On fallacies and normative reasoning: when people's judgements follow probability theory.

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Abstract

The systematic conjunction and disjunction fallacies seen in people's probability judgments appear to show that people do not reason according to the rules of probability theory. In an experiment examining people's judgments of the probability of different medical conditions, we find evidence against this view. In this experiment people's probability judgments closely followed the fundamental 'addition rule' of probability theory. This close match to probability theory comes alongside frequent occurrence of the conjunction and disjunction fallacies in those same probability judgments. These results support a model where people reason about probability via probability theory but are subject to random variation or noise in the recall of items from memory. In this model the effect of random variation is cancelled out by the mathematical form of the addition rule, producing agreement with probability theory; however, noise is not cancelled out for conjunctive or disjunctive comparisons, producing conjunction and disjunction fallacy responses.

Keywords: Probabilistic reasoning; conjunction fallacy; rationality

Introduction

Probability theory provides a calculus of chance describing how to make optimal predictions under uncertainty. Up to the 1960s the standard view in psychology was that people's probabilistic reasoning essentially followed probability theory. However, various systematic biases in people's probability judgements (many identified in the 1970s and 1980s by Tversky, Kahneman and colleagues) led researchers to conclude that, in fact, people do not follow probability theory but instead estimate probabilities using various heuristics. While these heuristics often yield reasonable judgments, they can also produce strong biases in people's probabilistic reasoning in certain cases (Tversky and Kahneman, 1973).

In this paper we return to the view that people follow probability theory when reasoning about uncertainty. We present a simple model where people estimate probabilities according to probablity theory, but are subject to random error or noise in recall from memory: we show that this model can explain various systematic biases seen in people's probabilistic reasoning. Importantly, this model predicts that bias will be 'cancelled' for certain combinations of people's probability estimates, and so those combinations should agree closely with probability theory. In an experiment testing this prediction by gathering people's probability estimates for various medical conditions, we find extremely close agreement with probability theory, demonstrating the predicted cancellation.

In the first section of the paper we give some background by briefly describing four biases in people's probabilistic reasoning. In the second section we describe our model and show how it explains these biases. In the third section we show that this model predicts that bias will be cancelled when probability estimates are combined according to probability theory's addition rule, which requires that

$$P(A_1) + P(A_2) - P(A_1 \textit{ and } A_2) - P(A_1 \textit{ or } A_2) = 0$$

for all events A_1 and A_2. In the fourth section we describe an experiment testing this prediction and showing that when people's probability estimates are combined via the addition rule, the result is 0 just as required by probability theory. The final section draws some conclusions from this result.

Background:Biases in Probabilistic Reasoning

We consider four biases in probabilistic reasoning: conservatism, subadditivity, the disjunction fallacy, and the conjunction fallacy. Perhaps the best known of these is the conjunction fallacy. Probability theory's 'conjunction rule' requires that, for any pair of events A_1 and A_2 where $P(A_1) \leq P(A_2)$ the relationship

$$P(A_1 \textit{ and } A_2) \leq P(A_1)$$

must hold. This follows from the fact that A_1 *and* A_2 can only occur if A_1 itself occurs. People reliably violate this requirement for some events, giving probability estimates for conjunctions that are greater than the estimates they gave for one or other constituent of that conjunction. Tversky & Kahneman's original demonstration of this fallacy (Tversky and Kahneman, 1983) concerned Linda:

> "Linda is 31 years old, single, outspoken, and bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice, and participated in anti-nuclear demonstrations"

Participants in Tversky & Kahneman's study read this description and were asked to rank various statements by their probability. Two of these statements were

- (A_1) Linda is a bank teller.
- (A_1 *and* A_2) Linda is a bank teller and an active feminist.

Tversky & Kahneman found that more than 80% of participants ranked A_1 *and* A_2 as more probable than A_1, showing a strong bias toward violating the conjunction rule. A large number of subsequent studies have shown that this conjunction fallacy occurs reliably in people's probabilistic judgments, although studies controlling for various extraneous factors suggest that typical conjunction fallacy rates are lower than those seen in the Linda example: between 20% and 40%. These studies also show that (1) the fallacy occurs most frequently when the objective probabilities $P(A_1 \textit{ and } A_2)$ and $P(A_1)$ are close together, and declines as the difference between these probabilities increases, and (2) the fallacy occurs most frequently when $P(A_1)$ is low and both $P(A_2)$ and $P(A_1|A_2)$ are high (for reviews see Costello, 2009a, Gavanski and Roskos-Ewoldsen, 1991, Fantino et al., 1997). These patterns are seen in the Linda example, where the description of Linda is designed so that $P(A_2)$ (the probability that Linda is a feminist) is very high: this necessarily implies that $P(A_1 \textit{ and } A_2)$ will be close to $P(A_1)$ (since Linda is almost certainly a feminist, if she is a bank teller she is almost certain to be a feminist bank teller).

A similar sort of fallacy occurs for disjunctions. Again, probability theory's 'disjunction rule' requires that, for any event A_2 and any disjunction A_1 *or* A_2

$$P(A_2) \leq P(A_1 \textit{ or } A_2)$$

must hold. This follows from the fact that if A_2 occurs, then A_1 *or* A_2 necessarily occurs. People reliably violate this requirement for some events, giving probability estimates for disjunctions that are greater than the estimates they gave for one or other constituent. Again, this fallacy occurs most frequently when the objective probabilities $P(A_2)$ and $P(A_1 \textit{ or } A_2)$ are close together, and declines as the difference between those probabilities increases (Costello, 2009b).

Our third bias, subadditivity, is similar to the conjunction fallacy in that it involves people's probability estimates violating a required upper bound. Let $A_1 \ldots A_n$ be a set of n mutually exclusive events, and let $A = A_1 \vee \ldots \vee A_n$ be the disjunction (the 'or') of those n events. Then probability theory requires that

$$\sum_{i=1}^{n} P(A_i) = P(A)$$

Experimental results show that people reliably violate this requirement, and in a characteristic way. On average the sum of people's probability estimates for events $A_1 \ldots A_n$ is reliably greater than their estimate for the probability of A, with the difference (the degree of 'subadditivity') increases reliably as n increases. An additional, more specific pattern is also seen: for pairs of mutually exclusive events A_1 and A_2 whose probabilities sum to 1 we find that the sums of people's estimates for A_1 and A_2 are normally distributed around 1, and so on average this sum is equal to 1 just as required by probability theory. This pattern is sometimes referred to as 'binary complementarity' (Tversky and Koehler, 1994).

Our final bias, conservatism (or 'underconfidence') involves comparison between people's subjective probability estimates for events and the true probablities for those events. In experiments where people are shown events occurring with a certain probability $P(A)$ (known to the experimenter) and are asked to generate subjective estimates for those events, a reliable pattern of deviation between known and estimated probabilities occurs: people's estimates tend to avoid the boundary probabilities of 0 and 1. More specifically, the the closer the true probabiliity $P(A)$ is to 0, the more people's estimates are greater than $P(A)$, while the closer $P(A)$ is to 1, the more people's estimates are less than $P(A)$. Differences between true and estimated probabilities are lowest when $P(A)$ is close to 0.5 and increase as $P(A)$ approaches the boundaries of 0 or 1 (Erev et al., 1994).

Our Model of Probabilistic Reasoning with Noisy Recall

In our model of probabilistic reasoning we assume a rational reasoner with a long-term episodic memory. For simplicity we assume that, apart from random variation, the reasoner is 'perfect': that is, if the reasoner were not subject to random variation then their probability estimates for any event would be equal to the true probability of that event. We take $P(A)$ to represent the true probability of some event A, $P_E(A)$ to represent a reasoner's estimate of that probability, and $\overline{P_E}(A)$ to represent the mean or expected value of $P_E(A)$ (the average estimate of the probability of event A).

We assume a simple form of long-term memory containing m episodes where each recorded episode i contains a flag $f_i(A)$ that is set to 1 if i contains event A and set to 0 otherwise, and the reasoner estimates the probability of event A by counting these flags. We assume a minimal form of transient error, in which there is some small probability d that when the state of some flag $f_i(A)$ is read, the value obtained is not the correct value for that flag. We take $C(A)$ to be number of flags that were read as 1 in some particular query of memory, and T_A be the number of flags whose correct value is actually 1. Note that due to random error $C(A) = T_A$ does not necessarily hold.[1]

Our reasoner computes a probability estimate $P_E(A)$ by querying episodic memory to count all episodes containing A and dividing by the total number of episodes, giving

$$P_E(A) = \frac{C(A)}{m}$$

Random variation afffects $P_E(A)$ when it causes some flag $f_i(A)$ be read incorrectly. The expected value of $P_E(A)$ is

$$\overline{P_E}(A) = \frac{T_A(1-d) + (m - T_A)d}{m}$$

[1] This type of sampling error is only one of many possible sources of nose. While we use this simple form of sampling error to motivate and present our model, our intention is to demonstrate the role of noise - from whatever source - in causing systematic biases in probability estimates.

(since on average $1-d$ of the T_A flags whose value is 1 will be read as 1, and d of the $m-T_A$ flags whose value is 0 will be read as 1). Since we assume that if the reasoner were not subject to random variation then each estimate $P_E(A)$ would be equal to $P(A)$, we have

$$P(A) = \frac{T_A}{m}$$

and so

$$\overline{P_E}(A) = P(A) + d - 2dP(A) \quad (1)$$

From this expression we see that the average value of $P_E(A)$ deviates from $P(A)$ in a way that systematically depends on $P(A)$. This deviation matches the pattern of conservatism seen in people's probability estimates: if $P(A) = 0.5$ this deviation will be 0, if $P(A) < 0.5$ then since d cannot be negative we have $\overline{P_E}(A) > P(A)$ with the difference increasing as $P(A)$ approaches 0, and if $P(A) > 0.5$ then $\overline{P_E}(A) < P(A)$ with the difference increasing as $P(A)$ approaches 1.

This deviation also explains the patterns of subadditivity seen in people's probability estimates. Recall that subadditivity applies to a set of n mutually exclusive events A_1 to A_n where $A = A_1 \vee \ldots \vee A_n$ is the disjunction (the 'or') of those events. Then from probability theory we have

$$P(A_1) + \ldots + P(A_n) = P(A)$$

From Equation 1 the expected value of the sum of people's probability estimates for events A_1 to A_n is given by

$$\sum_{i=1}^{n} \overline{P_E}(A_i) \quad = \quad \sum_{i=1}^{n} (P(A_i) + d - 2dP(A_i))$$

and using the fact that $P(A_1) + \ldots + P(A_n) = P(A)$ this gives

$$\sum_{i=1}^{n} \overline{P_E}(A_i) = P(A) + nd - 2dP(A)$$

Taking the difference between this expression and that for $\overline{P_E}(A)$ in equation (1) we get

$$\sum_{i=1}^{n} \overline{P_E}(A_i) - \overline{P_E}(A) = (n-1)d$$

(where n is the number of individual mutually exclusive events). This difference increases as n increases, producing subadditivity as seen in people's probability judgments. In the case of two mutually exclusive events A_1 and A_2 whose probabilities sum to 1, from Equation (1) we get

$$\overline{P_E}(A_1) + \overline{P_E}(A_2) = P(A_1) + P(A_2) + 2d - 2d(P(A_1) + P(A_2)) = 1$$

producing binary complementarity as in people's judgments.

We now turn to the conjunction and disjunction fallacies. In our model these fallacies are a consequence of random variation in individual probability estimates for constituent probabilities $P(A_1)$ and $P(A_2)$ and in conjunctive and disjunctive probabilities $P(A_1 \textit{ and } A_2)$ and $P(A_1 \textit{ or } A_2)$. Taking $P(A_1) \leq P(A_2)$, the conjunction fallacy occurs in our reasoners probability estimates when

$$P_E(A_1) < P_E(A_1 \textit{ and } A_2)$$

Since the reasoner is subject to random variation we can write the mean or average estimates as

$$\overline{P_E}(A_1) = P(A_1) + d - 2dP(A_1)$$

$$\overline{P_E}(A_1 \textit{ and } A_2) = P(A_1 \textit{ and } A_2) + d - 2dP(A_1 \textit{ and } A_2)$$

and we see that the reasoner's individual estimates for $P(A_1)$ and $P(A_1 \textit{ and } A_2)$ will vary randomly around those means. More specifically the reasoner's estimates for those probabilites at any given moment will be equal to

$$P_E(A_1) = \overline{P_E}(A_1) + e_{(A_1)}$$

$$P_E(A_1 \textit{ and } A_2) = \overline{P_E}(A_1 \textit{ and } A_2) + e_{(A_1 \textit{ and } A_2)}$$

where $e_{(A_1)}$ and $e_{(A_1 \textit{ and } A_2)}$ represent positive or negative random deviation from the mean at that time. The conjunction error will occur when

$$\overline{P_E}(A_1) + e_{(A_1)} < \overline{P_E}(A_1 \textit{ and } A_2) + e_{(A_1 \textit{ and } A_2)}$$

or, substituting and rearranging, when

$$(P(A_1) - P(A_1 \textit{ and } A_2))(1-2d) < e_{(A_1 \textit{ and } A_2)} - e_{(A_1)}$$

holds. Given that $e_{(A_1 \textit{ and } A_2)}$ and $e_{(A_1)}$ vary randomly and can be either positive or negative, this inequality can hold in some cases. The inequality is most likely to hold when $P(A_1) - P(A_1 \textit{ and } A_2)$ is low, and so this model predicts that the closer the conjunctive probability $P(A_1 \textit{ and } A_2)$ is to the lower constituent probability $P(A_1)$, the more likely the conjunction fallacy is to occur. Since we have

$$P(A_1) - P(A_1 \textit{ and } A_2) = P(A_1) - P(A_1|A_2) \times P(A_2)$$

the model also predicts that the lower $P(A_1)$ is, and the higher $P(A_2)$ and $P(A_1|A_2)$ are, the more likely the conjunction fallacy is to occur. Both these patterns are just as seen experimental studies of the conjunction fallacy.

Reasoning in just the same way for disjunctions, we see that the disjunction fallacy will occur when

$$\overline{P_E}(A_1 \textit{ or } A_2) + e_{(A_1 \textit{ or } A_2)} < \overline{P_E}(A_2) + e_{(A_2)}$$

or, substituting and rearranging as before, when

$$(P(A_1 \textit{ or } A_2) - P(A_2))(1-2d) < e_{(A_2)} - e_{(A_1 \textit{ or } A_2)}$$

holds. Again, since the error terms here vary randomly and can be either positive or negative, this inequality can hold in some cases, and is is most likely to hold when $P(A_1 \textit{ or } A_2) - P(A_2)$ is low: this model predicts that the closer the higher constituent probability $P(A_2)$ is to the disjunctive probability $P(A_1 \textit{ or } A_2)$, the more likely the disjunction fallacy is to occur. Again, this is just as seen in people's probability judgments.

The Addition Rule: Predicting Unbiased Probability Estimates

Our simple account where probability estimates are normatively correct but influenced by random noise can explain various patterns of bias in people's probability judgements, and also explain some specific situations in which those biases vanish (when probabilities are close to 0.5, for conservatism; and when two complementary probabilities sum to 1, for subadditivity). We now present a third situation in which this account predicts that bias will disappear.

Consider an experiment where we ask people to estimate, for any pair of events A and B, the probabilities of A, B, A *and* B and A *or* B. For each participant's estimates for each pair of events A and B we can compute a derived sum

$$X_E(A,B) = P_E(A) + P_E(B) - P_E(A \text{ and } B) - P_E(A \text{ or } B)$$

We can make a specific prediction about the average value of $X_E(A,B)$ for all events A and B. This value will be

$$\overline{X_E}(A,B) = \overline{P_E}(A) + \overline{P_E}(B) - \overline{P_E}(A \text{ and } B) - \overline{P_E}(A \text{ or } B)$$

Substituting from Equation 1 and rearranging we get

$$\overline{X_E}(A,B) = (1-2d)(P(A) + P(B) - P(A \text{ and } B) - P(A \text{ or } B))$$

However, probability theory's addition rule requires that

$$P(A) + P(B) - P(A \text{ and } B) - P(A \text{ or } B) = 0$$

for all events A and B, and we see that $\overline{X_E}(A,B) = 0$. Our prediction, therefore, is that on average $X_E(A,B)$ will be equal to 0 for all pairs of events A and B. More specifically, we predict that values of $X_E(A,B)$ will be symetrically distributed with a peak around the mean of 0.

An Experiment

We previously tested this addition rule prediction in experiments examining people's judgements of probability for everyday events (different types of weather). For these weather events, we found close agreement between people's probability estimates and the prediction (Costello and Watts, 2014). These weather events have not typically been used in studies examining people's probabilistic reasoning, however; and there therefore may have been some characteristics of those events that made people more likely to follow probability theory. In this section we describe a test of this prediction in an experiment examining people's judgements of the probability of different medical conditions. Such judgments have been used in a number of previous studies, particularly those examining the conjunction fallacy.

Materials and procedure

This experiment asked 21 UCD students to complete a survey asking them to estimate the likelihood of different medical conditions or combination of conditions. To construct the materials we selected 8 different medical conditions: Alzheimer's disease, Arthritis, Cardiovascular disease, Depression, Diabetes, Osteoporosis, High blood pressure, and Glaucoma. From these we selected 16 pairs of conditions and used each pair to construct conjunctive and disjunctive conditions such as: Alzheimer's disease *and* Arthritis; Cardiovascular disease *and* Depression; Alzheimer's disease *or* Arthritis; Cardiovascular disease *or* Depression; and so on. This gave 16 matched conjunctions and disjunctions, and 40 conditions in total: 8 single conditions, 16 conjunctive conditions, and 16 disjunctive conditions. Every participant was asked to estimate the probablity of every one of these 40 conditions occurring in a particular population (people in Ireland over 65 years of age).

Previous research has shown that the conjunction and disjunction fallacies ocucr less often when people are asked about event frequencies as opposed to event probabilities. To control for this we asked one group of participants ($N = 11$) to give estimates in terms of frequencies and the other group ($N = 10$) to give estimates in terms of probabilities. Examples of the questions are

(probability format) Imagine a randomly selected person in Ireland over 65 years of age. What are the chances, as a percentage (0% - 100%), that this person presents with Alzheimer's disease?

(frequency format) Imagine 100 randomly selected people in Ireland over 65 years of age. How many of these people (0 - 100) present with Alzheimer's disease?

For each group 40 questions were printed in a paper booklet, accompanied by instructions explaining the task. Question order was randomised. The task took around 30 minutes to complete.

Results

In analysing people's responses in this experiment we considered two factors: the extent to which people committed conjunction and disjunction fallacies, and the extent to which people followed probability theory's addition rule.

Fallacy occurrence. Both the conjunction fallacy and the disjunction fallacy occurred reliably in the frequency format group (where there was a 16% rate of conjunction fallacy occurrence and a 20% rate of disjunction fallacy occurrence) and in the probability format group (a 17% rate of conjunction fallacy occurrence and a 20% rate of disjunction fallacy occurrence). There was no significant difference in fallacy occurrence between the two groups, and so we collapsed the groups in further analyses. As predicted, there was a significant negative correlation between the rate of conjunction fallacy occurrence for a given conjunction A_1 *and* A_2 and the average difference between $P_E(A_1 \text{ and } A_2)$ and $P_E(A_1)$ ($r = -0.78, p < 0.01$). Also as predicted, there was a significant negative correlation between the rate of disjunction fallacy occurrence for a given disjunction A_1 *or* A_2 and the average difference between $P_E(A_1 \text{ or } A_2)$ and $P_E(A_2)$ ($r = -0.77, p < 0.01$).

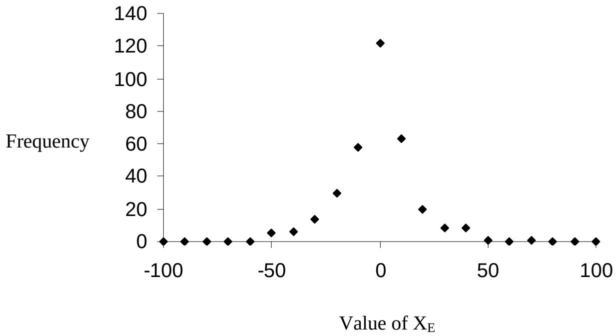

Figure 1: This scatterplot shows the frequency of occurrence of values of X_E in the experiment. Values of X_E are grouped into 'bins', each containing 10 values of X_E from $v-5 \ldots v+5$ for v from -100 to 100 in steps of 10 (probablity estimates in the experiment were given on a scale of 0 to 100). The critical point here is that these values are symmetrically distributed around 0 as predicted in our model.

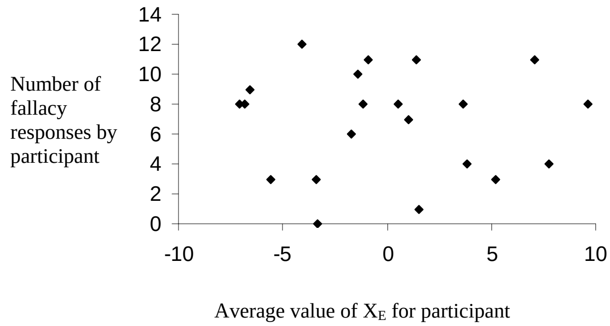

Figure 2: This scatterplot shows the total number of (conjunction and disjunction) fallacy responses for each participant against the average value of X_E for that participant's responses. The critical point here is that fallacy responses are relatively frequent even for participants whose average X_E values are close to 0.

Addition rule. For each participant we computed a value for the expression X_E for every set of conjunctions, disjunctions and constituents. For each participant there were thus 16 values of X_E, one for each pair of conditions used, giving $21 \times 16 = 336$ values for X_E. Our prediction is that these X_E values should vary randomly around a mean of 0 (the value required by probability theory for these X_E expressions). This prediction was strongly confirmed: the mean value of X_E across all responses was -0.018 ($SD = 16.5$) on the 100 point scale used in the experiment (that is, within one fiftieth of a unit of the predicted value), with a sample median of 0 and a sample mean of 0. The predicted mean of 0 lies within the 99.9% confidence interval of this observed mean. Figure 1 gives a summary representation of the distribution of these values of X_E. These values are symmetrically distributed around 0, as predicted by the model.

This result demonstrates an overall cancellation of bias across the 4 terms in X_E, where each term is subject to its own individual degree of bias: $P_E(A)$ and $P_E(B)$ being subject to conservatism (and with that bias cancelling only when 'binary complementarity' holds; that is, only when $P(A) = 1 - P(B)$), and $P_E(A \textit{ and } B)$ and $P(A \textit{ or } B)$ being subject to conjunction and disjunction fallacy effects. For the heuristics account to explain this overall cancellation it is not enough to say that people overestimate $P(A \textit{ and } B)$ and underestimate $P(A \textit{ or } B)$: it is necessary to calibrate the varying degrees of bias affecting estimates for $P(A)$, $P(B)$, $P(A \textit{ and } B)$ and $P(A \textit{ or } B)$. Further, to 'know' that the bias in these 4 probabilities should cancel requires access to the rules of probability theory (as embodied in the addition law): access which, in the heuristics view, people do not have. This cancellation result is thus hard for the heuristics view to explain.

Relation between fallacies and the addition rule. To examine the relationship between the addition rule expression X_E and the occurrence of the conjunction and disjunction fallacies, we obtained the average X_E value produced by each participant and compared it with the total number of fallacy responses produced by that participant. Figure 2 shows a scatterplot of these two measures. There was no relationship between these two measures ($r = -0.03, p > 0.1$); it is clear from the scatterplot that participants' whose average X_E value was close to 0 were just as likely to produce fallacy responses as those whose X_E value was further from 0.

Discussion and Conclusions

We have presented a simple model where people estimate the probability of some event A by estimating the proportion of instances of A in memory, but are subject to random errors in the recall of instances. These random errors in recall produce randomly varying, noisy estimates for the probability of A, and cause these estimates to be systematically biased in a way that depends on the value of $P(A)$. This systematic bias explains a range of biases and errors observed in people's probability estimation: conservatism, subadditivity, the conjunction fallacy and the disjunction fallacy.

We used this simple model to construct a probabilistic expression (the addition rule) that cancels the bias in estimates for one event against the bias in estimates for another. Our experimental results show that, even though there were systematic patterns of bias in people's probability estimates (producing reliable rates of conjunction fallacy and disjunction fallacy responses), when people's individual responses were combined in the addition rule, the results showed no systematic bias and gave a very close match to the requirements of probability theory. It is hard to explain this result without assuming that people's probabilistic reasoning in some way embodies the addition rule.

Our model has a number of advantages over other recent accounts of the conjunction fallacy, such as Busemeyer et al. (2011)'s model based on the logic of quantum theory, and Crupi et al. (2008)'s model based on causal support and confirmation. First, our model gives a general account for a range of different biases, not just the conjunction fallacy. Second, our model makes explicit predictions about cases where bias will vanish: as far as we can see, other models do not make such predictions. Third,without the observed patterns of conjunction fallacy occurrence in people's probability estimates, there would be no reason for suggesting that people reason about probability using the logic of quantum theory or causal support. There is, however, a clear reason for suggesting that people use probability theory: probability theory provides the optimal mechanism for such probabilistic reasoning. Because our model is based on probability theory, it has an *a priori* motivation which other models lack.

While our results demonstrate that people's probability estimates follow probability theory (when bias due to noise is cancelled) we do not think people are consciously aware of the equations of probability theory when estimating probabilities. Indeed we doubt whether the participants in our experiment were aware of the probablity theory's requirement that the addition law expression should equal 0 or would be able to apply that requirement to their estimations. Instead we propose that people's probability judgments are derived from a 'black box' module of cognition that estimates the probability of an event A by retrieving (some analogue of) a count of instances of A from memory. Such a mechanism is necessarily subject to the requirements of set theory and therefore embodies the equations of probability theory.

We expect this probability module to be based on observed event frequencies, and to be unconscious, automatic, rapid, relatively undemanding of cognitive capacity and evolutionarily 'old'. Support for this view comes from that fact that people make probability judgments rapidly and typically do not have access to the reasons behind their estimations, from evidence that event frequencies are stored in memory by an automatic and unconscious encoding process (Hasher and Zacks, 1984), and from results showing that animals effectively judge probabilities (for instance, of obtaining food from a given source) and that their probabilities are typically close to optimal (Kheifets and Gallistel, 2012).

These results pose a particular challenge to the view that people estimate probabilities using various heuristics. Notice that the heuristics view assumes that people estimate probabilities using heuristics that in some cases yield reasonable judgments (that is, judgments in accordance with probability theory) but in other cases lead to systematic error. To give evidence against the heuristics view it is therefore not enough to show that some of people's probability judgments agree with probability theory (that is expected in the heuristics view). Instead, our results challenge the heuristics view because they show that that, even when people's probability estimates show systematic bias (relatively frequent occurrence of the conjunction and disjunction fallacies), when those estimates are combined to form expressions that cancel out the biasing effects of noise, the results are on average strikingly close to those required by probability theory.

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