Infants' Early Understanding of Coincidences

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Abstract

Coincidences are surprising events that can provide learners with the opportunity to revise their theories about how the world works. In the current research, we investigate whether infants are truly sensitive to coincidences, detecting these events even when they cannot be predicted mere probabilities. In addition, we explore whether this sensitivity is translated into action, encouraging infants to engage in activities that enable them to revise their theories. Results from 2 experiments demonstrate that infants display a sensitivity to coincidence similar to adult intuitions, and they selectively explore objects that produce anomalous data that better supports an alternative theory than their prior theory.

Keywords: coincidence; probabilistic reasoning; theory revision.

Introduction

As scientists, we are sometimes met with surprising results in our research. At this point, it can be a struggle trying to reconcile the evidence with our theories – do we chalk these results up as experimenter error, or do we discard our theories in favor of an alternative one? Sometimes, such results were a mere coincidence; they go away upon a review of our procedures or an increase in sample size.

Other times, however, unexpected results have led to important scientific discoveries. For example, the discovery that cholera was caused by infected water, rather than the prevailing notion that the disease was transmitted by bad air, was due to a suspicious coincidence observed by John Snow, a physician, in 1854. After a particularly violent outbreak of cholera in the city of London, Snow noticed that the location of the victims were all tightly clustered around a water pump on Broad Street (Snow, 1855). Using this suspicious finding, Snow convinced the local council to remove the handle of the water pump, and this action has since been commonly credited with stopping the further spread of cholera. Such discoveries abound in the history of science, emphasizing the detection of suspicious coincidences as key to causal discovery (Owens, 1992; Nickerson, 2004) and rational inference (Horwich, 1982).

This detection of coincidences is not limited to scientific research, as adults use it to make inferences in daily life as well. They also act on these observations appropriately, taking the nature of these coincidences into consideration. Imagine a situation in which one leaves our apartment in a hurry on the way to work. While waiting for the elevator, we meet three of our neighbors, and each one of them is carrying an umbrella. Do we dismiss this observation as a mere coincidence? Probably not – we are likely to discard our belief that today is just like any fair weather day in California, rushing back to grab an umbrella too.

According to Griffiths & Tenenbaum (2007), our reaction at the elevator can be properly understood through a Bayesian framework, in which coincidences are formally defined as events that provide better support for an alternative theory, as compared to a currently favored causal theory. Whether a surprising observation should be taken as compelling evidence or dismissed as a mere coincidence is thus dependent on two factors: the prior probability of the alternative theory, and the strength of coincidence as given by the likelihood ratio. The likelihood ratio quantifies the support that the evidence provides for the alternative theory over the original theory. Griffiths & Tenenbaum (2007) also show empirically that adults evaluate coincidences in ways that are consistent with this framework – they take into account prior probabilities and likelihood ratios when thinking about unexpected evidence. For example, adults judged the results of a test of psychokinesis (low prior probability) as mere coincidence and that of a test of genetic engineering (high prior probability) as evidence, even when the data provided the same support for the two alternative theories. However, when the strength of the coincidence is manipulated to be sufficiently high in the case of psychokinesis, adults find it increasingly hard to dismiss the observations as just chance results. Therefore, adults act in ways consistent with an ideal Bayesian learner, evaluating and acting on observed coincidences in rational ways.

Our reaction in the elevator situation can thus be easily understood through the Bayesian lens. Given that there are many more fair weather days than rainy days in California, we should favor the null hypothesis due to its higher prior probability: today is a fair weather day, just like most days. However, the surprising observation that our neighbors are all carrying their umbrellas gives a high likelihood ratio, strongly suggesting an unexpected causal structure: today is a rainy day. We have thus detected a coincidence: the observed event provides better support for an alternative theory of rainy weather, as compared to our prior theory of fair weather. In this example, the alternative theory also has a sufficiently high prior probability, pushing us to discard our original theory, and to intervene by retrieving an umbrella from our apartment.

Detecting such coincidences is important for learners, as these events are often great opportunities for us to revise our current theory of how the world works. This opportunity is especially essential for children, whose accounts of the inner workings of the world are under major construction and revision (Carey, 1985, 2009; Gopnik & Meltzoff, 1997). Given that many causal relationships in the world are probably novel to young children, coincidences are rich sources of information for how their theories should be revised, and thus one should predict that children should pay great attention to coincidences (Griffiths & Tenenbaum, 2007). In this paper, we investigate this prediction by asking whether young infants are sensitive to such coincidences, and whether this sensitivity translates into action that can help them update their theories about how the world works.

Recent research exploring the development of probabilistic reasoning has provided ample evidence that 6- to 12-month-old infants are sensitive to differences in probabilities (Denison, Reed & Xu, 2012; Teglas, Girotto, Gonzalez & Bonatti, 2007; Xu & Garcia, 2008). For example, Teglas et al. (2007) showed that in a lottery machine-like setup that consisted of 1 yellow and 3 blue objects bouncing around, 12-month-old infants were more "surprised" to see a yellow object (low probability) exiting the machine, than when a blue object (high probability) did.

However, detecting coincidences is not quite the same as assessing the relative probabilities of different events. In some cases, events can have equal probabilities, but we do not consider them equally surprising. Take the instance of five rolls of a 6-sided dice. The probability of seeing the sequence "2, 1, 4, 3, 1" is $\left(\frac{1}{6}\right)^5$ = 0.00013. Although low in probability, this event is unsurprising to most adults. In contrast, the probability of seeing the sequence "1, 1, 1, 1, 1" is again $\left(\frac{1}{6}\right)^5$ = 0.00013, but this time we are astonished, becoming suspicious of the dice and the roller of the dice. These intuitions cannot be simply explained by the proposal that learners are actually evaluating and comparing the probabilities of "kinds" of events (e.g. the probability that the sequence consists of different numbers vs. the probability that the sequence consists of the same number), instead of single events (e.g. the probability that the specific sequence is "2, 1, 4, 3, 1"). This proposal is problematic as it is unclear what exactly counts as a "kind" of event, and what a learner should do when there are many possible "kinds" to consider (e.g. running sequences such as "1, 2, 3, 4, 5" or alternating sequences such as "1, 2, 1, 2, 1" are suspicious too, but do not fit into the two earlier-mentioned "kinds"; see Griffiths & Tenenbaum, 2007 for a more comprehensive review). Instead, we consider the sequence "1, 1, 1, 1, 1" to be a suspicious coincidence because it provides better support for the alternative theory that the dice is weighted towards "1," rather than our original theory that the dice is fair.

As such, there exists a gap in our knowledge of whether infants are truly sensitive to coincidences – we know that they are surprised by the occurrence of low-probability events, but do they detect coincidences even when the mere probabilities of different events are exactly equal? To investigate this question, we designed an experiment analogous to the dice roll example detailed earlier. 8-month-old infants were familiarized to a box containing 6 different colored balls. An experimenter then tossed out a ball from the box, seemingly with no control over the outcome of the event. The ball was then returned to the box, and this event was repeated 3 more times. Using a violation-of-expectation paradigm, we measured the amount of time infants looked at a trial where the same colored ball fell out each time (e.g. yellow, yellow, yellow, yellow), as well as a trial where a different colored ball fell out each time (e.g. blue, green, red, yellow). Note that each specific sequence shown had the same exact event probabilities: $\left(\frac{1}{6}\right)^4$.

We also designed a second experiment with an exploration measure, examining whether this sensitivity to coincidences translates to action, such that the detection of a suspicious coincidence could potentially have consequences on children's learning. In this experiment, we showed 13-month-old infants two different boxes each containing 6 different colored balls. One of the boxes always generated the same sample each time, and the other always generated a different sample each time. The two boxes were then offered to the infants to play with freely, and we measured the amount of time they played with each box.

Experiment 1

In Experiment 1, we investigated whether infants were sensitive to coincidences that cannot be predicted by the computation of the mere probabilities of events. If infants shared adult intuitions, they should look longer at the event in which the same colored ball fell out of the box each time under random sampling, than when a different colored ball fell out each time. We also included a Baseline condition to assess infants' intrinsic preferences for these two events.

Method

Participants Forty infants (21 males and 19 females, *M* = 8; 6 [months; days], *R* = 7; 3 to 9; 1) were tested. All were recruited from Berkeley, California, and its surrounding communities. An additional 7 infants were tested but excluded due to fussiness (N = 5) or experimenter error (N = 2). Infants who participated in the experiment were required to be exposed to English a minimum of 50% of the time. Infants received a small gift for their participation.

Materials A total of 36 colored balls (7 cm in diameter) were used. The balls came in 6 colors: red, purple, blue, green, yellow and orange.

A small white box (28 cm x 10 cm x 7.5 cm) constructed from foam core was used in the Free Play phase of the experiment (see Procedure). The box contained 3 different colored balls.

A small, transparent Plexiglas container with an open top (16.5 cm x 7.5 cm x 9 cm) was used to display the sampled ball during the test trials.

A large box (30 cm x 26 cm x 21 cm) was used to display the population of 6 different colored balls during the familiarization and test phase. The box was rectangular, with a Plexiglas window to show the population of balls, and two hidden back compartments. One compartment was used to hold the 4 sample balls to be tossed out later during the test trials, while the other compartment was to contain the balls that were being returned to the box after each toss. From the infants' perspectives, the box appeared as one single unit, filled only with 6 different colored balls. The Plexiglas display window was covered with a fabric curtain to ensure that the population would be hidden from sight while each sampled ball was being tossed out.

Apparatus The testing room was divided in half by curtains spanning its width and height. The curtains had a cut-out above a puppet stage that measured 94 cm x 55 cm (width x height). The experimenter sat behind the stage with her upper body and head visible to the infant. There was a black back curtain attached to the stage, such that the experimenter is hidden from view when it is dropped. An observer, present to code the infant's looking times, sat in a corner of the room and was not visible to the infant. She watched the infant on a TV monitor and coded the infant's looking behavior online using JHAB (R. Casstevens, 2007). The observer was blind to the order of the test trials.

Infants sat in a high chair about 70 cm from the center of the stage. Each parent sat next to their infant facing the opposite direction, and was instructed to avoid looking at the stage. Two video cameras were used to record each experimental session, one to record the infant's looking behavior, and another to record the experimenter's presentation of the trials.

Design and Procedure Each infant was randomly assigned to a Sampling condition or a Baseline condition. Both conditions consisted of a Calibration phase, a Free Play phase, a Familiarization phase and a Test phase.

Sampling Condition To calibrate each infant's looking window, a squeaky toy and/or keys were used in the Calibration phase to direct the infant's attention to the outside parameters of the stage.

In the Free Play phase, the infant was shown a white box containing three different colored balls. She was encouraged to play with the balls for approximately 30 seconds, and the experimenter ensured that the infant touched every ball. This phase was to allow the infants to become familiar with the balls used in the experiment.

The Familiarization phase that followed consisted of two trials. To begin each trial, the experimenter placed the large box on the stage with its front curtain down. Then, she lifted the curtain to reveal a population of 6 different colored balls, saying "See this?" She proceeded to shake the box side to side 4 times, and then set the box back to the center of the stage. While the infant was looking at the stage, the experimenter said "Look, [baby's name], look!" and dropped the back curtain, hiding herself from view of the infant. The observer began timing upon hearing the second "look". Trials ended when the infant looked away for 2 consecutive seconds.

The large box was removed from the stage between each familiarization trial, and the back curtain was lowered to conceal the experimenter. These trials were included to familiarize the infants to the population of balls in the large box, as well as to the general procedure of the experiment. The familiarizations lasted about 2 minutes for each trial.

The Test phase consisted of two test trials, a Uniform trial and a Variable trial. On each test trial, the experimenter placed the large box and the small Plexiglas container on the left and right side of the stage (infant's view) respectively. The two objects were placed 8 cm apart. The experimenter then lifted the front curtain of the large box, saying "What's this?" She lowered her head and directed her eye gaze at the box for 1 second, in order to remind the infants of the population of balls in the large box. She then picked up the box and shook it 4 times. After the box was set back down, the experimenter lowered the front curtain to conceal the box's display window. Then, the box was lifted and tilted to its side, allowing one ball to fall out into the small Plexiglas container. Although it appeared that the ball had fallen out from the population of balls at random, the ball actually fell out of the back compartment of the box, which contained balls that had initially been set up by the experimenter. The experimenter then directed her gaze towards the "sampled" ball in Plexiglas container, saying "Look at that!" After 1 second, the ball was returned into the box. This process of revealing the population, shaking the box and tossing a ball out was repeated 3 more times, to make a total of 4 "sampled" balls. When the 4^{th} ball was tossed out, the experimenter said "Look, [baby's name], look!" and dropped the back curtain of the stage. The observer began timing upon hearing the second "look," and ended the trial after the infant looked away for 2 consecutive seconds. Between trials, the stage was cleared and the back curtain was lowered as well. Each test trial lasted for approximately 2 minutes.

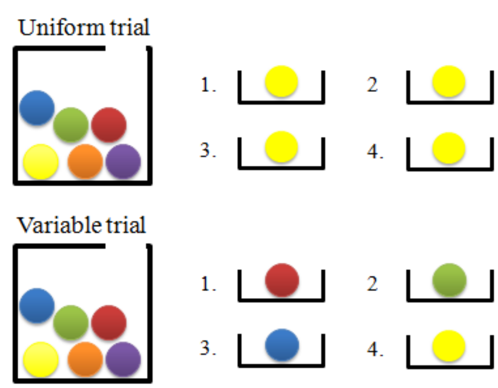

Figure 1: Schematic representation of the two trials.

Each infant participated in a Uniform trial and a Variable trial (See Figure 1). In the Uniform trial, the 4 "sampled" balls were all of the same color (e.g. 4 yellow balls), while in the Variable trial, the 4 "sampled" balls were all of a different color (e.g. 1 red ball, 1 green ball, 1 blue ball, and 1 yellow ball). The last ball that was tossed out in the Variable trial was always identical in color to the balls used in the Uniform trial, to ensure that any difference in looking time was not due to a preference for balls of a certain color. Trial order and the colors of the sampled balls were appropriately counterbalanced across infants.

Baseline Condition The procedure in the Baseline condition was identical to the Sampling condition, except that instead of having the 4 balls being tossed out from the large box, the balls were individually taken out from and returned to the experimenter's pocket. The Baseline condition provided a measure of the infants' pattern of looking times for 4 balls of the same color vs. 4 balls of different colors.

Results

Preliminary analyses found no effects of gender, median age-split (whether the infants were younger or older than the median age of the group), or test trial order (Uniform trial first vs. Variable trial first) on looking times. Subsequent analyses were collapsed over these variables.

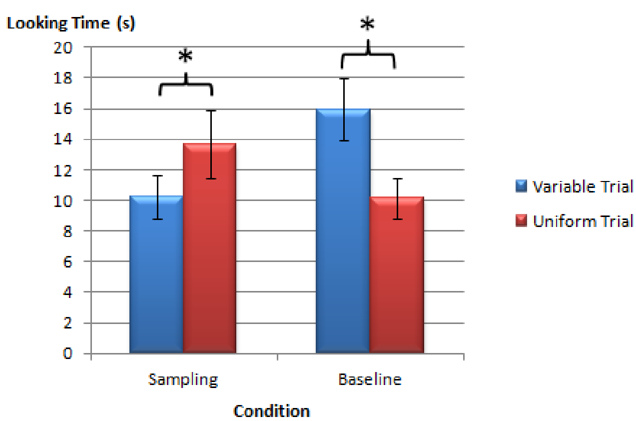

Figure 2: Mean looking times in the Sampling condition and the Baseline condition. Error bars represent SE.

Looking times for the test trials were analyzed using a 2 x 2 repeated-measures ANOVA with Condition (Sampling vs. Baseline) as the between-subjects factor and Trial Type (Uniform vs. Variable) as the within-subjects factor. There was a significant interaction between Condition and Trial Type, $F(1, 38) = 11.58$, $p = .002$, $\eta_p^2 = .23$. There were no main effects found.

To break down the interaction, we conducted follow-up t-tests exploring the effect of Trial Type (Uniform vs. Variable) for each Condition separately (See Figure 2 for mean looking times). In the Sampling condition, infants looked significantly longer in the Uniform trial (M = 13.68s, SD = 9.87) than the Variable trial (M = 10.22s, SD = 6.35), $t(19) = 2.49$, $p = .02$, $d = .42$. Thirteen out of 20 infants in this condition looked longer in the Uniform trial, Wilcoxon signed-ranked test: $z = 1.93$, $p = .05$. In contrast, infants in the Baseline condition looked significantly longer in the Variable trial (M = 15.96s, SD = 9.02) than the Uniform trial (M = 10.14s, SD = 6.01), $t(19) = 2.48$, $p = .02$, $d = .76$. Fifteen out of 20 infants in this condition looked longer in the Variable trial, Wilcoxon signed-ranked test: $z = 2.35$, $p = .02$.

Discussion

In the Sampling condition, infants looked reliably longer when 4 balls that were tossed out at random were all of the same color, than when 4 balls were all of different colors, even though the sequences had equal event probabilities. Hence, infants found it surprising when samples that were being generated from a uniform distribution over the long run were identical, i.e. when 4 randomly generated balls (with replacement) all shared the same color, even though they came from a population of 6 different colored balls. This pattern of looking time was reversed in the Baseline condition, where infants looked longer when 4 balls of different colors were produced from the experimenter's pocket instead. Thus, our findings cannot be attributed to a preference for sequences of identical events. These results support the claim that infants are sensitive to coincidences, even when such suspicious coincidences cannot be predicted by evaluating the mere probabilities of particular events.

Experiment 2

In Experiment 2, we used an exploration measure to examine whether the infants' sensitivity to coincidences translates into action with consequences on their learning. We predicted that infants should play longer with the box that generated the same colored ball each time under random sampling as compared to a box that generated a different colored ball each time.

Method

Participants Fifteen infants (10 males and 5 females, M = 13; 3 [months; days], R = 12; 18 to 13; 29) were tested. All were recruited from Berkeley, California, and its surrounding communities. An additional 3 infants were tested but excluded for not playing with any of the boxes during the test trial. Infants who participated in the experiment were required to be exposed to English a minimum of 50% of the time. Infants received a small gift for their participation.

Materials The materials used in Experiment 2 were identical to those used in Experiment 1, except that the large box containing the population of balls was replaced with two new boxes. Similar to the large box, these two boxes (29 cm x 23 cm x 22 cm) each had a Plexiglas window to display a population of 6 different colored balls, as well as two hidden back compartments. One of the boxes had its surface painted white, with a black fabric curtain covering the display window, while the other box had its surface painted black, with a white fabric curtain covering its window. This design was to enhance infant's discrimination of the two boxes, without biasing the infant towards any of the boxes.

Design and Procedure Infants were tested individually in a forced-choice paradigm. Each infant sat on her parent's lap on the floor approximately 1.2 meters from a puppet stage. Parents were instructed to hold on to their infant, and to avoid influencing their child in any way. They were also told that they would be asked to set their infant on the floor directly in front of their lap when the experimenter gives the instruction, "Do you want to come and play?" towards the end of the experiment. Each experimental session consisted of a Free Play phase, a Demonstration phase, and a Test phase. Two video cameras recorded the infants' and experimenter's behavior during the session.

Free Play Phase This phase was identical to that of Experiment 1.

Demonstration Phase To begin the Demonstration phase, the experimenter placed the two large boxes on the stage about 20 cm apart, with their front curtains down. One of the boxes was a Uniform box, containing 4 balls of the same color hidden in its back compartment. The other box was a Variable box, containing 4 hidden balls of different colors instead. The experimenter also placed a transparent container in the space in front of the center of the two boxes. She then drew the infant's attention to the box on the left, saying "What's in this box?" The front curtain of this box was subsequently lifted, revealing a population of 6 different colored balls. The procedure that followed was identical to an individual test trial in Experiment 1, in which the experimenter seemingly tosses out 4 colored balls from the box at random, one after another with replacement. The only exception was that the 4^{th} ball was returned to the box after 1 second, as looking behaviors were not of interest in Experiment 2. After this 4^{th} ball was returned to the box, the experimenter said "All done!" She then pointed to the box on the right, and said "Let's see what's in this box!" The experimenter then repeated the steps performed on the previous box. This phase lasted approximately 3 minutes.

The boxes that were assigned as the Uniform or Variable box, as well as the colors of the sampled balls, were appropriately counterbalanced across infants.

Test Phase Each infant completed one test trial. The experimenter brought the two large boxes forward and set them down on the ground about 1 m from the infant, saying "Do you want to come and play?" Parents were instructed to let go of their infant if they had not done so at this point. When the infant touched one of the boxes, the experimenter started a timer and the test trial ended after 60 seconds.

Coding Infants were coded for the amount of time in which they were in contact with each of the boxes.

Results

All of the infants' behaviors were coded offline. Preliminary analyses found no effects of gender or demonstration order (Uniform box first vs. Variable box first) on infants' exploration of the boxes. Subsequent analyses were collapsed over these variables.

Preliminary results show that infants played significantly longer with the Uniform box (M = 25.02s, SD = 26.06) than the Variable box (M = 7.02s, SD = 11.78), $t(14) = 2.08$, p = .05, d = 0.95.

Discussion

As predicted, infants played reliably longer with the Uniform box than the Variable box. These results replicate the findings in Experiment 1, demonstrating the infants are sensitive to coincidences that cannot be predicted by mere probabilities. In addition, our preliminary results indicate that infants do translate this sensitivity into action, selectively exploring the box that generated data which was indicative of a suspicious coincidence.

General Discussion

We provide some suggestive evidence that infants are sensitive to coincidences, detecting these anomalous events even when they cannot be predicted by their mere event probabilities. In Experiment 1, infants were presented with a box that ostensibly generated balls under random sampling, creating an expectation that a sequence of tosses should result in a sampling distribution that is even across the 6 colors present in the box. Infants were surprised when the box consistently produced samples that were identical instead. This finding is impressive, considering that the two different sequences that infants saw in Experiment 1 had equal probabilities of occurring. Experiment 2 replicated this novel finding with an older age group through an action measure, and extended the finding by demonstrating that the sensitivity that infants show for coincidences translates into action, as infants preferentially explored a box that produced a sequence of four of the same colored balls, as compared to a box that produced a sequence of four different colored balls.

We speculate that the obtained differences in looking exploration times arise because infants are evaluating the data that they receive according to how well it supports different underlying causal models. For example, a sequence of four yellow balls is surprising because the event provides better support for the alternative theory that the box is rigged, rather than the original theory of random sampling. However, another interpretation of our results is possible, namely that infants (and adults) may consider the uniform sequence "yellow, yellow, yellow, yellow" to be lower in probability than the variable sequence "blue, green, red, yellow." Therefore, the results obtained may be due to a misunderstanding of event probabilities, rather than a consideration of alternative theories. More empirical work is thus necessary to parse these interpretations apart. That being said, our results continue to provide evidence that young infants are sensitive to anomalous data, and will selectively explore the source of these anomalies.

One might also raise the representativeness heuristic as an alternative account of these results (Kahneman & Tversky, 1972). By this account, infants preferred the box that generated the same sample each time because the samples (e.g. 4 yellow balls) were very dissimilar to the population from which they were drawn (i.e. 6 different colored balls). However, recent research examining infants' probabilistic reasoning has rendered this interpretation unlikely, as looking patterns were predicted by probabilistic reasoning and not by the representativeness heuristic when these two interpretations were pitted against each other (Denison & Xu, 2010).

Besides understanding how infants' representation of the presented events led to differences in their looking and exploration times, of interest in these studies is also why such differences arose. We believe that the ideas advanced by Griffiths & Tenenabaum (2007) may shed light on this issue: infants pay attention to the coincidences that they encounter in the world, as these surprising events are likely to be rich sources of information for theory revision. By selectively investigating these events, children provide themselves with an opportunity to make a discovery that can enable them to revise their theories.

The present results thus provide tentative support for a growing set of findings demonstrating that infants and young children attend to the generative process of the data they observe and effectively consider between different models for the inputs that they receive (e.g. Gerken, 2010). Our results also bring additional insight to recent research demonstrating a Goldilocks effect in infant's allocation of attention, which found preferential attention to visual sequences that are neither too simple, nor too complex (Kidd, Piantadosi & Aslin, 2012). In this research, experimenters found that 7-month-old infants are likely to look away earlier for events that are highly predictable. However, predictability is probably not the only determinant of how infants allocate their attention – although a sequence of 4 yellow balls in our experiments was highly predictable, infants paid greater attention to this event because it was inconsistent with their prior expectations about random sampling. Thus, we suggest that infants' consideration of the generative process for the observed data may also play an important role in their allocation of visual attention.

In summary, our experiments provide some suggestive evidence that infants may be sensitive to coincidences. Furthermore, this sensitivity translates into action, as infants preferentially explored the source of such anomalous data. However, many open questions remain: How did infants represent the events presented in our experiments? Why did infants show the obtained differences in looking and exploration time? What is the relationship between infants' looking times and their exploration? Future research examining these questions will provide us with a better grasp of infants' understanding of coincidence, and how children eventually come to have an accurate idea of how the world works by adulthood.

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