference between tasks in the number of participants with negative slopes (12 *vs.* 11, $p > .9$, Fisher Exact test).

General Discussion

Are we pierced by a number line? Three experiments suggest that the answer is *yes*. We established and then twice-replicated a novel effect: Negative numbers were associated with the space behind the body, and positive numbers, the space in front. These sagittal spatial dispositions, it seems, require the presence of negative numbers; in two experiments, we failed to find evidence of sagittal dispositions for positive numbers on their own.

There was also a recurring main effect of stepping direction, with consistently faster responses *backward*. This was likely due to our particular experimental set-up, though we cannot exclude the possibility that undergraduate students have a targeted aversion to numbers.

We turn now to two issues. First, how might these spatial dispositions relate to other spatial-numerical associations? Second, where might such dispositions come from, and what impact might they have?

Relation to other spatial dispositions

The multiplicity of linear construals of number—horizontal, vertical, and now sagittal—evokes the varied spatial construals of time (e.g Núñez & Cooperrider, 2013). In *external* spatial representations of time, the spatial axis does not include the body—such as when temporal sequences are conceptualized as a left-to-right path in front of the speaker. All previously documented mental number lines fall into this category: they involve paths outside the body, whether left-to-right, right-to-left, or bottom-to-top. In *internal* spatial representations, by contrast, the body is part of the representation. Time, for instance, can be conceptualized as running from back to front, with "now" identified with the body. The SNL appears to be the first linear representation of number of this sort, with the body dividing positive from negative integers and perhaps anchored to zero. Núñez and Cooperrider suggest that, for time, external representations may require extensive cultural scaffolding, while internal representations may appear more spontaneously. That seems unlikely for the integers, where even an internal representation—an SNL—is likely to require extensive cultural support.

Origins and implications

Where could this SNL come from? On a deflationary account, participants may merely have guessed the purpose of the experiment. While we cannot rule this out entirely, the repeated null findings for the natural numbers, for which there are well known spatial associations, make it unlikely the effect was driven entirely by participants surmising the study's purpose. The SNL could derive from conventional expressions (e.g. "count backward or forward"), though this is unlikely given how infrequent they are. An SNL could also emerge from explicit analogical reasoning. The sagittal axis and the integers share considerable structure: a single dimension; a privileged reference point; transitive relations between elements. These similarities might make them particularly ripe for structural alignment. Lastly, both integers and time might tap into a more general association between sagittal space and *sequences*, especially sequences with a privileged reference point (e.g. now, zero). Sagittal dispositions might therefore exist for various sequences of all kinds (e.g. temperature, intimacy).

The existence of an SNL may have a variety of implications. Spatial dispositions may scaffold the acquisition of number concepts, supporting children's early sense-making. The SNL might thus be a productive target for educational intervention, much like recent interventions that improved arithmetic by training associations between numbers and horizontal space (e.g. Siegler & Ramani, 2009). There is evidence, however, that *external* spatial dispositions like the horizontal SNARC play a negligible role during complex mathematics (Cipora & Nuerk, 2013), perhaps because they are displaced by dispositions associated with arithmetic or algebraic manipulation (Marghetis et al, 2014; Goldstone et al, 2010). We thus hypothesize that *internal* spatial construals like the SNL may play a critical role in more advanced mathematics.

Conclusion

Three studies demonstrated that negative numbers are associated spontaneously with the space behind the body, positive numbers with the space in front. These spatial dispositions were only evident when the task involved both positive and negative number. This is, to our knowledge, the first evidence of a sagittal number line for the integers.

Reasoning about integers, therefore, is not entirely abstract, but induces systematic dispositions to act. We may even be tempted to say about the integers what Bourdieu said about honor: that they are "nothing other than the cultivated disposition, inscribed in the body schema and the schemes of thought" (Bourdieu, 1977:15). Of course, this goes too far. The integers outstrip the dispositions we have internalized from a lifetime of experience; they depend on notational systems, axioms, diagrammatic practices, a sociotechnical ecosystem. But these spatial dispositions—our mathematical *habitus*—may nevertheless play a central role in enacting our mathematical conceptual systems.

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