

# Multi-modal Symbolic Representations of Number: Everything you ever wanted to know about Mental Abacus, but were afraid to ask

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## Introduction

How do different modalities like language, gesture, and mental imagery interact to support symbolic representation? We investigate the unique case study of “mental abacus” (MA). By visualizing a physical abacus, supporting computations with gesture, and translating responses in a verbal format, MA users perform arithmetic calculations with astonishing speed and accuracy. We explore the nature of this expertise, and how different systems interact to create a multimodal symbolic system.

Our symposium presents the results of a large-scale investigation of MA by an interlocking group of researchers. A brief **Introductory Presentation** will frame MA as a case study of how modalities interact to support efficient mathematical computations. **Paper 1** then explores how gesture interacts with visual working memory to facilitate MA computations. Results indicate that gesture movements correspond to specific mental computations, and that interfering with movements disrupts mathematical operations. Further experiments show that this interaction across modalities arises at the level of motor planning (and that actual movements play no role in supporting computations).

**Papers 2 and 3** then both analyze results from a 3-year, longitudinal study of more than 200 children, randomly assigned to receive 3 hours/week of either MA training or supplemental standard mathematics curriculum. Paper 2 reports a significant benefit of MA for arithmetic measures, suggesting that ordinary children in a low SES K-12 setting can attain relative levels of expertise. Further, it shows that this expertise stems from the optimal use of existing cognitive capacities. Paper 3 investigates the relationship between mathematics expertise and non-symbolic number representation in this same population, exploring whether mathematics expertise is associated with either better ability to perceive approximate numerosity or to map approximate quantity to specific number words.

Finally, **Paper 4** uses MA as a case study in visual attention. Specifically, it investigates how users allocate attention over the abacus while doing computations, and whether math expertise is associated with attentional expertise as well. This study finds that, while the structure of the abacus exploits visual-attentional biases observed even in naïve sub-

jects, knowledge of MA introduces additional biases that facilitate abacus processing. Together, these papers show how visual and motor resources can be leveraged to support computations that would otherwise be impossible. Additionally, they show that learning this visuospatial technique yields substantial benefits in mathematical performance.

## Abacus gesture in the mind, not the hands: Brooks et al.

Mental Abacus (MA) users move their hands as if they are moving abacus beads, even after years of practice solving problems without the physical device. This phenomenon provides a unique opportunity to explore the role of gesture on a task where gestures are easily mapped onto internal visual representations. We exploit this case study to test the mechanisms by which gesture interacts with visual imagery.

We first present evidence that children’s gestures grow larger and more precise in response to problems that are particularly difficult. These findings suggest that gesture plays a more important role when the task is more complex.

Second, we isolate the mechanism by which gesture affects visual imagery by testing the contributions of visual feedback, proprioceptive feedback, and motor planning on math performance. For MA, the critical mechanism for gestures effect appears to be the planning of motor movements, rather than visual or proprioceptive feedback from actual movements. While blindfolding participants or requiring them to hold their hands still did not impact performance ( $t_s < 1$ ,  $p_s > 0.1$ ), motor interference caused a drastic and significant decrease in performance ( $\beta = -2.45$ ,  $t = -3.88$ ,  $p < 0.001$ ). This result constrains hypotheses about gestures role in facilitating visual imagery. It also supports previous suggestions that gestures can be internalized over time, in absence of physical movements.

## Learning mental abacus improves mathematics achievement: Barner et al.

Mental abacus experts exhibit amazing arithmetic ability (Frank & Barner, 2012). However, the nature of this expertise is poorly understood, and it is unknown whether MA training can produce such advantages in large groups of school-aged children, or only in select individuals. We evaluated this question, while also testing whether learning mathematics in

a visuo-spatial format (MA) improves non-mathematical cognitive capacities.

We conducted a 3-year longitudinal curriculum intervention, and asked whether learning MA improves childrens (1) arithmetic ability; (2) mathematical concepts not directly trained by MA; and (3) non-mathematical cognitive functioning. In addition to testing effects of MA training, this was also among the first large-scale longitudinal studies to test developmental relations between working memory, non-symbolic number, and symbolic mathematics. We randomly assigned 203 first-graders to either learn MA or receive additional curriculum-based math for 3 hours a week over three years. Children received a battery of mathematical and cognitive tasks annually including standardized math tests (WIAT, WJ-III), tests of conceptual math knowledge (place value), Ravens Matrices, tests of working memory capacity (verbal and spatial), and non-symbolic number sense.

MA training substantially improved arithmetic achievement and mastery of math concepts, relative to standard math training. However, we found no transfer to non-mathematical capacities like spatial working memory and number sense. Nevertheless, childrens spatial working memory skill at initiation of the study predicted subsequent ability to benefit from MA, suggesting that children with strong spatial working memory may be best equipped to exploit the abacus visual representation format.

### **Does intervening on symbolic math affect number discrimination and estimation?:**

**Sullivan et al.**

Humans can represent and manipulate numerical quantities nonverbally, by using a system known as the Approximate Number System (ANS). The precision of a persons ANS and their ability to make speeded estimates both predict symbolic math performance (Halberda, Mazzocco, & Feigenson, 2008; Booth & Siegler, 2006). These findings suggest that non-verbal numerical abilities are importantly related to symbolic math success. However, it remains uncertain what mediates these correlations.

As described in Talk #2 (Barner et al.), we conducted a longitudinal study of the effect of MA training on math skill, nonverbal number skill (ANS acuity), and estimation ability. We replicated past findings that ANS acuity and estimation ability predict math performance. However, we also found that (1) improved math achievement across a battery of standardized tasks did not improve ANS acuity or estimation skill, and that (2) ANS acuity at the beginning of the study did not mediate differential gains in math achievement due to the intervention. At least for this case study, these data suggest that ANS and math achievement are not inextricably linked.

### **Does expertise direct the allocation of attention within visual arrays? The case study of mental abacus: Wagner et al.**

The abacus is a symbolic system that represents number via the arrangements of beads into columns. Here, we explore whether learning to use an abacus alters how children allocate visual attention when processing visual arrays, or if, alternatively, abacus structure exploits pre-existing biases of visual attention without requiring expertise.

On an abacus, beads are organized into columns which denote place value (increasing from right to left). Beads in each column are moved toward a horizontal beam to be in-play and represent a digit between 0 and 9. When a columns beads are each out-of-play, the column represents 0, and affects cardinality when the zero is trailing (e.g., the 0 in 250) but not when it is leading (e.g., 025).

We asked whether experts and novices differ in how they allocate attention to in-play and out-of-play beads, and to beads representing trailing and leading zeroes. To do so, we used a visual search task, in which search elements were overlaid on top of abacus beads. Both experts and nave subjects detected targets faster on in-play than out-of-play beads (both  $p < .001$ ). However, while experts detected targets faster on beads representing trailing than leading zeros ( $p < .005$ ), naïve subjects didn't ( $p = .52$ ). These results suggest that while abacus structure exploits attentional biases that can be observed even in nave subjects, experience using the abacus also introduces additional biases that may facilitate abacus processing.

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