

# Full Day Tutorial on Quantum Models of Cognition and Decision

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## General Purpose

This *full day* tutorial is an exposition of a rapidly growing new alternative approach to building computational models of cognition and decision based on quantum theory. The cognitive revolution that occurred in the 1960's was based on classical computational logic, and the connectionist/neural network movements of the 1970's were based on classical dynamical systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamical systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the disjunctive axiom of Kolmogorov probability; quantum reasoning does not obey the principle of monotonic reasoning. It turns out that humans do not obey these restrictions either, which is why we consider a quantum approach.

This tutorial will provide an exposition of the basic assumptions of classical versus quantum theories. These basic assumptions will be examined, side-by-side, in a parallel and elementary manner. The logic and mathematical foundation of classical and quantum theory will be laid out in an accessible manner that uncovers the mysteries of both theories. We will show that quantum theory provides a unified and powerful explanation for a wide variety of paradoxes found in human cognition and decision ranging from attitude, inference, causal reasoning, judgment and decision, conceptual combinations, memory recognition, and associative memory. This tutorial introduces and trains cognitive scientists on this promising new theoretical and modeling approach.

## Presenters

Zheng (Joyce) Wang is an associate professor at The Ohio State University. She was Co-Editor for a special issue on quantum cognition that appeared in *Topics in Cognitive Science* (2013, Vol. 5 (4)). Her work on quantum cognition has been funded by NSF and AFOSR. Jerome Busemeyer is Provost Professor of Psychological and Brain Sciences at Indiana University. He is Editor of *Decision* and Associate Editor of *Psychological Review*, and was Editor of *Journal of Mathematical Psychology*. His research interests include decision-making and dynamic modeling. Jennifer Trueblood is an assistant professor at the University of California, Irvine. She has published articles on the topic of quantum cognition in *Psychological Review* and *Cognitive Science*. Her work on quantum cognition has been funded by NSF.

## Previous Tutorials and Symposia

The tutorial has been presented at the Cognitive Science meetings in Nashville (2007), Washington DC (2008), Amsterdam (2009), Sapporo (2012), and Berlin (2013), with about 30 to 50 participants each time. The ratings from participants after the tutorial were all very positive. Also, this tutorial follows a symposium on quantum cognition presented at the Cognitive Science meeting 2011, and these papers appeared as a special issue in *Topics in Cognitive Science*. A similar tutorial was presented at the 3rd and 4th Annual Meetings on Quantum Interaction in Saarbruecken, Germany (2009) and Aberdeen, Scotland (2010) with about 40 participants, the Society for Mathematical Psychology (2012), and BRiMS (2013).

## Participants Background

This tutorial will introduce participants to an entirely new area and no previous experience or background with quantum theory will be assumed. **No background in physics is required.** In fact, except for a few simple examples to motivate the idea, little or no reference to physics will be made during main part of the tutorial. What is required is an elementary background in classical logic

and probability. During the tutorial, we will review basic concepts of linear algebra needed for quantum theory (e.g., vectors, projectors, unitary transformations).

## Material to be Covered

1. *First, we will examine major differences between classical versus quantum theories of probability.* The concept of superposition is introduced and distinguished from classical probability mixtures. The important issue of measurement in classical and quantum systems will be compared and examined. The key to this section will be several dramatic empirical examples illustrating empirical violations of the classical laws of probability (e.g., conjunction, disjunction, and total probability) and the parsimonious explanation of all these violations by quantum theory. (1-1.5 hours)

2. *Then we will examine the differences between classical and quantum dynamical systems.* The basic idea of a Markov processes will be introduced and compared with quantum processes. (Cognitive architectures and many neural networks can be represented as Markov processes). A parallel development of Markov and quantum processes will be shown. The concept of a state will be distinguished for Markov and quantum systems. The effects of measurement on the state of the system are compared for Markov and quantum systems. A key goal is to show when and how quantum processes depart from Markov processes, and how we can empirically test whether a system is Markov or quantum. (1-1.5 hours)

3. *Next, we will use a concrete example to show how to build computational models based upon quantum theory.* We will present the details of MATLAB and R programs used to compute the choice probability and response time predictions of a dynamic quantum model that has been developed to explain three ongoing research programs in cognitive and decision making: violations of the “sure thing principle” of rational decision theory, violations of dynamic consistency in decisions, and interference of categorization on decisions. (1 hour)

4. *In the fourth part, we will introduce quantum computing and information processing ideas.* The concepts of a bit and a qubit will be contrasted. The concept of a conjunction of properties used in classical information processing theory will be related to the concept of a tensor product space used in quantum theory. The controlled U-gate will be introduced and compared with a production rule. The linear transformation of states used by quantum theories will be related to the distributed representation and content addressable properties of connectionist/neural networks. The concept of fuzzy representation and probabilistic representation will be discussed and compared for fuzzy set, Bayesian, and quantum theories. The idea of an entangled state will be described. Bell’s inequality will be introduced, and violations found in conceptual combinations are reviewed. The dramatic implications of violations of this inequality for classical theories will be discussed. (1 hour)

5. *This part will present the details of MATLAB programs used to perform quantum computing for some complex information processing tasks.* This includes pattern recognition and planning event-dependent action sequences under uncertainty. Basic tools of quantum computing will be used including Kronecker products to perform U-gate operations, and partial traces for measurement of components of a complex system. (1 hour)

6. *Finally, we will review the progress in quantum cognition research in general, and propose future directions.* (30 minutes)

See the references and the web page below for some of the material to be covered and relevant background material:

[http://mypage.iu.edu/~jbusemey/quantum/Quantum\\_Cognition\\_Notes.htm](http://mypage.iu.edu/~jbusemey/quantum/Quantum_Cognition_Notes.htm)

## References

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