Half Day Tutorial on Using Quantum Probability Theory to Model Cognition

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General Purpose

This tutorial introduces why and how to build cognitive models using quantum probability (QP) theory. In the tutorial, we will show that QP is inherently consistent with deeply rooted psychological conceptions and intuitions. It offers a fresh conceptual framework for explaining some puzzling empirical findings of cognition, and provides a rich new source of alternative formal tools, compared to classical probability (CP) theory, for cognitive modeling.

CP models, including Bayesian models, have had an enormous influence in cognitive science (e.g., Griffiths et al., 2010). Such formal models are appealing for many reasons. First, CP theory provides an integrated, coherent, self-consistent set of principles, which can be flexibly applied in any inductive inference situation. Second, such approaches are more falsifiable. Core principles of CP theory are inter-dependent, and identifying an empirical violation of one principle in a setting could invalidate the applicability of CP theory as a whole in that setting. Third, CP principles are intuitive. In the words of Laplace (1816, cited in Perfors et al., 2011), "probability theory is nothing but common sense reduced to calculation."

However, human cognition often goes against the description and prescription from CP theory. In one of the most influential empirical traditions in cognitive psychology, Kahneman, Tversky, and colleagues have reported persistent, clear violations of CP principles in decision making (e.g., Tversky & Kahneman, 1974). For example, consider the famous conjunction fallacy. Participants are told of a person, Linda, looking very much like a feminist and unlike a bank teller. Then, they are asked to judge probabilities of some events. Violating CP rules, people think the probability that Linda is a bank teller *and* a feminist is higher than the probability that she is just a bank teller. According to CP theory, it is a fallacy to think P(A and B)>P(A). Importantly, even when we become aware of our "fallacy," we cannot shake off the impression that Linda is indeed more likely to be a bank teller and a feminist, than to be just a bank teller.

Important findings like this have led to intense and extensive controversy about the mechanisms which guide human cognition and decision making. The inspiration for exploring QP theory in cognitive modeling partly arises as a way to resolve this controversy.

The physical theory of quantum mechanics is a marriage between a framework for how to assign probabilities to events and assumptions regarding the nature of the physical world. We can call the former QP theory (or just quantum theory). Can it be applied outside of physics? The motivation for doing so is twofold. First, QP theory is a highly rigorous framework for probabilistic inference. It has been developed over several decades by some of the most brilliant scientists of all time (e.g., Bohr, Dirac, von Neumann, Planck) and has been intensely scrutinized ever since. Thus, the application of QP theory in cognitive modeling has exactly the same *formal* advantages as that of CP theory. Second, quantum theory allows us to consider the possible relevance in cognitive modeling of several novel concepts. For example, in quantum theory, a cognitive system can be in a superposition state. This means that relative to a question or measurement, the system is in an indefinite state, with all definite states having potential to be expressed. This provides an intrinsic formal representation of the conflict, ambiguity, or uncertainty that people experience in cognitive processes. For another example: states can be entangled, which means a change in one part of the system inexorably and instantaneously affects another part. Entanglement is a form of extreme association, which can be helpful for formalizing important cognitive processes, such as holism, cognitive dissonance, and social projection.

Fundamental quantum conceptions, such as superposition, entanglement, interference, and complementarity, have no *formal* counterparts in cognitive theory. We are part of a growing group of researchers who have been intensely exploring their applicability in understanding human cognition. Quantum theory reveals alternative intuitions in probabilistic models of cognition. The quantum cognition research program aims to explore whether these alternative intuitions can explain paradoxical findings in decision, memory, and other areas of cognitive processing.

The tutorial introduce the basic principles of quantum theory, in the context of well-known empirical findings in psychological literature. The basic elements of QP theory will require only some knowledge of linear algebra. *No background in physics or quantum theory is assumed.* The tutorial will be self-contained. It will show how probability computations can be carried out in quantum theory, how one can build quantum cognitive models, and what the nature of probabilistic intuition is in such models. The tutorial will be useful to all researchers interested in modeling cognition.

Previous Tutorials and Symposia

Similar tutorials have been presented regularly at the CogSci meetings in Nashville (2007), Washington DC (2008), Amsterdam (2009), and Sapporo (2012), and the Society of Mathematical Psychology meeting (2012). Around 30-50 participants attended each of the tutorials, with an increasing number of attendees over the years. We have been invited to present short workshops at various universities, such as University of Osnabruck, university of Cincinnati, and Cornell University. At the 2011 CogSci meeting, we co-organized a symposium covering recent progress in the quantum cognition research program. Other tutorials were organized for the annual meetings of Quantum Interaction (since 2009; about 40 participants).

Presenters

The main presenters, Pothos and Wang, have both contributed extensively to the quantum cognition research program. They both have multiple publications on quantum cognitive models in psychological journals targeting a broad audience. Their presentation will be rigorous, clear, relevant, and accessible. Notably, Pothos has recently co-authored a *Behavioral & Brain Sciences* target article, summarizing progress with the quantum cognition research program. Wang has co-edited a special issue of *Topics in Cognitive Science* that synthesizes current research on quantum cognitive models. Also, both Pothos and Wang have good experience with traditional cognitive models and are currently associate editors for the *Frontiers in Cognitive Science* journal. Finally, Busemeyer is one of the pioneers of the quantum cognition research programme and has extensive relevant publication and editorial experience.

Material to be covered

The tutorial will be organized in two parts: (1) an introduction to the key concepts and mathematical modeling tools in QP theory; and (2) an overview of successful cognitive applications, with concrete examples of cognitive models and corresponding MATLAB codes.

In the first part, we will provide a working definition of QP theory. What is it? Why should it be relevant to a cognitive scientist? What are its main characteristics in comparison to CP theory? We will then introduce the basic elements of QP theory (state vector, Hilbert spaces, how to compute simple and conjunctive probabilities) using simple illustrative models of well-known decision and judgment fallacies. We will explain the differences in how probability is computed in the classical vs. quantum way and how these differences give rise to QP theory's unique properties (superposition, incompatibility, interference).

An important question we will address is: is it possible to achieve some sort of isomorphism between (limited cases of) QP and CP models and, if yes, at what price?

We will then introduce structured representations and the idea of entanglement, another unique feature in QP. Time evolution in quantum models will be compared with time evolution in classical models and we will discuss how interference effects can arise in the former, but not the latter, correspondingly leading to violations of the law of total probability, or not.

In second part, we will review successful applications of QP to explain puzzling empirical results in human cognition and decision. We will present some simple MATLAB code illustrating the implementation of QP models in example situations. Perhaps contrary to the common impression of being mysterious and difficult, quantum cognitive models are intuitive. They can be very simple as well, based mostly on linear algebra. We will focus on recent quantum cognition work on probabilistic judgment, measurement order effects, memory, and conceptual combination. What these areas have in common is that they all led to empirical insights which have been hard to reconcile with a CP perspective. Yet, as we will discuss, the unique properties of QP have enabled natural, compelling, and falsifiable accounts of these empirical results. Finally, the tutorial will outline directions for future research.

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