

# Using a Hybrid Cognitive Architecture to Model Children's Errors in an Analogy Task

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## Abstract

We model the performance of children on the Goswami and Brown (1990) analogy task, paying close attention to the distribution of errors children made on the task. This distribution follows a very particular pattern which, as we show, may be simulated by assuming a lack of development in the richness of children's concepts of physical causation. This modeling is done using the hybrid cognitive architecture CLARION, and a method of representing structured knowledge within CLARION's dual-process system.

**Keywords:** analogy; analogical reasoning; reasoning; Piaget; CLARION; cognitive modeling; cognitive architectures

## Introduction

Analogical reasoning is a core component of adult human thought; researchers in cognitive science and artificial intelligence are coming to realize this increasingly (Gentner & Forbus, 2011; Licato, Bringsjord, & Govindarajulu, 2013). But the ability to reason analogically was once seen as a higher cognitive ability that did not emerge fully until Piaget's formal-operations stage of cognitive development. While exploring his concept of reflective abstraction, Piaget performed a set of experiments with children that supported his suspicion that analogy was linked to higher-level cognitive processes (Piaget, Montangero, & Billeter, 2001). Subsequent work by Sternberg and Nigro (1980) and Goldman et al. (1982) added to Piaget's suspicion, eventually transforming it into the hypothesis that 9- and 12-year-olds solve analogies not using analogical reasoning, but by simple low-level association (Goswami & Brown, 1990).

But Piaget himself was not necessarily convinced that analogical reasoning was impossible with young children. He observed cases in which children in the sensorimotor and preoperational stages displayed basic analogical reasoning, even if he thought many of them only provided "approximate analogies from moment to moment." He offered an explanation for the apparent lack of analogical ability in the children he worked with:

[W]e can say that the failure to construct relations between relations, or forms of forms, at the more primitive stages comes about because there are as yet no stable elementary relations (and we mean two-term relations here, not four-term relations). Consequently there are no simple forms that can be expressed in terms of stable classes (Piaget et al., 2001, p.143).

## Goswami and Brown's Experiment

Nevertheless, Goswami and Brown (1990) were not convinced that Sternberg and Nigro (1980) and Goldman et al. (1982) properly controlled for Piaget's apparent belief that two-term stable elementary relations did not exist in young children. For example, one preoperational child brought the pair *cork:bottle* together with *lid:pot* under the justification "cause there's something that comes out of the pot" (Piaget et al., 2001, p.142). Piaget et al. call this a momentary approximation rather than a stable analogy, since the *lid* object could easily be replaced with an *oven*, with the justification "sometimes you heat it with the oven." The relation which links the two pairs, upon which the determination of an analogical match should be made, is easily abandoned by the child.

Goswami and Brown (1990) suspected that the results would be different if two changes were made: First, children should instead be tested only with very basic two-term causal relations; and second, the experiment should verify first that children actually have a stable understanding of those relations. Simple objects with which children were likely to be familiar were also introduced, such as *playdoh:cut playdoh* and *apple:cut apple*. The relation used here, that of cutting an object, is typical of the basic, physical transformations used in the experiment.

The experiment was designed to rule out the possibility of mere association matches being made over the correct, analogical matches. For each analogy problem, an **a**, **b**, and **c** term were given as pictures, in the typical  $a : b :: c : ?$  proportional-analogy style. Children were asked to choose

between the following, where the examples we provide are from the *playdoh:cut playdoh::apple:?* problem:

- **d** - Correct choice (*cut apple*).
- **e** - Correct transformation, wrong object (*cut bread*).
- **f** - Wrong transformation, correct object (*bruised apple*).
- **g** - Mere appearance match of the *c* term (*ball*, where the ball's size and shape are similar to those of the apple).
- **h** - Semantic/category match of the *c* term (*banana*).

The analogy experiment was divided into two phases. In the first phase (the induction phase), children were simply asked to choose the correct answer, and were told whether the choice they made was correct or not. If it was incorrect, the experimenter would show the child which was the correct answer, and not explain any further. In the second phase (the explanation phase), after choosing, the child would be given an explanation of why the correct answer applies, whether or not the choice the child made was correct.

Their results were positive: A clear demonstration of analogical ability was shown in children as young as 3 or 4 years of age, with a significant difference between them, and the performance by 6-year-olds was nearly perfect. No other age groups were tested. Perhaps more relevant to our current work, however, is the distribution of errors made by the 3- and 4-year-olds, which can be seen in Table 1.

Table 1: Percentage of children who passed criterion, and errors made by children on the Goswami and Brown (1990) analogy task. Errors presented as a percentage of total errors. Error data from the 6-year-old age group were not provided in the original paper, as the number of errors was deemed too small to be significant. Reprinted with permission.

Percentage Passing Criterion (Children)					
Age (years)		Induction	Explanation		
3		28	52		
4		73	90		
6		95	100		

Errors Made (Children)					
Age (years)		E	F	G	H
3	Induce	21	45	18	16
	Explanation	23	39	17	21
4	Induce	11	66	10	13
	Explanation	30	63	3	3

This distribution of errors is interesting, because it might be able to provide us some insight into better modeling of analogical reasoning in cognitive architectures. What simulation-level details can account for the error distribution

seen in Table 1? This question is the focus of our work in this paper.

The remainder of this paper will first briefly describe CLARION, the cognitive architecture chosen for this modeling task. We then need to touch on CLARION's NACS subsystem, and the style of structured knowledge representation which gives CLARION a unique ability to model many of the phenomena that are important in this task. We describe the steps we took to model the error distribution seen in Table 1, and then close with a brief discussion.

## CLARION

CLARION (Sun, 2002) is an integrative cognitive architecture that has a dual-process structure; that is, it consists of two levels: explicit (top level) and implicit (bottom level). CLARION has been able to model a wide variety of cognitive phenomena while maintaining psychologically plausible data structures and algorithms (Sun, 2001; Sun & Zhang, 2004, 2006). The dual-process representational system of CLARION allows for both localist concepts, such as those presented in the Goswami and Brown (1990) experiment, and distributed knowledge, which is a natural way of representing low-level associations between concepts (Sun, 2002). This makes it an ideal choice for our purposes.

The architecture is further divided into four *subsystems*, each with explicit and implicit levels, which specialize in different aspects of cognition: the Motivational Subsystem (MS), the Metacognitive Subsystem (MCS), the Action-Centered Subsystem (ACS), and the Non-Action-Centered Subsystem (NACS). Work described in this paper will only be using the NACS.

### NACS — the Non-Action-Centered Subsystem

The NACS contains general knowledge about the world that is not contained in the ACS. Whereas the ACS is meant to capture the knowledge that directly control actions while interacting with the world, the knowledge in the NACS is often more deliberative and used for making inferences. The top level of the NACS is called the General Knowledge Store (GKS), and it contains localist chunks that can be linked to each other using Associative Rules (ARs).

The bottom level of the NACS is called the AMN, or the Associative Memory Network, and it contains implicit associative knowledge encoded as dimension-value pairs (DV pairs). Each GKS chunk is connected to a set of DV pairs in the AMN with weights that can be adjusted over time. This unique structure gives CLARION the ability to define a *directed* similarity measure between two chunks  $c_1$  and  $c_2$  which is derived from the amount of overlap between the DV pairs connected to the two chunks (Sun, 1995; Tversky, 1977; Sun & Zhang, 2004). However, in this paper we will be using a simplified similarity function based on the number of dv pairs connected to  $c_1$  and  $c_2$ :

$$S_{c_1 \rightarrow c_2} = \frac{|c_1 \cap c_2|}{|c_2|^{1.0001}} \quad (1)$$

Note that it is possible for the denominator in Equation 1 to be zero, in which case the entire equation is given the default value of 1. It is the measure in Equation 1 which we will use in this paper to determine low-level associative similarity between chunks.

The Associative Rules (ARs) link groups of chunks to other chunks in the GKS, and consist of a set of condition chunks  $c_1, c_2, \dots$  and a single conclusion chunk  $d$ . For any given AR, each condition chunk  $i$  has a weight  $W_i$  such that  $\sum_i W_i = 1$ .

The chunks in the GKS and DV pairs in the AMN have activation levels which can be set by CLARION's other subsystems. Activations can also spread through the NACS using the chunk-DV pair connections and the top-level ARs. The manner in which this activation spreads can be restricted, as other subsystems can temporarily disable Rule-Based Reasoning (activation spreading through ARs) or Similarity-Based Reasoning (activation spreading through chunk similarity), or perform activation propagation as some weighted combination of both of these reasoning types. These abilities are detailed further in Sun & Zhang (2004, 2006), in which these mechanisms are shown to be psychologically plausible.

Structural knowledge in CLARION is represented through combining ARs with Cognitively Distinguished Chunks, or CDCs. CDCs are depicted as star-shaped in our diagrams (Figure 1). Associative rules link the CDCs to the chunks in the structure. For example, the *WHOLE* CDC links object nodes to proposition nodes. In Figure 1, which depicts the proposition *CHASES(DOG CAT)*, the *WHOLE* CDC is part of two ARs (depicted in the Figure as an arrow with multiple tails and one head).

A *COMPONENT* CDC is also defined, such that for every rule involving a *WHOLE* CDC, a complementary rule going in the other direction is created with a *COMPONENT* CDC. Whole chunks are always pictured above component chunks. Next, we introduce *Ordinal CDCs*, which are also pictured in Figure 1 as *1ST*, *2ND*, etc. Ordinal CDCs simply preserve the roles objects play within propositions in a general way that does not name the roles specifically (contrast this with the LISA model (Hummel & Holyoak, 2003), which has distinct role units for every type of role.).

## Analogical Reasoning in CLARION Using Templates

Analogical and deductive reasoning in CLARION can be done using the *template* form. Templates are groups of chunks that both specify what constitutes an acceptable form match and how to transform the input when such a match is found. Chunks can also exist in templates that have zero semantic content. These are called "blank chunks," and will be used when matching templates to other structures. In Figure 2, blank chunks are pictured as chunks without any labels.

Given some template and a set of structured chunks, before performing analogical reasoning using the template system, a set of source chunks has to first be collected. These chunks can then be transformed into a template using an algorithm

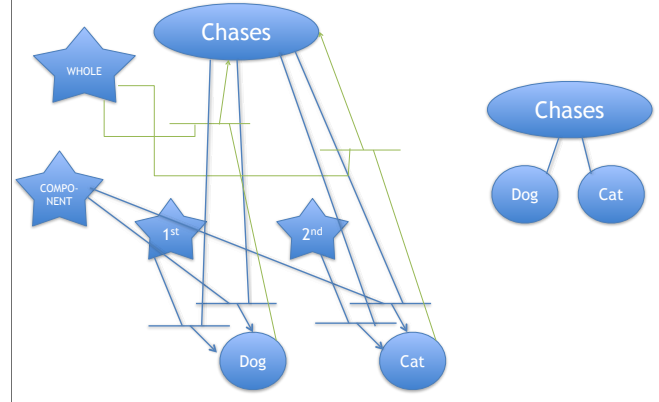


Figure 1: A knowledge structure representing the proposition *CHASES(DOG, CAT)*. On the right is the simplified version, which omits the CDCs and many of the ARs, though they are there (just not pictured). Note that in both versions, only chunks are pictured. The bottom-level DV pairs which each chunk is connected to are omitted for simplicity.

that tries (in parallel) different transformations of the source chunks into templates. For the purposes of this paper, a simple version of this algorithm is used that simply transforms the object-level chunks (objects **a** and **b**) to blank chunks.

Actually determining whether the template applies to the chunks is a nontrivial algorithmic problem. We solve this by using an Ant Colony Optimization (ACO) algorithm based on (Sammoud, Solnon, & Ghédira, 2005). The algorithm starts by collecting the template and target chunks, and drawing *eligibility links* between pairs of chunks that can potentially be mapped to each other. It determines this by using the similarity metric defined in Equation 1. If that similarity is above a certain minimum similarity level  $\epsilon$ , then an eligibility link is drawn. Varying  $\epsilon$  allows us to approximate the rigidity of the matching requirements, so that a lower value encourages more creative matchings, while a higher value forces more structural consistency between the structures being matched. Eligibility links are automatically drawn from blank chunks to other chunks in the target.

The eligibility links are then selected by the ants in a bottom-up fashion, starting with the object-level chunks. As each level is completed, the eligibility links remaining in the upper levels are re-evaluated. This is because the choices made at the lower levels may not be consistent with choices on the upper levels (lest structural consistency between the template and target structures be violated).

The ACO algorithm which matches templates to targets can then be used. Our ACO implementation is described in more detail, along with a more thorough explanation of templates, in (Licato, Sun, & Bringsjord, 2014). The authors are not unaware of the level of compressed descriptions in this paper regarding the simulation, but due to space many technical details had to be omitted from this paper.

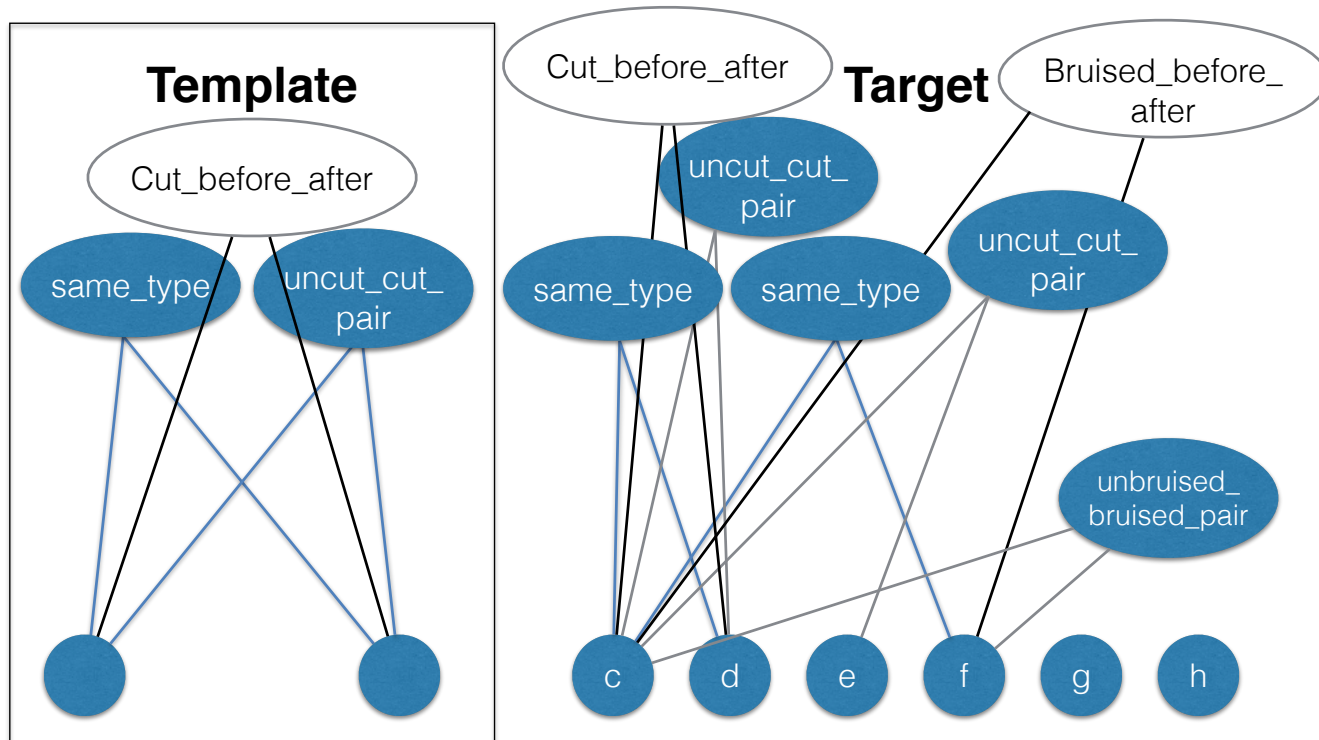


Figure 2: The template and target representation used in our simulation. DV pairs are not pictured here. Units pictured with dotted lines are only present for the simulation of 6-year-old reasoning, whereas the rest of the units were present for all simulated age groups.

### Matching Children's Errors

The period between 3- and 4-years of age is one of rapid cognitive development in many areas, among these the ability to understand, and reason about, physical causality (Das Gupta & Bryant, 1989; Frye, Zelazo, & Palfai, 1995). The increase in this particular ability is reflected in the data from Table 1 (Goswami & Brown, 1990). The question before us, then, is: Exactly what changes occur at CLARION's level of abstraction that plausibly explain increased understanding of physical causality? The obvious first answer is quantitative: We can either increase  $\epsilon$  (recall, this is the minimum similarity required for an eligibility link to be drawn in the matching algorithm), or we can increase the number of ants and iterations used in the ACO algorithm. Initial testing, however, showed that a tweaking of these parameters was not enough to explain the distribution of errors seen in Table 1. In particular, it did not replicate the fact that **f** and **e** errors are by far the most common, in that order. Something else had to be explored in addition to the quantitative features. As it turns out, there are some clues in the psychological literature that hint of a *qualitative* change as well. That change is a development in the conceptual representations used by the children.

The idea of qualitative change over time is a fundamental one in Piaget's constructivist philosophy (Piaget, 1977). Several systems implement constructivist views by recruiting

new units as a normal part of the learning process, such as the neurologically plausible model DORA (Doumas, Hummel, & Sandhofer, 2008), and those built on cascade-correlation neural networks (Shultz, 2003; Shultz & Sirois, 2008). Such systems increase their representational power dynamically, leading to qualitative changes in the expressivity of the knowledge representation used by the system, and giving them clear advantages over static models (Shultz, 2012). The cascade-correlation model has also been used to effectively model Piagetian tasks (Shultz, Mareschal, & Schmidt, 1994; Shultz, 2003).

It makes sense, then, that a qualitative change in the representations used might offer an explanation for the Table 1 data. Assuming a constructivist (as opposed to a Fodorian nativist (Fodor, 2008)) theory of cognitive development, concepts such as those used in the Goswami and Brown (1990) experiments are built and enriched from more primitive constructs. It should not be too controversial to suggest, then, that perhaps part of what explains the rapid increase in understanding of physical causation after age 4 is a development in the relevant conceptual constructs themselves. In other words, perhaps the 3- and 4-year-olds are reasoning using a primitive version of the concepts being tested, which are not fully chunked, not fully explicit, and still decomposable into their constituent concepts; whereas the 6-year-olds, who per-

formed at or close to ceiling, have much richer conceptual representations.<sup>1</sup>

We will use the *playdoh:cut playdoh::apple:?* problem as an example. The relevant relationship being tested is *Cut*, a rich concept consisting of: before and after states, a division in the object in the after state characterized by a clean slice, which was likely done with a sharp blade, etc. If the *Cut* concept is built from observations, it is possible that these primitive concepts are those from which the *Cut* concept is built. If so, does the representational network of concepts used by children still make use of these primitive concepts?

The abundance of **e** and **f** errors might be explained via the use of primitive conceptual structures. Instead of structuring our simulation such that items **c** and **d** are linked by a single predicate *Cut*, we link them with two separate concepts: *Same\_Type* (since they are both apples), and *Cut\_Uncut\_Pair*, which simply links two objects such that one is uncut and the other is. Now **c** (an uncut apple) and **e** (cut bread) can be linked with the *Cut\_Uncut\_Pair* predicate, and **c** can be linked with **f** (a bruised apple) by way of the *Same\_Type* predicate. Furthermore, we can link **c**, **f** with an *Unbruised\_Bruised\_Pair* predicate and have that predicate share DV pairs with *Cut\_Uncut\_Pair* to reflect the semantic similarity between bruising and cutting.

This representation is pictured in Figure 2. Note that the source analog (the **a** and **b** chunks and all relationships between them) was transformed into a template such that the object-level chunks were blank. We were then able to execute the model by matching this template to the representation consisting of objects **c** through **h**. Most of the time, the chunks corresponding to **c** would be mapped to **a**, and whichever object's chunks mapped to **b** was taken to be the answer selected by the system. Also note that there are additional chunks which were only present for the representations used for the 6-year-old group's simulation, which is meant to reflect more highly developed *Cut* and *Bruise* concepts in the 6-year-olds.

In order to reflect object-level similarities shared between the **c** and **g** objects, we simply created a set of DV pairs that were uniquely shared by the chunks corresponding to **c** and **g**. A parallel move was made for **c** and **h**. A parameter  $\delta$  was introduced, which represents how frequently the system resorts to using the sort of low-level associative reasoning children were thought to reason with in place of analogy (Sternberg & Nigro, 1980). In CLARION, this type of reasoning is called Similarity-Based Reasoning (SBR) (Sun & Zhang, 2004). Our simulation used SBR to select an answer whenever either a randomized variable was greater than  $\delta$ , or analogical reasoning produced a mapping that was inconsis-

tent or incomprehensible; this is meant to represent instances where analogical reasoning fails and the subject must resort to other reasoning methods.

**Simulation** Recall now that there are four quantitative parameters that were varied in this simulation: the number of ants and rounds used by the ACO algorithm, the  $\epsilon$  value representing the minimum similarity required for an identity link, and the  $\delta$  value which set the frequency of SBR.

For the 3-year-old group, a simulation of 20,000 iterations was done using one ant, one round, and  $\epsilon$  was randomly chosen from the  $[0.15, 0.3]$  range (one selection was made for each simulated subject) to introduce variation. The value of  $\delta$  used was 0.15. Another simulation of 20,000 iterations was done to simulate the 4-year-old group, which used five ants, five rounds,  $\epsilon \in [0.25, 0.4]$ , and  $\delta = 0.05$ . Finally, for the 6-year-old group, the parameters were almost exactly the same as the 4-year-old group; the only changes were the addition of relationship chunks as pictured in Figure 2, and the setting of  $\delta = 0$ . The results are presented in Table 2.

Table 2: Percentage of correct choices and errors made by the simulation. Errors are presented as a percentage of total errors.

#### Percentage Passing Criterion (Simulation)

Age (simulated years)	Passed
3	41
4	74
6	98

#### Errors Made (Simulation)

Age (simulated years)	E	F	G	H
3	23	45	16	16
4	24	56	10	10
6	0	99	1	0

## Conclusion

Significant features of the human data were preserved in the simulation. For example, **f** errors were more frequent than **e** errors, and the ratio of correct answers to incorrect answers lines up nicely with the human data. We feel that this imparts a certain level of plausibility to our assumptions, but nevertheless, it would be interesting to see if the assumptions made in this paper hold up in other situations. The hand-chosen values of the quantitative parameters aside, perhaps more interesting is our simulation's assumption that part of what explains the difference in performance between 3-, 4-, and 6-year-olds is a qualitative difference in the representational structures used. If toddlers really do use concepts that are not as "well-chunked" as those of older children, and instead use concepts composed largely of separable, primitive components, subsequent experimentation might be able to support or refute this difference. We encourage researchers to exam-

<sup>1</sup>This idea was inspired by the quote reprinted in this paper's introduction, in which Piaget suggests there are "no simple forms that can be expressed in terms of stable classes" (Piaget et al., 2001). Perhaps the idea that in toddlers, concepts are still in a form which is predominantly a loose tying-together of primitive conceptual structures, rather than a solid and robust collection of rich concepts, was what Piaget had in mind.

ine this possibility, by designing experiments to detect and understand the nature of these proto-concepts.

Regardless, the modeling of qualitative increases in representational strength is a promising direction, which our work will continue to explore.

## References

- Das Gupta, P., & Bryant, P. (1989, October). Young Children's Causal Inferences. *Child Development*, 60(5), 1138-1146.
- Doumas, L. A., Hummel, J. E., & Sandhofer, C. (2008). A Theory of the Discovery and Predication of Relational Concepts. *Psychological Review*, 115, 1-43.
- Fodor, J. A. (2008). *LOT 2: The Language of Thought Revisited*. Oxford Univ. Press.
- Frye, D., Zelazo, P. D., & Palfai, T. (1995). Theory of Mind and Rule-Based Reasoning. *Cognitive Development*, 10, 483-527.
- Gentner, D., & Forbus, K. (2011). Computational Models of Analogy. *Wiley Interdisciplinary Reviews: Cognitive Science*, 2(3), 266-276.
- Goldman, S., Pellegrino, J., Parseghian, P., & Sallis, R. (1982). Developmental and Individual Differences in Verbal Analogical Reasoning. *Child Development*, 53, 550-559.
- Goswami, U., & Brown, A. L. (1990). Melting Chocolate and Melting Snowmen: Analogical Reasoning and Causal Relations. *Cognition*, 35(1), 69-95.
- Hummel, J. E., & Holyoak, K. J. (2003). Relational Reasoning in a Neurally-plausible Cognitive Architecture: An Overview of the LISA Project. *Cognitive Studies: Bulletin of the Japanese Cognitive Science Society*, 10, 58-75.
- Licato, J., Bringsjord, S., & Govindarajulu, N. S. (2013). How Models of Creativity and Analogy Need to Answer the Tailorability Concern. In T. R. Besold, K.-u. Kühnberger, M. Schorlemmer, & A. Smaill (Eds.), *Proceedings of the ijcai 2013 workshop on computational creativity, concept invention, and general intelligence*. Beijing, China.
- Licato, J., Sun, R., & Bringsjord, S. (2014). Structural Representation and Reasoning in a Hybrid Cognitive Architecture. In *Proceedings of the 2014 international joint conference on neural networks (ijcnn)*.
- Piaget, J. (1977). *The Essential Piaget* (H. E. Gruber & J. J. Voneche, Eds.). New York, New York, USA: Basic Books, Inc.
- Piaget, J., Montangero, J., & Billeter, J. (2001). The Formation of Analogies. In R. Campbell (Ed.), *Studies in reflecting abstraction*. Psychology Press.
- Sammoud, O., Solnon, C., & Ghédira, K. (2005). An Ant Algorithm for the Graph Matching Problem. In *5th european conference on evolutionary computation in combinatorial optimization (evocop 2005)*. Springer.
- Shultz, T. R. (2003). *Computational Developmental Psychology*. Cambridge, Massachusetts: The MIT Press.
- Shultz, T. R. (2012). A Constructive Neural-Network Approach to Modeling Psychological Development. *Cognitive Development*, 27, 383-400.
- Shultz, T. R., Mareschal, D., & Schmidt, W. C. (1994). Modeling cognitive development on balance scale phenomena. *Machine Learning*, 16, 57-86.
- Shultz, T. R., & Sirois, S. (2008). Computational Models of Developmental Psychology. In R. Sun (Ed.), *The cambridge handbook of computational psychology* (pp. 451-476). New York, New York, USA: Cambridge Univ Press.
- Sternberg, R. J., & Nigro, G. (1980). Developmental Patterns in the Solution of Verbal Analogies. *Child Development*, 51(1).
- Sun, R. (1995). Robust Reasoning: Integrating Rule-Based and Similarity-Based Reasoning. *Artificial Intelligence*, 75(2).
- Sun, R. (2001). From Implicit Skills to Explicit Knowledge: A Bottom-Up Model of Skill Learning. *Cognitive Science*, 25(2), 203-244.
- Sun, R. (2002). *Duality of the Mind: A Bottom Up Approach Toward Cognition*. Lawrence Erlbaum Associates, Mahwah, NJ.
- Sun, R., & Zhang, X. (2004). Accounting for Similarity-Based Reasoning within a Cognitive Architecture. In *Proceedings of the 26th annual conference of the cognitive science society*. Lawrence Erlbaum Associates.
- Sun, R., & Zhang, X. (2006). Accounting for a Variety of Reasoning Data Within a Cognitive Architecture. *Journal of Experimental and Theoretical Artificial Intelligence*, 18(2).
- Tversky, A. (1977). Features of Similarity. *Psychological Review*, 84(4), 327-352.