Cooperation Decreases with Development of Number Sense

Ellen E. Furlong (ellen.furlong@yale.edu)
Department of Psychology, PO Box 208205
New Haven, CT 06511USA

John E. Opfer (opfer.7@osu.edu)
Department of Psychology, 245 Psychology Bldg
Columbus, OH 43210 USA

Abstract

Cooperation among children can appear haphazard, a finding often attributed to deficient social skills and moral reasoning. Here we took a game theoretical approach to understand development of cooperation, using the prisoner's dilemma to test an alternative source of age-differences in cooperative behavior—how children and adults represent the numerical magnitudes of payoffs for cooperating versus not. We found that as incentives increased solely in numerical magnitude, speed of incentive comparisons decreased and cooperation increased. Further, though children tended to be more cooperative than adults, effect of age on cooperation was moderated by speed of incentive comparison. We conclude that representations of numeric value constrain how economic rewards affect cooperation and that children's greater cooperativeness may be attributed to a poor sense of numerical value.

Keywords: Cooperation; Numerical Cognition; Cognitive Development.

Introduction

Development of cooperation—how it begins, how it changes over time, and what factors promote it—has invited speculation for at least 350 years. According to Rousseau (1754/2007), cooperation is our birthright, society breeds competition; according to Hobbes (1651/2008), we are naturally competitive, society promotes cooperation. Although scientists champion neither position, nearly all look to the same factors—social constructs—to explain development of cooperation (Miles, Hare & Tomasello, 2006; Warneken & Tomasello, 2006; 2007; Warneken, Chen & Tomasello, 2006). Research on the role of social constructs (i.e., theory of mind, communication, fairness norms, trust, social tolerance) on development of cooperation finds support for both views—development breeds either competition or cooperation, depending on the context (i.e., Damon, 1975; Lane & Coon, 1972; Piaget, 1932; Warneken & Tomasello, 2006; 2007; Warneken, Chen & Tomasello, 2006).

One possible way to explain the role of context on development of cooperation is to consider that cooperation may result, not only from developing social skills, but also from how cooperative incentives are mentally represented (Furlong & Opfer, 2009). This role for incentive structure has been explored by game theory, which predicts circumstances under which organisms are likely to cooperate and tests these predictions using games such as Prisoner's Dilemma (PD). Following this approach, we propose a novel and surprising influence on cooperation— how children represent numeric value. In the following sections we: (1) follow Hobbes' lead and provide a game theoretical analysis linking incentive structure to cooperative behavior, (2) explain how developing representations of number affect representation of incentive structures, and (3) show how this analysis accurately predicts Rousseau's claim that cooperation would decrease with age and experience.

Game Theory Links Incentives to Cooperation

Insight into why cooperation depends heavily on contextual factors comes from game theory, which makes predictions about the incentive structures under which organisms are likely to cooperate. Incentive structures in which small immediate costs of cooperation are offset by large immediate benefits, known as mutualisms, commonly lead to cooperation. Even simple organisms—such as fish and ants—readily engage in cooperation under mutualist incentive structures (Bronstein, 2001; Mesterton-Gibbons & Dugatkin, 1992; Trivers, 1971).

While cooperative mutualisms occur readily throughout the animal kingdom, reciprocity--in which short-term costs of cooperation are exchanged in expectation of long-term benefits--is relatively scarce. Indeed, in many cases, these exchanges can be explained by simpler mechanisms such as kin selection, where cooperation does not occur in expectation of any future exchange (i.e., Maynard-Smith, 1965; Trivers, 1971; Stevens et al, 2005).

Biologists typically account for high mutualism rates and low reciprocity rates by arguing that mutualism poses relatively few risks (costs are immediate and relatively low and benefits are immediate and relatively high), whereas the additional temporal element of reciprocity makes it fairly risky (costs are immediate and high and future large benefits are tenuous and may never realize; Maynard-Smith, 1965; Stevens et al, 2005; Trivers, 1971). The likelihood of cooperation depends, therefore, on the relation between benefits and costs—in other words, its incentive structure.

How incentive structure can affect cooperative behavior is often examined using the prisoner's dilemma game (Clements & Stephens, 1995; Noe, 2006; Rapoport & Chammah, 1965; Valev & Chater, 2006). The prisoner's dilemma can be conceptualized in this way: Suppose two children, Bonnie and Clyde, have agreed to charge $3 per

glass in competing lemonade stands. If Bonnie cooperates and charges $3, she'll earn $3; however, if she reneges and drops her price, she may be able to sell more lemonade for a cheaper price (say, 2 cups for $2.50 each yielding $5). If both renege, their prices will drop until they sell lemonade at cost—$1 per cup. If Clyde drops his price, but Bonnie does not, Bonnie will lose her clients to Clyde and earn nothing (Figure 1a).

Generally, if players meet only once, they maximize rewards by defecting; however, if players interact repeatedly, they maximize rewards by cooperating (Axelrod & Hamilton, 1981; Rapoport & Chammah, 1965). Sadly, even in iterated dilemmas, people and animals tend to defect (i.e., Baker & Rachlin, 2002; Dawes & Thaler, 1998). However, reciprocal dilemmas can elicit mutualistic behavior simply by manipulating incentives—for example, by changing the reward for mutual cooperation from $3 to $6 and the temptation to defect from $5 to $8, cooperation rates increase (Figure 1b; Rapoport & Chammah, 1965; Valev & Chater, 2006).
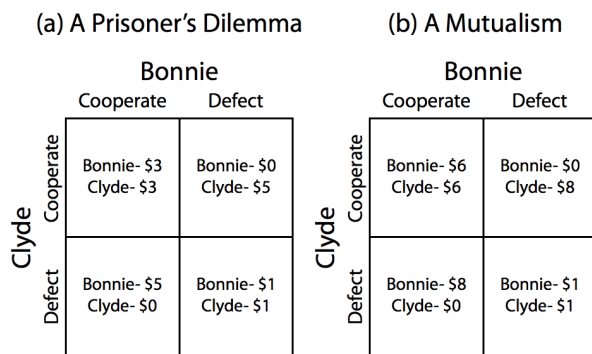

Figure 1. Payoff matrices characteristic of Prisoner's Dilemma (where cooperation is rare) and Mutulism (where cooperation is common)

Representation of Incentive Structure Depends on Representation of Numeric Value

Why manipulating incentives results in mutualistic behavior might be explained by how the brain represents numeric quantity. Specifically, as numeric values increase, discriminability decreases; thus, while participants quickly determine that $5 > 3$, they are slower to determine that $8 > 6$ (Moyer & Landauer, 1967; Starkey & Cooper, 1980).

This numeric size effect fits into a broader literature suggesting non-symbolic numeric quantities may be represented logarithmically: that the brain overestimates differences among small quantities and compresses differences among large quantities (Dehaene, 1997; Nieder & Miller, 2003). Therefore, the difference between 3 and 5 feels larger than the difference between 6 and 8.

The suggestion that subjective incentives in the brain may be quite different than objective incentives in the real world

is not new. This is the chief insight of prospect theory (Kahneman & Tversky, 1979): choices and framing of incentives may affect their subjective value. As Bernoulli (1738/1954) famously observed, "a gain of 2000 ducats is more significant to a pauper than to a rich man though both gain the same amount." The framing of incentives—in this case, the initial endowment—may affect decisions about those incentives.

Although most theories of decision-making rely on prospect theory, in which economic value is subject to size effects (Kahneman & Tversky, 1979), we argue that numeric value, independent of economic value, can affect cooperative behavior. This hypothesis leads to an interesting implication—namely, converting a reciprocal dilemma into a mutualism may not require manipulating economic values of incentives; rather, it may be accomplished by manipulating numeric values alone.
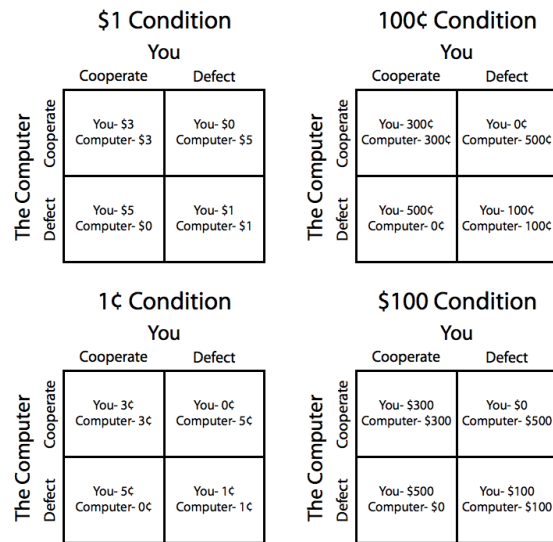

Figure 2: Payoff matrices used by Furlong & Opfer (2009).

This surprising hypothesis was recently tested in a series of experiments in which adult participants played one of four prisoner's dilemma games, identical except for incentive structure (Figure 2; Furlong & Opfer, 2009). As observed in previous studies, subjects in the baseline ($1) condition showed relatively high rates of defection and low rates of cooperation. When rewards were increased a hundred-fold to $100, however, subjects showed the opposite behavior—low rates of defection and high rates of cooperation. This finding could be explained by the standard economic value model—perhaps subjects cooperated more in the $100 condition simply because there was more at stake. On the other hand, subjects may have cooperated more simply because 100 is a larger number than 1. In support of the latter explanation, subjects playing for 100¢, which is economically equivalent to playing for $1 and numerically equivalent to playing for $100, showed identical behavior to those playing for $100. Similarly, subjects in the 1¢ condition behaved identically to those in the $1 condition, even though the increase from 1¢ to $1

represents a one hundred fold increase in economic value. Thus, cooperative behavior changed in response to numeric value, but not in response to economic value.

These numeric-magnitude effects are consistent with the idea that numbers associated with payoff values are represented logarithmically. That is, the linear model predicts defection whenever the ratio between the reward for mutual cooperation and the temptation to defect is less than 1, and because 300¢/ 500¢ equals $3/$5, changing numeric values would not matter. This preservation of ratio information does not obtain if numeric values are scaled logarithmically, as ln(300)/ln(500) is approximately 1 (i.e., temptation to defect and cooperate are nearly equal), whereas ln(3)/ln(5) is approximately .68 (i.e., temptation to defect is higher than temptation to cooperate).

Hypothesized Effects of Developing Number Representations on Cooperation

In Furlong and Opfer's (2009) work, big numbers increased cooperation—a finding predicted by the way the mind represents non-symbolic quantities to increase logarithmically with actual value. When representing symbolic quantities, however, important developmental differences emerge (see Opfer & Siegler, 2012, for review). For young preschoolers, numeric symbols are meaningless stimuli. For example, 2- and 3-year-olds who count flawlessly from 1-10 have no idea that the number 6 is greater than the number 4, nor do children of these ages know how many objects to give an adult who asks for 4 or more (Le Corre et al., 2006; Opfer, Thompson, & Furlong, 2009; Sarnecka & Carey, 2008). As young children gain experience with the symbols in a given numerical range and associate them with non-verbal quantities in that range, they initially map them to a logarithmically-compressed mental number line (Berteletti et al., 2010; Booth & Siegler, 2006; Opfer, Thompson, & Furlong, 2010; Siegler & Booth, 2004; Siegler & Opfer, 2003; Thompson & Opfer, 2010). Over a period that typically lasts 1-3 years for a given numerical range (0-10, 0-100, or 0-1000), their mapping changes from a logarithmically compressed form to a linear form, in which subjective and objective numerical values increase in a 1:1 fashion. Use of linear magnitude representations occurs earliest for the numerals that are most frequent in the environment, that is the smallest whole numbers, and it gradually is applied to increasingly large numbers.

The logarithmic-to-linear shift in children's representations of symbolic quantities expands children's quantitative thinking profoundly. It improves (1) children's ability to estimate the positions of numbers on number lines (Siegler, Thompson, & Opfer, 2010), (2) to estimate the measurements of continuous and discrete quantities (Booth & Siegler, 2006; Laski & Siegler, 2007; Thompson & Siegler, 2010), (3) to categorize numbers according to size (Laski & Siegler, 2007; Opfer & Thompson, 2008), (4) to remember numbers that they have encountered (Thompson & Siegler, 2010), and (5) to estimate and learn the answers to arithmetic problems (Booth & Siegler, 2006). All of these abilities also have important educational roles, leading to use of linear representations of number being highly correlated with mathematics achievement and a broadly effective target of instructional interventions. Thus, children's representations of symbolic quantities—like those used in the payoff matrices of prisoner's dilemma games—change dramatically with age and experience.

Developmental differences in representations of symbolic magnitudes have important implications for how children and adults are likely to respond to economic incentives. That is, if representations of numeric quantity affect cooperative decisions, adults--who are least likely to use logarithmic representations of symbolic quantity—should show the smallest effect of numeric value on cooperative behavior, whereas young children—who are most likely to use logarithmic representations—should show the largest effect of numeric value on cooperative behavior. This is a somewhat surprising and counter-intuitive prediction: because behavioral variability typically decreases with age, effect sizes generally increase with age. To test this hypothesis, we explored the effects of numeric and unit changes on cooperation in third-grade children, fifth-grade children and adults engaged in a prisoner's dilemma game.

Method

Participants

Undergraduate students (23 males, 25 females; M=19.58 years of age, s=1.43), third-grade students (19 males, 29 females; M=9.33 years of age, s=.33) and fifth-grade students (25 males, 23 females; M=11.06 years of age, s=.43) from largely middle-class schools were randomly assigned to play one of four iterated prisoner's dilemma games (IPDs) identical except for payoff structure (Figure 2). All participants received a sticker (children) or course credit (adults) for participating.

Design and Procedure

Participants played IPDs against computers using a "Tit-for-Tat" (TFT) strategy – initially cooperating and thereafter mirroring the participant's behavior on the preceding trial. Participants received no instruction on strategy but were told they were going to play a game called "rock/paper" in which they could earn pretend money (rock was defect and paper cooperate). They were further instructed that the goal was to earn as much money as possible, and that the amount of money they earned depended on how they and the computer played the game. Participants could click on an
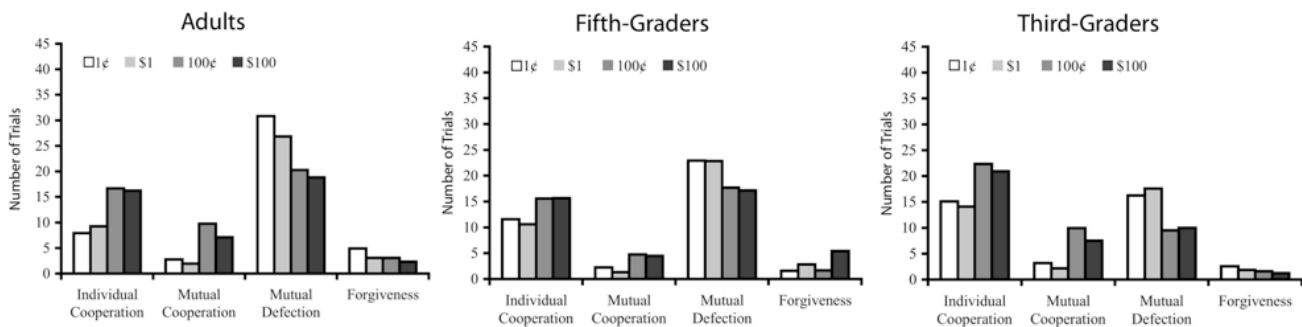
Figure 3. Cooperation in adults, 5th graders and 3rd graders in the prisoner's dilemma game.

icon of a piece of paper (cooperate) or a hand in a fist (defect) to make their choice. Once they made their choice the computer's 'choice' was presented as well as a running total of each player's score. Each participant was allowed as much time as they wanted to complete each of 45 trials.

The design was a 2 (unit: dollars or cents) X 2 (number: 1 or 100) factorial design resulting in four games, identical except for payoff structure (Figure 2) – a numerically small dollars condition ($1), a numerically large cents condition (100¢), a numerically small cents condition (1¢), and a numerically large dollars condition ($100).

We measured four indices of cooperative behavior—individual cooperation (total number of trials in which the participant cooperated), mutual cooperation (number of trials in which participant and computer engaged in cooperation together), mutual defection (number of trials in which participant and computer defected together) and forgiveness, a measure of number of trials to cooperate after the computer's first defection.

To ensure children understood the monetary conversion, children were asked, "how many pennies are in a dollar?" Only one child (a third-grader) answered this question incorrectly; his data were excluded from analyses.

Additionally, subjects participated in a computerized number discrimination task in which they were presented with two numbers (i.e., 3 and 5) and asked to press one of two keys to indicate which was the larger as quickly and accurately as possible. Combinations of the numeric values presented to participants in the numeric discrimination task were identical to the prisoner's dilemma task.

Results and Discussion

First, we explored effects of number, unit and value on cooperative behavior in all three age groups. This analysis is followed by an exploration of the magnitude of the effect of numeric value on cooperation across ages. Finally, we explore the relation between numeric representation in the number comparison task with cooperative behavior in the prisoner's dilemma task.

Two (units: dollars, cents) by two (number: 1, 100) MANOVAs were conducted on the four indices of cooperation. No age groups showed a main effect of unit,

nor did any age group show an interaction of unit with number on their cooperative behavior. Further, no age group showed an effect of economic value (1¢, $1 or $100) on cooperative behavior.

Cooperation in all three groups, however, varied with number (Figure 3; Adults: $F[4, 41]=2.66$, $p=.046$; 5th graders: $F[4, 41]=5.09$, $p=.002$; 3rd graders: $F[4, 41]=3.89$, $p=.009$). Specifically, numerically greater rewards increased individual cooperation (Adults: $F[1, 44]=10.06$, $p=.003$; 5th graders: $F[1, 44]=10.42$, $p=.002$; 3rd graders: $F[1, 44]=13.49$, $p=.001$) such that changing rewards from 3¢ to 300¢ increased cooperation rates, but an economically identical change from 3¢ to $3 did not. The same pattern was evident in rates of mutual cooperation, where numerically large rewards elicited more mutual cooperation than numerically small rewards (Adults: $F[1, 44]=7.18$, $p=.01$; 5th graders: $F[1, 44]=9.46$, $p=.004$; 3rd graders: $F[1, 44]=8.49$, $p=.006$). Further, numerically large rewards elicited less mutual defection than numerically small ones (Adults: $F[1, 44]=9.18$, $p=.004$; 5th graders: $F[1, 44]=6.05$, $p=.02$; 3rd graders: $F[1, 44]=9.75$, $p=.003$). While no effect of number was observed for forgiveness in adults and 5th graders, 3rd graders did show an effect of number on forgiveness ($F[1, 44]=5.94$, $p=.02$), requiring fewer trials to 'forgive' their partner for large numeric values than for small numeric values.

A 3 (age: 3rd grade, 5th grade, adult) X 2 (number: 1, 100) MANOVA also revealed main effects of age ($F[8, 272]=5.92$, $p<.001$) and number ($F[4, 135]=8.30$, $p<.001$) on cooperation. This effect was observed for individual cooperation ($F[2, 138]=9.57$, $p<.001$) and mutual defection ($F[2, 138]=19.05$, $p<.001$). Results for mutual cooperation ($F[2, 138]=2.18$, $p=.11$) and forgiveness $F[2, 138]=2.41$, $p=.09$) trended toward significance. Post-hoc tests revealed 3rd graders had more individual cooperation than both 5th graders and adults (ps < .01). This pattern held true for mutual defection (5th graders: $p=.001$; adults: $p < .001$). Fifth graders and adults did not differ from each other on individual cooperation ($p=.82$) but they trended to differ on mutual defection ($p=.06$). No differences were found between the groups on mutual cooperation or forgiveness.
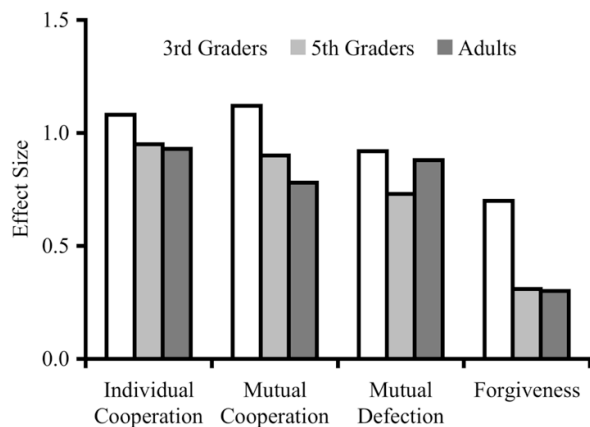
Figure 4: 3rd graders showed larger effects of number on individual and mutual cooperation and forgiveness than 5th graders and adults.

We next compared values of Cohen's *d*, a measure of effect size, for each of the four indices of cooperation in each of the three age groups. We expected to find a larger effect of number (a larger value of *d*) in third-grade children than in fifth-grade children and adults. This predicted pattern was indeed observed for three of our four measures of cooperation: 3rd graders showed a larger effect of numeric value on individual cooperation (*d*=1.08), mutual cooperation (*d*=1.12) and forgiveness (*d*=0.70) than 5th graders (individual cooperation: *d*=0.95; mutual cooperation: *d*=0.90; forgiveness: *d*=0.31) or adults (individual cooperation: *d*=0.93; mutual cooperation: *d*=0.78; forgiveness: *d*=0.30). Effect sizes for mutual defection were roughly equal across all three groups (3rd: *d*=0.92; 5th: *d*=0.73; adults: *d*=0.88; Figure 4).

We hypothesized that the age related increases in effect size were due to number representations, and that the 5th graders were already demonstrating adult-like number cognition. Data from individual participants were analysed to determine whether their reaction times in the number discrimination task were best fit using a linear difference between the two comparison numerals (i.e., 5 − 3) or a logarithmic difference (i.e., ln[5] − ln[3]). This allowed us to classify participants are relying on a more linear or more logarithmic representation.

As predicted, as age increased reliance on a linear representation increased as well ($\chi2(2)$=4.88, p=.08); 57% of 3rd graders were best fit by the linear model; 69% of 5th graders and 77% of adults were best fit by the linear model. A 2 (representation type: logarithmic or linear) X 2 (number: 1 or 100) MANOVA on cooperation revealed a significant effect of representation type on cooperation (F[4, 96]=2.78, *p*=.03) such that participants best fit by the logarithmic model showed greater individual cooperation (F (1, 104)=3.41, *p*=.06) and mutual defection (F (1, 104)=7.40, $p < .01$) than participants best fit by the linear model.

Conclusion

What is the nature of human cooperation? Are we Rousseauian, naturally cooperative, or are we Hobbesian, naturally competitive? The answer may be Both: we start life cooperative (a la Rousseau), but become competitive with age and experience (a la Hobbes). Our cooperative decisions may be shaped, however, not just by changing social influences, but also by developing numeric representations.

Consistent with this perspective, adults--who represent numbers relatively precisely--showed more individual and mutual cooperation and less mutual defection in response to large numbers, not large economic values. Not only did third- and fifth-grade children also demonstrate this pattern, but age related changes in number representation were associated with changes in cooperation: 3rd graders showed a larger effect of number than the older children and adults in individual cooperation, mutual defection and forgiveness.

Further, numerical representations predicted individual cooperation, mutual cooperation, defection and forgiveness rates; subjects who relied more on logarithmic representations demonstrated higher rates of individual and mutual cooperation, lower rates of mutual defection, and took less time to forgive their partner than subjects who relied more on linear representations. These results suggest that logarithmic representations may make it harder to discriminate incentives, resulting in them being treated more like a cooperative mutualism than a reciprocity.

Our results may be able to shed light on previous findings that children appear Hobbesian or Rousseauian depending on the context (Damon, 1975; Lane & Coon, 1972; Piaget, 1932; Warneken & Tomasello, 2006, 2007; Warneken, Chen & Tomasello, 2006). Perhaps these inconsistencies in cooperation can be explained by how costs and benefits are represented in the minds of children. Children may be more likely to cooperate in tasks in which they perceive the costs to be minimal and/or the benefits large (e.g., holding a door open for a stranger), but may be less likely to cooperate in tasks in which they perceive the costs to be large and the benefits minimal (e.g., providing another child with a reward out of one's own stock).

Thus, while it may not be possible to definitively resolve the Hobbes-Rousseau debate, the combination of game theory and psychology of number may make it possible to predict which circumstances incentive cooperation. Put simply, children may be more Rousseauian when costs and benefits are hard to discriminate, but more Hobbesian when they are easily discriminable.

References

Axelrod, R. & Hamilton, W.D. (1981). The evolution of cooperation. *Science, 211*, 1390-1396.

Baker, F. & Rachlin, H. (2002). Teaching and learning in a probabilistic prisoner's dilemma. *Behavioural Processes, 28*, 211-226.

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical Estimation in Preschoolers. *Developmental Psychology, 46*, 545-551.

Bernoulli, D. (1954). Exposition of a new theory on the measurement of risk. In Louise Sommer (Ed. & Trans). Econometica, 22, 22-36 (Original work published 1738).

Booth, J., & Siegler, R. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology , 41*, 189-201.

Bronstein, J.L. (1994). Our current understanding of mutualism. *Quarterly Review of Biology, 69*, 31-51.

Clements, K.C. & Stephens, D.W. (1995). Testing models of non-kin cooperation: Mutualism and the Prisoner's Dilemma. *Animal Behaviour, 50*, 527-535.

Damon, W. (1975). Early conceptions of positive justice as related to the development of logical operations. *Child Development, 46*, 301-312.

Dawes, R. & Thaler, R. (1988). Cooperation. *Journal of Economic Perspectives, 2*, 187-197.

Dehaene, S. (1997). *The Number Sense: How the Mind Creates Mathematics*. New York: Oxford University Press.

Furlong, E.E. & Opfer, J.E. (2009). Cognitive constraints on how economic rewards affect cooperation. *Psychological Science, 20*, 11-16.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*, 1457-1465.

Hobbes, T. (2008). Leviathan. In J.C.A. Gaskin (Ed. & Trans). New York: Oxford World Classics. (Original work published 1651).

Kahneman, D. & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263-291).

Lane, I.M. & Coon, R.C. (1972). Reward allocation in preschool children. *Child Development, 43*, 1382-1389.

Le Corre, M., Van de Walle, G., Brannon, E.M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130–169.

Maynard-Smith, J. (1965). The evolution of alarm calls. *American Naturalist, 99*, 59-63.

Meles, A., Hare, B. & Tomasello, M. (2006). Chimpanzees recruit the best collaborators. *Science, 311*, 1297-1300.

Mesterton-Gibbons, M. & Dugatkin, L.A. (1992). Cooperation among unrelated individuals: Evolutionary factors. *Quarterly Review of Biology, 67*, 267-281.

Moyer, R.S. & Landauer, T.K., (1967). Time required for judgments of numerical inequality. *Nature, 215*, 1519-1520.

Nieder, A. & Miller, E.K. (2003). Coding of cognitive magnitude: Compressed scaling of numerical information in the primate prefrontal cortex. *Neuron, 37*, 149-157.

Noe, R. (2006). Cooperation experiments: coordination through communication versus acting apart together. *Animal Behaviour, 71*, 1-18.

Opfer, J. E., & Thompson, C. (2008). The trouble with transfer: Insights from microgenetic changes in the representation of numerical magnitude. Child Development, 79, 788-804.

Opfer, J. E., Thompson, C., & Furlong, E. (2010). Early development of spatial-numeric associations: Evidence from spatial and quantitative performance of preschoolers. *Developmental Science, 13*, 761-771.

Peters, E., Vastfjall, D., Slovic, P., Mertz, C.K., Mazzocco, K., & Dickert, S. (2006). Numeracy and decision making. *Psychological Science, 17*, 407-413.

Piaget, J. (1932). *The Moral Judgment of the Child*. New York: The Free Press.

Rapoport, A. & Chammah, A.M. (1965). *Prisoner's Dilemma: A Study in Conflict and Cooperation.* Ann Arbor: The University of Michigan Press.

Rousseau, J.J. (2007). *A Discourse of on the Origin of Inequality and A Discourse on Political Economy.* In G.D.H. Cole (Ed. & Trans). Stillwell, KS: Digireads.com (Original work published 1754).

Siegler, R.S. & Booth, J.L. (2004). Development of numerical estimation in young children. Child Development, 75, 428-444.

Sielger, R.S. & Opfer, J.E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. Psychological Science, 14, 237-243.

Siegler, R.S., Thompson, C.A. & Opfer, J.E. (2009). The logarithmic-to-linear shift: One learning sequence, many tasks, many time scales. Mind, Brain & Education, 3, 143-150.

Starkey, P. & Cooper, R.G. (1980). Perception of numbers by human infants. Science, 210, 1033-1035.

Stevens, J.S., Cushman, F.A. & Hauser, M.D. (2005). Evolving the psychological mechanisms for cooperation. Annual Review of Ecological Systems, 36, 499-518.

Thompson, C.A. & Opfer, J.E. (2010). How 15 hundred is like 15 cherries: Effect of progressive alignment on representational changes in numerical cognition. Child Development, 81, 1768-1786.

Thompson, C.A. & Siegler, R.S. (2010). Linear numerical magnitude representations aid children's memory for numbers. *Psychological Science, 21*, 1274-1281.

Trivers, R.L. (1971). The evolution of reciprocal altruism. Quarterly Review of Biology, 46, 35-57.

Valev, I. & Chater, N. (2006). Game relativity: How context influences strategic decision-making. Journal of Experimental Psychology: Learning, Memory, and Cognition, 32, 131-149.

Warneken, F. & Tomasello, M. (2006). Altruistic helping in human infants and young chimpanzees. Science, 311, 1301 – 1303.

Warneken, F. & Tomasello, M. (2007). Helping and cooperation at 14 months of age. Infancy, 11, 271-294.

Warneken, F., Chen, F. & Tomasello, M. (2006). Cooperative activities in young children and chimpanzees. Child Development, 77, 640-663.