# A Mathematical Formalization of Fuzzy Trace Theory

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**Abstract**

In this paper, we develop a novel formalization of Fuzzy Trace Theory (FTT), a leading theory of qualitative risky decision-making. Our model is the first to explicitly formalize and integrate the concepts of gist and the gist hierarchy. Domain knowledge constrains the space of possible decision problems, explaining which gists are chosen in which contexts. We test our model against risky-choice framing and Allais paradox problems, and manipulations of these problems. Our results also confirm new predictions regarding how problem manipulations can enhance or attenuate framing effects.

**Keywords:** Decision-making; mathematical model; risky choice; framing effect; Allais paradox; gist

## Introduction

In this article, we introduce a mathematical model of Fuzzy Trace Theory (FTT), a leading theory of decision-making under risk, which assumes that decision-makers use a qualitative "gist" representation of a stimulus, in parallel with a precise verbatim representation (Reyna, 2012). We integrate memory and decision-making research to formalize how decision options and probabilities are mentally represented. By "formalization," we mean a mathematical description, and extension, of a verbal theory. We focus our analysis on risky choice tasks. We explain experimental evidence from several classic decision problems and experimental manipulations of these problems (e.g., Allais, 1953; Kühberger & Tanner, 2010; Peters & Levin, 2008; Reyna, 2012; Tversky & Kahneman, 1981). We then use our mathematical theory to make and test novel predictions. This paper provides the first explicit formalization of the concepts of gist, the gist hierarchy, and the fuzzy-processing preference (described below).

## Gist and Verbatim

The central tenet of Fuzzy Trace Theory (FTT) is that people encode, store, retrieve, and forget memories that are characterized by different levels of detail. We refer to these levels as "gist" and "verbatim." These representations are encoded separately and roughly in parallel (Brainerd et al., 2009). A gist representation captures the meaning, or "essence," of a stimulus, and is therefore a symbolic mental representation. Gists representations are simple, qualitative (for reviews, see Reyna, 2012) and are grounded in experience. In contrast, a verbatim representation of a stimulus retains its surface form. Examples include memory representations of exact words, numbers, and pictures. Even though verbatim representations reproduce the details of a given stimulus, they are also symbolic representations.

Rivers et al. (2008) illustrate the differences between gist and verbatim representations using the following scenario: Consider an adolescent who must decide between attending a party where alcohol will be served to minors (which the adolescent perceives as a fun but risky option) and attending a friend's sleepover where alcohol is not served (which the adolescent perceives as a fun but "safe" option in the sense that there is no risk of getting in trouble for underage drinking). Suppose that the adolescent thinks that the party will be more fun than the sleepover; however, the adolescent faces a small risk (e.g., a 10% chance) of being caught drinking at the party. An expected-value framework might characterize the options as follows:

1. A 100% chance of an amount of fun at the sleepover.
2. A 90% chance of twice as much fun and a 10% chance of no fun (getting caught) at the party.

A verbatim representation of these two options would be what is described above, that is, a precise description of outcomes and their probabilities. Outcomes and probabilities need not be fully explicit for a mental representation to be "verbatim"; verbatim representations encode the literal content of information or experience, however limited that might be. Option 2 is preferable based on explicit outcomes and probabilities; in many instances, the odds are with the risk-taking adolescent. However, a categorical gist representation of these two options is:

1. Some fun with certainty at the sleepover.
2. Some chance of some fun and some chance of no fun at the party.

The gist representation encourages risk avoiding (option 1) because the possibility of "no fun" is confined to the risky option (option 2). Research on risky choices suggests that decision makers represent decision options in both ways simultaneously – i.e., in terms of specific verbatim outcomes and probabilities (when those are known or estimated) and as qualitative gist representations. Figure 1a shows a visual representation of this choice in a two-dimensional Euclidean space (the "decision space"), whereas Figures 1b and 1c shows how points in this space are mapped to gists, represented as curves within the decision space ("constraints"). Finally, Figure 1d indicates which gist will be chosen given multiple interpretations.

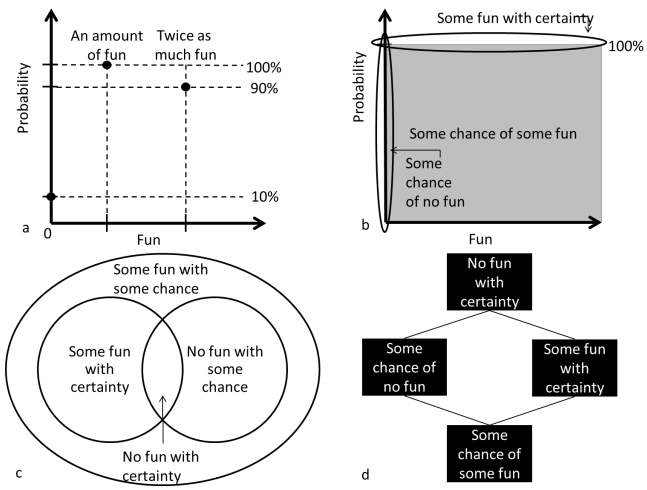


**Figure 1**. a) Visual representation of the choice faced by an adolescent decision-maker. Each point in this space represents a decision option (i.e., a fixed amount of fun with a fixed probability). b) The gist representation of the adolescent decision problem. All points in the grey box are interpreted as "some chance of some fun," all points in the horizontal oval are interpreted as "some fun with certainty," and all points in the vertical oval are interpreted as "some chance of no fun." Note that there are portions of the space where the ovals and grey box overlap each other. c) Venn diagram representing overlapping gists for the adolescent problem. d) A lattice representation of the gists in the adolescent decision problem (the "constraint lattice"). Higher elements in the lattice are preferred interpretations. Links indicate that all of the points in the higher gist category are contained within the lower gist category.

## The Hierarchy of Gist

A second tenet of FTT is that decision-makers prefer to operate on the simplest gist that can be extracted from information, which are often qualitative and categorical representations, whenever possible. This preference increases with experience in a domain (e.g., Reyna et al., 2013). Although more precise and quantitative representations are simultaneously generated, they are only relied on when necessary. A more precise representation may compete with the gist representation if they endorse very different decisions. In general, FTT assumes that subjects prefer to use the least precise representation of a problem that enables a decision to be made – i.e., they have a fuzzy-processing preference (Reyna & Brainerd, 2011).

### Categorical Comparisons

When two decision complements fall into different qualitative categories (e.g., "some fun" vs. "no fun"), these categories are compared. As we will show below, each of these categories is associated with a valence. Thus, the category that is more highly valued will be chosen.

### Ordinal Comparisons

When two decision options' complements have the same gist, finer-grained distinctions are required. For example, if two complements both involved risk, representing them both as "Some chance of some fun and some chance of no fun" would not distinguish them. Under these conditions, decision-makers revert to ordinal (e.g., more vs. less) decision-making.

For example, consider a hypothetical choice between:

1. A 90% chance of an amount of fun and a 10% chance of no fun (getting caught) at one party.
2. A 90% chance of twice as much fun and a 10% chance of no fun (getting caught) at a second party.

Both of these options would be represented as "some chance of some fun and some chance of no fun. These two options have the same categorical gist, but they can also be represented ordinally as:

1. Some chance of less fun and some chance of no fun (e.g., at one party).
2. Some chance of more fun and some chance of no fun (e.g., at a second party).

Here, option 2 would be preferred to option 1. According to FTT and consistent with empirical evidence, decisions rely on these ordinal representations when options cannot be distinguished by categories, (e.g., some fun vs. no fun).

### Interval Comparisons

Although ordinal representations are more precise than categorical ones, ordinal comparisons are still not always sufficient to make a choice. For example, one could imagine a choice between:

1. A 90% chance of an amount of fun and a 10% chance of no fun (getting caught) at one party.
2. A 60% chance of twice as much fun and a 40% chance of no fun (getting caught) at a second party.

An ordinal interpretation comparing these options is:

1. $\text{Less}_1$ fun is $\text{more}_2$ likely, and no fun is $\text{less}_3$ likely
2. $\text{More}_1$ fun is $\text{less}_2$ likely, and no fun is $\text{more}_3$ likely

Note that such a representation requires only that outcomes within a complement be compared if they have the same categorical representation (e.g., "some fun with some chance"). Thus, not all pairwise comparisons between complements are necessary. We use subscripts to clearly indicate which parts are being compared.

When ordinal comparisons lead to an indeterminate decision outcome, even more precise representations are used, such as comparing interval-level values. For example, the classical expected value is an interval representation, which we predict subjects will use. Using the interval-level numbers, the expected value of the first decision option in the original adolescent example is an amount of fun multiplied by 1.00 (i.e., probability of 100%). In contrast, the second decision option has an expected value of twice as much fun times 0.90, plus no fun times 0.10. This sum is equal to 1.8 times as much fun as the first decision option, if one assumes an interval-level scale of outcomes. Thus, an adolescent using a verbatim representation and multiplying

outcomes and probabilities would choose the risky decision option because it has a larger expected value.

## Values

The final tenet of FTT is that decisions are made on the basis of simple valenced (i.e., positive or negative) affect (e.g., Peters & Levin 2008). Thus, once options are represented in a categorical, ordinal, or interval fashion, the more positively valenced option is chosen (e.g., winning money is preferred to losing money; saving lives is preferred to losing lives). Consider the adolescent's gist representation described above:

1. Some fun with certainty
2. Some chance of some fun and some chance of no fun.

Given the value that some fun is preferred to no fun, we predict that the adolescent would choose option 1. Decision outcomes would differ for an ordinal decision maker, who would represent the problem as follows:

1. Less fun with certainty
2. Some chance of more fun and some chance of no fun.

An ordinal decision-maker would be unable to choose between these two options because more fun is preferred to less fun, but less fun is preferred to no fun. Finally, the interval, or verbatim, decision-maker would choose option 2 because it has 1.8 times the expected amount of fun as option 1 (described above). FTT thus demonstrates how problem representations along with positive or negative valenced dimensions can drive the decision outcome.

## The Model

We formalize FTT using algebraic tools originally developed to explain visual object perception and human concept learning (Feldman 1997; Jepson & Richards, 1993). These tools are based on lattice theory. Mathematically, a lattice (e.g., Figure 1c) is a kind of partial order on a set of elements, meaning that some (but not necessarily all) of these elements are ranked. Each element in the lattice stands for a decision category (such as "some chance of some fun," "some chance of no fun," etc.) Lattices have a common lowest element (called a "meet") and a common highest element (called a "join"). In our model, the requirement of a common meet ensures that all of the lattice elements are abstracted from the same world phenomena. The join of the lattice may be the empty set if our gist categories do not overlap. In our lattices, a link indicates that the decision category at the lower end of the link contains the category at the higher end.

### Categories

Feldman (1997) introduced a mathematical approach to qualitative categorization based on visual perception and artificial intelligence. The key to this approach is that an item is interpreted as if it were the category in which it falls (e.g., in our adolescent decision problem, twice the fun with a 90% chance is interpreted as "some fun with some chance" – a representation that captures an entire set of points in a space). This can be used to capture the notion of qualitative categories containing a range of values of which the specific stimulus values are just one of many examples (Reyna, 2012). Category boundaries are defined by constraints that are "non-accidental" (Jepson & Richards, 1992). A feature is non-accidental if it represents a psychologically special value in its category (e.g., Feldman, 2004) – for example, a 10% chance of no fun is a special case of "some chance of some fun" because "no fun" is a special case of "some fun." Constraints are "non-accidental" because the probability that any point in our space will fall on a constraint is functionally zero (Feldman, 1997, remarks that mathematically, it has measure zero). We draw upon Feldman's (1997) model to fully formalize of these ideas.

### The Decision Space

Assume a space $S$, each point of which corresponds to one complement in a potential decision option (e.g., "90% chance of twice as much fun"). Since we are studying risky decision problems, we restrict our analysis to a Euclidean space (e.g., Figure 1a), although there are a range of problems explained by FTT that are not captured by this rather restrictive assumption, which we leave to future work. Each point in $S$ may be parameterized by $R^d$ where $d$ is the dimension of the Euclidean space. This means that, each point in $S$ is indexed by a set of $d$ real parameters $s_1, s_2, \ldots s_d$, which are that point's coordinates in the space.

### Constraints

A set of points $p$ contained in $S$ obeys a constraint if they all the points in $p$ satisfy a single function $f$ expressed as $f_r(f_1,f_2,\ldots,f_d)=0$. If we assume that this function is *smooth* – i.e., it may be differentiated an arbitrary number of times – we can define a constraint as a *manifold* in our space. By manifold, we mean a subset of the space that has a dimension of at most $d$-1 (for example, a constraint in a 2-dimensional space could be mapped to a line, which is 1-dimensional). Thus, a constraint p in configuration space $S$ of dimension $d$ is a manifold that can be mapped to a space with dimension less than $d$. These manifolds are spaces in their own right, only with lower dimension. This means that one constraint may be embedded in another constraint. For example "some fun with certainty" is a 1-dimensional space (e.g, a line), in which "no fun with certainty," a 0-dimensional space (e.g., a point) is embedded. This leads to the creation of a hierarchy of manifolds ("some fun with some chance" contains "some fun with certainty," which contains "no fun with certainty," etc.) that will be represented by our lattice. We define the *constraint set* $C=\{c_1, c_2, \ldots, c_N\}$ as containing all the constraints explicitly mentioned in our decision problem. In our adolescent problem, $C$={no fun, certainty}. These constraints may intersect – e.g., a hypothetical decision option which guarantees no fun with certainty. On the other hand, the set may be empty, $C=\{\}$, if there are no constraints in the decision problem. We may use the constraint set to define a *constraint lattice* – a structure in which larger, more inclusive categories appear towards the

bottom, and smaller categories appear towards the top (cf. Jepson & Richards, 1993). Specifically, for a space $S$ with constraint set $C=\{c_1, c_2, ..., c_N\}$, the *constraint lattice* $L_{S,C}$ is the smallest set that contains $S$, contains each $c$ in $C$, and is closed under intersection (meaning that all possible combinations of overlapping constraints are included).

We formalize our extended version of the fuzzy processing preference by specifying that one always chooses the manifold in our space with the lowest dimension. Mathematically, this is identical to Feldman's (1997) "Maximum Codimension Rule." Given a point in our space and a set of possible gist interpretations for that point, we always choose the interpretation that is highest in the associated lattice structure.

### Formalizing Ordinal Decision-Making

FTT predicts that decision-makers revert to ordinal (e.g., more vs. less) decision-making when categorical distinctions cannot be made (i.e., all decision options fall into the same category). If the ordinal representation of a decision option is preferred along all dimensions of our space, and strictly preferred along at least one dimension, then that decision option is preferred overall. Otherwise, a decision cannot be made and we must revert to a more precise representation. In order to formalize this intuition, we again use a *partial order* – i.e., every pair of decision options may be less than, greater than, equal to, or unrelated to one another. For each k-dimensional category, $R_k$ in $L_{S,C}$, where $k \leq d$ we define the partial order, $\leq_R$, as follows:

- Since $S$ is a Euclidean space, then every dimension, d, in $R_k$, is associated with a *total order*, $\leq$ (i.e., every pair of decision options is either less than, greater than, or equal to one another).
- Given two points, $\boldsymbol{f}$ and $\boldsymbol{q}$, in $R_k$ We define $\leq_R$ as a product order on $R_k$ x $R_k$ (meaning that $\boldsymbol{f} \leq_R \boldsymbol{q}$ if and only if $f_1 \leq q_1$ , $f_2 \leq q_2$, ... $f_k \leq q_k$ ).

### Formalizing the Gist Hierarchy

We introduce the "gist hierarchy" as follows:

1. At the categorical level, each point is represented according to the extended fuzzy processing preference (i.e., as points in our space, $S$, interpreted according to the constraint lattice, $L_{S,C}$). All comparisons between points are made accordingly.
2. At the ordinal level, points $x$ and $y$ in the same category, $R_k$, in $L_{S,C}$, are compared according to the associated partial order of the category, $\leq_R$.
3. At the interval level, $x$ and $y$ are evaluated according to their expected values (i.e., a summation of each value multiplied by its respected probability).

Subjects choose the decision predicted by the categorical level. If it is indifferent, descend the gist hierarchy until the decision can be made. If no decision can be made at any level, subjects remain indifferent.

This model does not incorporate an error term, and instead predicts the modal outcome for each gamble; nevertheless, preliminary results suggest that a simple assumption of normally distributed noise should suffice. A more detailed discussion of the appropriate error term is outside the scope of this paper and is left to future work.

## Model Application

We begin by applying our model to the standard Asian Disease Problem (ADP; Tversky and Kahneman, 1981; 1986). The ADP is one of the literature's most widely replicated demonstrations of framing effects. The classic framing effect is that people avoid risks when options are framed as gains, but are risk seeking when those same options are described as losses. Framing effects challenge a fundamental axiom of economic theory (i.e., that preferences are coherent across different descriptions of the same options). Many experiments have confirmed the classic results across domains (e.g., Kühberger & Tanner, 2010). The text of the gain-framed standard ADP is:

"Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the program are as follows:

If Program A is adopted, 200 people will be saved

If Program B is adopted, there is a 1/3 probability that 600 people will be saved and a 2/3 probability that no people will be saved." (Kahneman & Tversky, 1981)

The loss-framed version of the same problem uses the same preamble but presents the decision options as:

"If Program C is adopted 400 people will die.

If Program D is adopted there is a 1/3 probability that nobody will die, and a 2/3 probability that 600 people will die." (Kahneman & Tversky, 1981)

The typical result (i.e., the framing effect) is that most people prefer the certain option in the gain frame (A), but they prefer the risky gamble option in the loss frame (D).

As per our mathematical formalization, there are two types of numbers that a decision-maker is required to understand. The first represents the number of people who are saved (or who die), and the second number represents the probability with which this outcome occurs. We represent these numbers in a 2-dimensional space, with the horizontal axis ranging from 0 live (or die) to 600 live (or die), and the vertical axis ranging from 0% to 100% probability. The certain option is located at (200, 1) because, if Program A is chosen, there is a 100% chance that 200 people will be saved. The first (non-zero) complement of the gamble option is located at (600, 1/3) since there is a 1/3 probability that 600 people will be saved; the second (zero) complement of the gamble option is located at (0, 2/3) since there is a 2/3 probability that 0 people will be saved.

### Empirically-Grounded Constraints

In practice, constraints are based upon innate and learned categories. For the domain of risky decision problems, common constraints are found in the literature on numerical cognition. Several independent findings support a

categorical distinction between "some" and "none." Beyond the relevant FTT findings (e.g., Reyna, 2012; Reyna et al., 2013), experimental and fMRI data have shown that subjects prefer to avoid winning nothing in a risky gamble, even if doing so lowers their overall expected utility (e.g., the "$P_{max}$" strategy of avoiding winning nothing as in Venkatraman & Huettel, 2012). Similarly, Tversky and Kahneman noted that "… very small probabilities can be either greatly overweighed or neglected altogether" (1992) consistent with the interpretation of very small probabilities as either "none" or "some." Similarly, zero is encoded into an "end stimulus" category that is separate from how other numbers are encoded (Pinhas & Tzelgov, 2012). Data indicate that, absent cues to the contrary, "all" and "certainty" are not subject to similar end effects (e.g., Holyoak & Glass 1978). Note that the word "all" does not appear in the standard ADP, neither does the word "certainty" nor the probability value "100%." Thus, our theory predicts that these values are interpreted as "some" (Reyna et al., 2013 performed a critical test of this prediction). Therefore, only the following constraint is used: {none saved} ("no chance" is not used because there are no points on the horizontal axis), i.e., an option in which no one is saved is qualitatively different than an option in which some are saved (Reyna, 2012).

Interpretations associated with higher levels on the lattice are preferred to those on lower levels (e.g., as in Figure 1d). This framing of the ADP contains three complements:

1. 200 saved –interpreted as "some chance that some are saved."
2. 600 saved with probability 1/3 – interpreted as "some chance that some are saved."
3. 0 saved with probability 2/3 – interpreted as "some chance that none are saved."

The decision-maker therefore faces the following choice:

a) Some chance that some saved
b) Some chance that some saved OR Some chance that none saved

Most decision-makers value human life; thus, relevant values are retrieved from long-term memory indicating that "some saved is better than none saved." Option a therefore weakly dominates option b. Similar logic applies to the loss framing of the ADP. Although the ADP was initially explained with Prospect Theory (Tversky & Kahneman, 1981), further tests support an FTT-based interpretation of the ADP's results (e.g., Kühberger & Tanner, 2010).

## Results

Our model successfully predicts each of the effects listed in Tables 1 and 2, including several variants of the ADP and the Allais Paradox gambles (Allais, 1953). Several of these (e.g., items 3 & 4) are not predicted by previous theories (e.g., Tversky & Kahneman, 1981; 1992).

**Table 1.** Overview of the 14 effects replicated by our model. Whereas IDs 1-12 are variants of the ADP, IDs 13 and 14 correspond to the Allais gambles (Allais, 1953).

| ID | Experimental Effect | Reference |
|---|---|---|
| 1 | A: 200 live vs. B: 1/3 * 600 live or 2/3 * none live | TK81 |
| 2 | C: 400 die vs. D: 2/3 * 600 die or 1/3 * none die | TK81 |
| 3 | A: 200 live vs. B: 1/3*600 live | R12, R13 |
| 4 | C: 400 die vs. D: 1/3*none die | R12, R13 |
| 5 | A: 200 live vs. B: 2/3*none live | R12, R13 |
| 6 | C: 400 die vs. D: 1/3*none die | R12, R13 |
| 7 | A: 200 live vs. B: 1/3*all live or 2/3*none live | BR14 |
| 8 | C: 400 die vs. D: 1/3*none die or 2/3*all die | BR14 |
| 9 | A: 200 live and 400 don't live vs. B: 1/3*600 live or 2/3*none live | KT10 |
| 10 | C: 400 die and 200 don't die vs. D: 1/3 * none die or 2/3 * 600 die | KT10 |
| 11 | A: 400 do not live vs. B: 1/3 * 600 live or 2/3 * none live | KT10 |
| 12 | C: 200 do not die vs. D: 1/3 * none die or 2/3 * 600 die | KT10 |
| 13 | A: $1m with certainty vs. B: 0.89*$1m or 0.1*$5m or 0.01*$0 | A53 |
| 14 | C: 0.89 * $0 or 0.11*$1m vs. D: 0.90*$0 or 0.10*$5m | A53 |

***Note:*** TK81 = Tversky & Kahneman, 1981; R12 = Reyna, 2012; R13 = Reyna et al., 2013; BR14 = Broniatowski & Reyna, 2014; KT10 = Kühberger & Tanner, 2010; A53 = Allais, 1953

**Table 2.** Gist representations of the 14 effects replicated by our model.

| ID | Experimental Effect | Outcome |
|---|---|---|
| 1 | A: some live WSC vs. B: some live or none live WSC | A |
| 2 | C: some die WSC vs. D: some die WSC or none die WSC | D |
| 3 | A: some live WSC vs. B: some live WSC | Indifferent |
| 4 | C: some die WSC vs. D: none die WSC | Indifferent |
| 5 | A: some live WSC vs. B: none live WSC | A |
| 6 | C: some die WSC vs. D: none die WSC | D |
| 7 | A: some live WSC vs. B: all live WSC or none live WSC | Attenuated |
| 8 | C: some die WSC vs. D: none die WSC or all die WSC | Attenuated |
| 9 | A: some live WSC and some don't live WSC vs. B: some live WSC or none live WSC | Indifferent |
| 10 | C: some die WSC and some don't die WSC vs. D: none die WSC or some die WSC | Indifferent |
| 11 | A: some don't live WSC vs. B: some live WSC or none live WSC | B |
| 12 | C: some don't die WSC vs. D: none die WSC or some die WSC | C |
| 13 | A: some $ with certainty vs. B: some $ WSC or some $ WSC or no $ WSC | A |
| 14 | C: no $ WSC or less $ WSC vs. D: no $ WSC or more $ WSC | D |

***Note:*** WSC = with some chance

## Discussion

Our approach builds on Tversky & Kahneman's Cumulative Prospect Theory (CPT; 1992) in that we hold losses and gains, rather than final assets, as the carriers of value. Unlike CPT, we do not distinguish between different degrees of these quantities by a value or decision-weight function in gist representations. Instead, decision options are perceived as gists that may be categorically distinct, or related in an ordinal fashion. CPT holds that calculations are performed to generate a weighted decision. In contrast, we hold that gist and verbatim representations of a stimulus are encoded simultaneously. Tversky and Kahneman's (1992) principle of diminishing sensitivity, which has historically been explained as satiation, can instead be explained as a result of categorical thinking. Comparisons made between two elements in the same category (i.e., two elements with the same gist) would be perceived as distinct but not different, yielding quantity insensitivity. Our framework demonstrates a potential theoretical unification of risky decision-making with elements of visual perception. Indeed, in their seminal paper on framing Tversky and Kahneman (1981) compared different frames with different perspectives on a visual scene. Our work extends this analogy between perception and explanation, demonstrating that the same mathematical formalism applies to both.

This theory is the first, to our knowledge, to provide an integrated formal model of gist, the gist hierarchy, and qualitative decision-making. Our mathematical model provides a novel extension to FTT by explaining gist-selection in terms of empirically grounded constraints – i.e., prior knowledge which imposes interpretive structure on the space of possible decisions. Our mathematical framework builds upon three basic tenets of FTT – the gist/verbatim distinction (formalized by our concept of constraints), the hierarchy of gist (formalized by our extended fuzzy processing preference and associated lattices), and preferences over these gist categories based on valenced affect. These three formalized tenets are used to predict 14 experimental effects.

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