Individuation vs. Aggregation Strategies for Processing Number Sets

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Abstract

How do people remember individual values from a set of numbers? Previous research has demonstrated seemingly conflicting findings. In some tasks, participants implicitly aggregate number sets (Morris & Masnick, in press), but in other tasks, participants recognized individual values, even for sets of eight, at levels greater than chance (Cravalho, Morris, Was, & Masnick, 2013). In the current paper, we investigated the possibility that these differences are driven by the strategies participants use to achieve different processing goals. The current paper describes three experiments in which participants were given the goal of correctly recognizing individual numbers presented in number sets of varying sizes (four, six, and eight). The results suggest that participants used individuation strategies in which they attended to diagnostic information while encoding numbers (e.g., the ones column in a set of numbers) and that we can explicitly individuate sets larger than four with the use of effective strategies.

Keywords: Numerical Processing; Strategies; Eye Tracking.

Introduction

How do people remember individual numbers when they are presented a set of numbers? Non-symbolic number representation, or “number sense”, occurs in an approximate number system (ANS) that underlies arithmetic operations such as estimation (Lemer, Dehaene, Spelke, & Cohen, 2003; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). The ANS may incorporate two systems, a parallel individuation system and an aggregation system (Hyde, 2011). Evidence for these points comes from two well-replicated findings. First, people individuate items, i.e., represent individual values within a set, for sets smaller than four (Scholl, 2001) and second, given sets larger than four, people aggregate items, i.e., average over sets retaining information about set features (Masnick & Morris, 2008; Scholl, 2001).

The difference between the two systems may be based on strategies (Hyde, 2011) and how we focus our attention (Chong & Treisman, 2005). For example, focusing on a scene would promote aggregation of information, while focusing on an object would promote individuation.

Previous research has suggested that when adults and children compare number sets, participants aggregated number sets and then compared summary values that included approximate means and variance (Masnick & Morris, 2008; Morris & Masnick, in press). Because participants were asked to compare sets, aggregation is consistent with the goal of the task itself. One open question is if the selection of aggregation or individuation strategies is related to the task goal. Cravalho, Morris, Was, and Masnick (2013) examined memory for individual numbers when processing number sets and found evidence that participants were likely to recognize individual values, even for sets of eight, at levels greater than chance. However, these experiments limited the amount of study time before recognition, displayed only two recognition options, and did not record objective data from which to identify processing strategies.

Aims and Predictions

We investigated the types of strategies participants would use when given the goal of correctly recognizing individual numbers presented in number sets of varying sizes. If the subject focused on the global properties of sets, they would aggregate the number set; conversely, if the subject used individuation strategies, they would process information about individual values that would allow them to be recognized correctly.

The independent variable was number set size (four, six, and eight). Dependent variables were number recognition, reaction time, confidence judgments, self-reports of strategy use (Exp. 1 and Exp. 2), and eye fixations (Exp. 3). We predicted that number recognition accuracy would be related to the properties of the set and strategy selection. More specifically, we predicted that smaller set size would be associated with individuation strategy use and larger sets would be associated with aggregation strategies. Further, we predicted that the use of a more diagnostic strategy (i.e., a strategy that focused attention onto critical information to achieve the processing goal) would lead to better number recognition performance than less diagnostic strategies (e.g. focusing on whole numbers). We expected different strategies to emerge as set size increases. For instance, encoding individual numbers (e.g., memorizing numbers) is expected to be associated with sets of four, while encoding set aggregates (e.g., scanning the numbers) is expected to be associated with sets larger than four. Finally, we predicted that confidence judgments would decrease as set size increased.

Experiment 1

Method

Participants. Participants (*N* = 21) were undergraduate students at a Midwestern state university. The average age was 20.43 (*SD* = 1.91), 76% of the participants were female.

Procedure. The number set task consisted of 126 trials presented using E-Prime® software. Each trial had four parts, described here in order of presentation (see Figure 1).

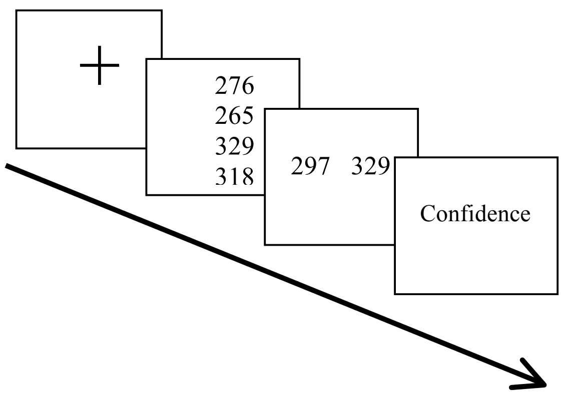

Figure 1: Number set task procedure for Experiment 1.

First, a fixation cross was presented for 500 MS to focus the participant's attention. Then, a number set was shown (details of the sets below). The number set was displayed until the participant hit the spacebar to move on to the next screen. Subsequently, two numbers were presented, an actual value from the set and the mean value from the set. The participant was then asked to indicate which of the two numbers was in the presented set by pressing a computer key. The two numbers and indication prompt remained on the screen for 4 s. After responding, participants were asked how sure they were of their answer. Confidence judgment response options were presented as follows: 1) 0-25%, 2) 26-50%, 3) 51-75%, & 4) 76-100%. The confidence prompt and response options remained on the screen until the participant chose an answer.

There were four sets of stimuli, each preceded by a set of instructions explaining the process outlined above to the participant. The first six number sets were practice trials and were not analyzed. Experimental trials included 40 sets of four numbers, 40 sets of six numbers, and 40 sets of eight numbers, with each set consisting of three digit numbers. Within each set size, 10 sets were drawn from one of two variance types, either 10% (low variance) or 20% (high variance) of the set mean. This helped eliminate patterns, such as repeating digits in numbers (e.g., "333), which may have influenced participant strategy. For half of the choice trials, participants were shown the set mean. For the remaining half, participants were shown the set median; although for sets of 4 the mean and median were identical.

The three blocks of experimental trials were presented sequentially, randomized within-participants. The presented location of the actual value and mean value was also randomized so that the actual value and mean value were presented on the left or right side of the screen in 50% of trials. After completing all number set trials, participants were surveyed about their strategy use during the task. Participants were presented nine strategy descriptions (see Table 1) and were asked to estimate how often they used each strategy during the experiment using the following scale: 1) never, 2) some trials, 3) most trials, or 4) always. The response options and prompt remained on the screen until the participant chose an answer. The strategy descriptions were derived from the authors' previous work on number set processing (Masnick & Morris, 2008; Cravalho, Morris, Was, & Masnick, 2013).

Table 1: Strategy Descriptions and Examples.

Strategy	Example
Look at the first digit.	The "1" in 125.
Look at the second digit.	The "2 "in 125.
Look at the third digit.	The "5" in 125.
Try to figure out the average.	Calculate mean value.
Find the biggest number.	Scan set for highest value.
Find the smallest number.	Scan set for lowest value.
Just get a sense of the numbers.	Scan set values.
Look for a number that is not like other numbers.	Find any value unlike other values.
Try to memorize specific numbers.	Memorize a few numbers.

Results and Discussion

Participants' highest recognition was for sets of four numbers (*M* = .84, *SD* = .14), then recognition declined as set size increased to six (*M* = .69, *SD* = .15) and again as set size increased to eight (*M* = .64, *SD* = .09).

Whether studying the number sets or viewing the response screens, reaction time (see Figure 2) increased from sets of four to sets of six and then decreased when set size increased to eight.

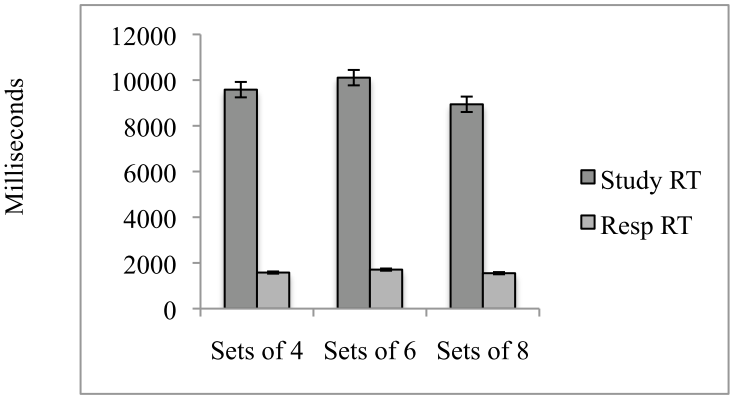

Figure 2: Mean study and response reaction times for Experiment 1.

Repeated measures ANOVA were used to analyze all main variables. There was a significant overall decline in number recognition as set size increased, $F(2, 40) = 34.69$, $p = .00$, $\eta^2 = .63$. As set size increased from four to six, there was a significant decrease in accuracy, $F(1, 20) = 62.10$, $p = .00$, $\eta^2 = .76$, as well as a significant decrease in accuracy as set size increased from six to eight, $F(1, 20) = 4.44$, $p = .04$, $\eta^2 = .18$.

There was not a significant overall decrease in study reaction times as set size increased ($F < 1$), nor was there significant decreases in reaction time according to contrasts between set sizes. However, there was a significant overall decrease in response reaction times as set size increased, $F(2, 40) = 3.52$, $p = .04$, $\eta^2 = .15$. Contrasts showed a significant decrease in reaction time as set size increased from four to six, $F(1, 20) = 7.24$, $p = .01$, $\eta^2 = .27$, then a significant increase in reaction time as set size increased from six to eight, $F(1, 20) = 6.23$, $p = .02$, $\eta^2 = .24$.

The most frequently used strategies, or those with mean ratings closest to "always" (assigned a value of 4 on our scale, with 1 indicating "never"), were to "just get a sense of the numbers" ($M = 3.29$, $SD = .96$), to "try to memorize specific numbers" ($M = 2.81$, $SD = .98$), and to "look at the third digit" ($M = 2.71$, $SD = 1.01$). These data reflect the likely pattern of participants trying to memorize numbers from the sets of four, then using the more diagnostic strategy of looking at the ones column of the numbers as set sizes exceeded four. This former strategy use would support the finding of Scholl (2001) that people individuate items for sets smaller than four. The general pattern of scanning to get a sense of the numbers was most likely used across all set sizes.

Confidence in number recognition (with a rating of 4 representing "76%-100%" confidence in one's answer) decreased from sets of four ($M = 3.49$, $SD = .53$) to sets of six ($M = 3.02$, $SD = .58$) and again when set size increased to eight ($M = 2.86$, $SD = .71$).

Supported by the medium to large effect sizes, the results demonstrate a clear effect of set size on all of the variables examined. Participants recognized the actual value more often given smaller set sizes (e.g., four) than larger set sizes (e.g., eight), but performed above chance regardless of set size. The confidence level of the participants fell along with their performance. Participants took more time to study and respond to sets of four than sets of eight, indicating that they were memorizing values. Whether studying the number sets or viewing the response options, sets of six yielded the longest reaction times. This may have been because the participants were memorizing the numbers or studying them down to the ones column, which would have taken longer than scanning and may have detracted from their confidence in their response.

Experiment 2

Some possible limitations of Experiment 1 are that the presence of unlimited study time and only two response options may have inflated task performance. Experiment 2 provided the same experimental conditions except that participants were given a limited amount of time to study the number sets and a third response option was added. The changes in study time and response options changes the task demands and likely changes the strategy used for the task (Cravalho et al., 2013).

Method

Participants. Participants ($N = 36$) were undergraduate students at a Midwestern state university. The average age was 19.72 ($SD = .91$), 81% of the participants were female.

Procedure. The changes to the number sets task for Experiment 2 were as follows. Instead of being given an unlimited amount of time to study the presented sets, each set was shown for 3 seconds. This change in Experiment 2 was made to standardize the experiment with previous work. To change the response stimuli, three numbers were presented when the participant was asked to indicate which number was in the set (see Figure 3). In addition to an actual value from the set and the mean value from the set, a number that did not appear in the set nor was the mean value of the set was presented. The presentation location of the actual value, the mean value, and the unseen value was also randomized so that the various values were presented in three possible positions on the screen in one third of the trials.

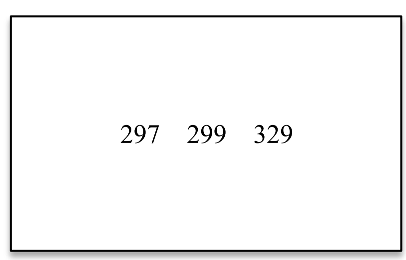

Figure 3: Sample response screen from Experiment 2.

Note. *Following the example in Figure 1, 297 is the mean value, 299 is the unseen value, and 329 is the seen value.*

Results and Discussion

Regardless of set size, participants correctly chose the number from the set on more than 50% of trials (see Figure 4). As set size increased from four to six, the proportions for the two incorrect choices (the mean value and the number not seen in the set) each doubled. As set size increased from six to eight, the proportion of mean value choices increased, whereas the proportion of number not seen in the set choices decreased slightly.

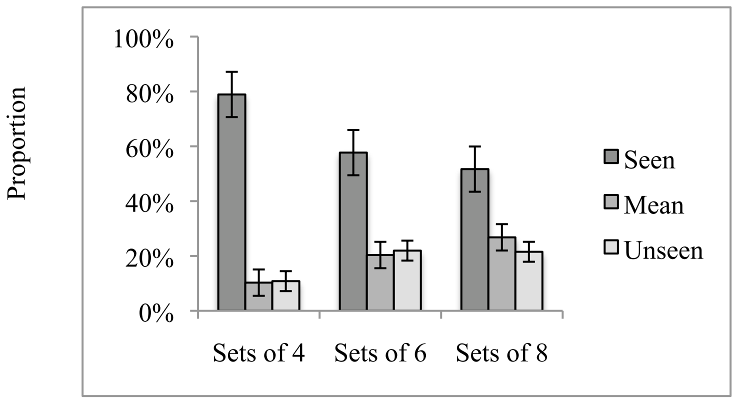

Figure 4: Proportion of response options for Experiment 2.

The general reaction time patterns (see Figure 5) were consistent across set size. Reaction time was the fastest when participants correctly chose the number from the set. For incorrect answers, reaction times were slower when participants chose the number not seen in the set than when they chose the mean value. The overall pattern for each response type was a decrease in reaction time as set size increased,

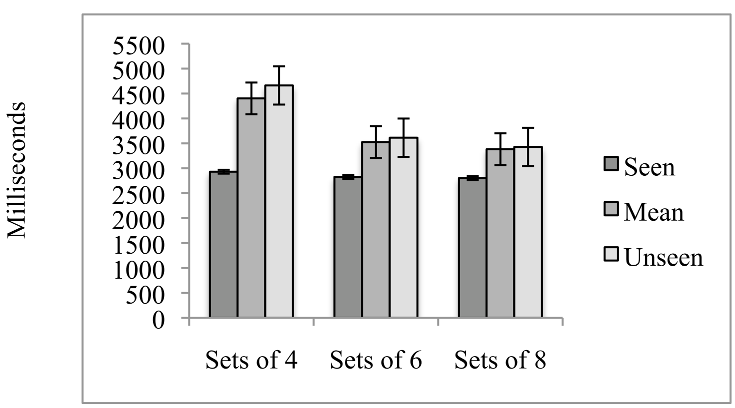

Figure 5: Mean reaction times for Experiment 2.

Repeated measures ANOVA were used to analyze all main variables. There was a significant overall decline in number recognition as set size increased, $F(2, 70) = 111.79$, $p = .00$, $\eta^2 = .76$. As set size increased from four to six, there was a significant decrease in number recognition, $F(1, 35) = 104.32$, $p = .00$, $\eta^2 = .75$, as well as a significant decrease as set size increased from six to eight, $F(1, 35) = 13.27$, $p = .00$, $\eta^2 = .28$. There was a significant effect of response type on number recognition, $F(1, 70) = 591.01$, $p = .00$, $\eta^2 = .94$, as well as a significant interaction between set size and response type, $F(4, 140) = 72.45$, $p = .00$, $\eta^2 = .67$. There was a significant interaction as set size increased from four to six, $F(1, 35) = 95.18$, $p = .00$, $\eta^2 = .73$, but there was not a significant interaction as set size increased from six to eight, $F(1, 35) = 3.59$, $p = .07$, $\eta^2 = .09$. Thus, as set size increased from four to six, participants erred more by choosing the unseen number, but as set size increased from six to eight, they erred more by choosing the mean value of the set.

There was a significant overall decrease in reaction times as set size increased, $F(2, 64) = 17.59$, $p = .00$, $\eta^2 = .36$. There was also a significant decrease in reaction time as set size increased from four to six, $F(1, 32) = 27.36$, $p < .01$, $\eta^2 = .46$, but not as set size increased from six to eight, $F(1, 32) = 1.22$, $p = .28$, $\eta^2 = .04$. There was a significant effect of response type on reaction time, $F(1, 64) = 51.65$, $p < .01$, $\eta^2 = .62$, as well as a significant interaction between set size and response type, $F(2, 128) = 6.17$, $p < .01$, $\eta^2 = .16$. There was also a significant interaction as set size increased from four to six, $F(1, 32) = 11.70$, $p < .01$, $\eta^2 = .27$, but not as set size increased from six to eight, $F(1, 32) = 1.18$, $p = .29$, $\eta^2 = .04$. Therefore, as set size increased from four to six, reaction time for incorrect answers decreased sharply, but as set size increased from six to eight, reaction time for incorrect answers decreased at a more gradual rate. Reaction time for correct answers varied slightly across all three set sizes.

The most frequently used strategies were again to "just get a sense of the numbers" (*M* = 3.39, *SD* = .73), to "try to memorize specific numbers" (*M* = 2.92, *SD* = .94), and to "look at the third digit" (*M* = 2.75, *SD* = .87). These data reflect the same strategy patterns seen in Experiment 1.

Confidence ratings decreased from sets of four (*M* = 3.20, *SD* = .36) to sets of six (*M* = 2.68, *SD* = .54) and again when set size increased to eight (*M* = 2.31, *SD* = .57). There was a positive relation between correct number recognition and confidence judgment for set size of four, $r(36) = .39$, $p = .02$, but not for set size of six or set size of eight. Zero-order correlations indicated a negative relation between choosing the mean (an incorrect response) and confidence judgment for set size of four, $r(36) = -.42$, $p = .01$, but not for set size of six or set size of eight. Finally, zero-order correlations indicated a negative relation between choosing the unseen number (an incorrect response) and confidence judgment for set size of six, $r(36) = -.37$, $p = .03$, but not for set size of four or set size of eight.

These results reveal a similar pattern of behavior as in Experiment 1, supported by a larger magnitude of effect sizes. One exception was that participants did not appear to use a diagnostic strategy for sets of six. This is surprising because such a strategy may have been ideal given the additional response option. One reason for this result may be the limited amount of time to study the numbers, unlike in Experiment 1 when they had an unlimited amount of study time. The negative relation between confidence and choosing the unseen number, indicating that they knew when they did not know the correct response, supports this assertion.

The strategy for sets of six discussed above is reflected by the significant interactions seen between sets size and response type for number recognition and reaction time. Although participants were accurate in their number recognition for sets of six, they took less time to choose an answer than for sets of four, signaling that they moved on to a more general strategy as set size grew.

Experiment 3

One explanation of the high task performance in Experiments 1 and 2 is that the task design allowed participants using an effective strategy to attend to and encode individual values without much explicit processing. A shortcoming of these previous experiments is that there was no objective evidence for participant strategies; participant strategies were inferred from their self-reports. The strategy that was most often reported was to "just get a sense of the numbers", which appears to have been used as a catch-all answer. Experiment 3 was a pilot study providing similar experimental conditions as the previous experiments but included the collection of eye tracking data as a dependent measure. The addition of these data allow for more direct insight into participant strategy use by providing direct evidence for the attention to information during encoding. It would also allow for triangulation with the previously reported strategies of memorization and looking at the third digit.

Method

Participants. Participants (N = 14) were adults living in northeastern Ohio, 57% of the participants were female.

Procedure. The number sets task was identical to that in Experiment 1 except as follows. The number sets task was presented on a Tobii® T-60XL eye-tracking monitor using Tobii® Studio. Tobii® Studio automatically notes the X and Y coordinates of gaze every 16.7 MS, providing an operational definition of a fixation. Participants were given a 9-point calibration before beginning the task. Areas of interest were defined around the hundreds, tens, and ones columns and around whole numbers. There were three practice trials. Experimental trials consisted of 10 sets of four numbers (each set presented for 2 s), 10 sets of six numbers (each set presented for 3 s), and 10 sets of eight numbers (each set presented for 4 s). The trials were not randomized within-participants and participants were not asked to give confidence judgments or surveyed about their strategies.

Eye Movement Data Processing. The first two authors reviewed video playback of the first six participants' fixations during the number sets task for reliability purposes. Fixations were coded when participants viewed number sets and when they viewed the response options.

Results and Discussion

Correct number recognition was highest for sets of four numbers (M = .89, SD = .10), and then declined as set size increased to six (M = .73, SD = .19) and eight (M = .73, SD = .20). A repeated measures ANOVA demonstrated a significant overall decline in number recognition as set size increased, $F(2, 26) = 5.26$, $p = .01$, $\eta^2 = .29$. As set size increased from four to six, there was a significant decrease in recognition accuracy, $F(1, 13) = 6.87$, $p = .02$, $\eta^2 = .35$, but not as set size increased from six to eight ($F < 1$).

Regardless of set size, or whether viewing a number set or the corresponding response options, most fixations were directed towards the tens column (see Figure 6). As set size increased, fixations directed towards the tens column decreased, whereas fixations directed towards the ones column increased. Another pattern was that the overall number of fixations increased along with set size.

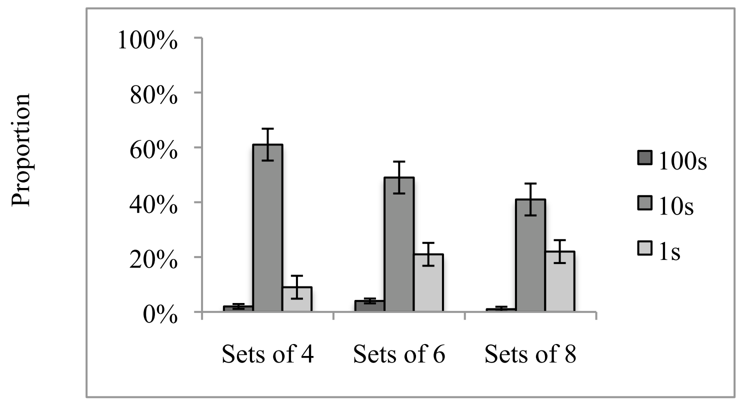

Figure 6: Proportion of eye fixations for Experiment 3.

These data more clearly depict the shifts in strategy. Regardless of set size, looking at the tens column was the dominant strategy. However, once set size had past the processing threshold of four (Scholl, 2001), more fixations occurred in the ones column, which corroborates with the strategy self-reports from Experiments 1 and 2. The third most dominant strategy was scanning both the tens and ones columns, which remained fairly stable in frequency across set size, supporting the importance of more diagnostic information when individuating number sets. This pattern, along with the highest use of scanning the tens and ones columns, indicates that participants were using the most diagnostic strategies (i.e., processing more specific cardinality information) when viewing sets of eight.

General Discussion

We investigated whether participants used aggregation or individuation strategies to recognize individual values from sets of three-digit numbers. We showed participants sets of varying size for unlimited (Exp. 1) and limited (Exp. 2 and Exp. 3) amounts of time and then asked them to recognize a number from the set amongst two (number seen and mean) or three options (seen, mean, and unseen).

Previous research provided evidence that participants aggregated over sets of numbers (Morris & Masnick, in press). Given these results, we expected to see evidence that participants erroneously recognized set means, rather than actual values, particularly when set sizes were larger than four. Our results suggest that task demands (e.g., recognizing a specific number in a set rather than comparing two sets) and strategies influence number recognition. For example, participants in all three experiments correctly recognized numbers from sets at levels greater than chance even as set size grew.

Our results provide evidence that participants were using diagnostic strategies that supported individuation rather than aggregation. For example, focusing on the tens or ones column when recognizing a three-digit number in a set of numbers may provide more information about unique features than the hundreds column.

In addition, reaction time for sets of six was related to a diagnostic strategy when time was not a factor, suggesting that we may have an easier time individuating larger sets when processing constrains are eased. In summary, our results suggest that sets larger than four are effectively individuated with diagnostic strategies and confidence in strategy use was indicative of number recognition performance. Future research should include the collection of eye tracking data with more variations of the number set task, including the inclusion of distracter tasks and new task goals.

References

Chong, S. C., & Treisman, A. (2005). Attentional spread in the statistical processing of visual displays. *Perception & Psychophysics, 67(1)*, 1-13.

Cravalho, P. F., Morris, B. J., Was, C.A., & Masnick, A.M. (2013). Representing Number Sets: Encoding Statistical Properties. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceedings of the thirty-fifth annual conference of the Cognitive Science Society* (pp. 352-357). Austin, TX: Cognitive Science Society.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S.. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284(5416)*, 970-974.

Hyde, D. C. (2011). Two systems of non-symbolic numerical cognition. *Frontiers in Human Neuroscience, 5*, 1-8.

Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia, 41(14)*, 1942-1958.

Morris, B. J., & Masnick, A. M. (in press). Comparing data sets: Implicit summaries of the statistical properties of number sets. *Cognitive Science: A Multidisciplinary Journal.*

Masnick, A. M., & Morris, B. J. (2008). Investigating the development of data evaluation: The role of data characteristics. *Child Development, 79(4)*, 1032-1048.

Scholl, B. J. (2001). Objects and attention: The state of the art. *Cognition, 80(1/2)*, 1-46.