Modeling a Cognitively Limited Network in an Agent-Based Simulation

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Abstract

We investigate how cognitive capacity limits the number of group relations that a person can maintain. The simulation experiment's results using ACT-R and its memory equations replicated an effect similar to that of Dunbar's (1998) number, or the average total number of group ties capable of being supported in memory. In our study, we also examined the influences of two spatial factors (navigation strategies and map configurations) on the growth of generative networks. Our results suggest three interesting conclusions: (a) a fixed-path navigation strategy increases the speed that networks can form; (b) a higher *grid ratio* (connectivity of the agents' world) provides more chances for agents to build relations, and thus increases the network generation speed; but (c) neither factor influenced the total relations that an agent could maintain, which implies that Dunbar's number primarily depends on internal cognitive factors and less on external factors.

Keywords: ACT-R, Cognitive modeling, Social-cognitive network, network formation.

Introduction

What does it mean to know someone? In colloquial English, this can imply anything from knowledge of someone's true character or secrets to a casual friendship. Nevertheless, this kind of knowing seems to imply more than a declarative association. I may know that Michelle Obama is Barrack Obama's wife, but I cannot say that I know either Michelle or Barrack Obama. Knowing in this context seems to imply knowledge not only of an individual's identity but also some knowledge of the significant relationships in their life, knowledge derived from direct interactions with that individual. I may get to *know of* Barrack Obama by reading his memoir but I get to *know* him in a social sense by speaking with him.

In this paper, we begin to explore what it means to know someone in a network, and how that knowledge influences our daily interactions. Drawing from Simon's (1991) work on bounded rationality in organizations and Dunbar's (1998) work examining the connections between cognition and language, we believe this form of knowledge reliably constrains organizations and moderates our behavior. Consequently, we seek to identify more concretely the mechanisms that underlie tie-formation, in other words the foundations of friendship. We begin by modeling the rate of tie formation in cognitively plausible generative networks.

Dunbar (1998) presents empirical evidence that suggests that human social networks are cognitively constrained. Chiefly, he argues that the neocortex size of humans limits the size of a fully connected human social network to about 150 ties. He defines a fully-connected social network as one where all members can not only attach an identity but also a relation to all other members (Dunbar, 1998, pp. 66-68). He further argues that this constraint underlies the small-world effect observed by Milgram (1967) and others. He distinguishes this number of group ties from the number of sympathy ties, the number of intimates a person encounters in a month (n=11-12), or the number of face-to-name matches a human can typically perform (n=1,500-2,000). Dunbar infers these numbers and the relationship between neocortex size and social network size from empirical studies of human and non-human primates. He then compared these findings with anthropological evidence, finding his predictions basically matched the anthropological data.

McCarty, Killworth, Bernard, Johnsen, and Shelley (2001) propose a far larger number (n=291) as an average network size. In part, this discrepancy is rooted in a difference in definitions. McCarty et al.'s (2001) definition of a social tie requires mutual identification as opposed to Dunbar's stricter definition of mutual identification and placement in the network. Also, McCarty et al. suggest other possible sources of discrepancy such as responder biases (number preferences and individual differences), size effects that influence the respondents' ability to accurately estimate the number of acquaintances associated with either very small or large subpopulations, and analysis errors arising from missing data or numerical biases introduced when combining studies. Nevertheless, neither of these potential sources of error nor the difference in definition seem to entirely account for the wide discrepancy in these estimates because, while McCarty et al. allude to ecological effects, neither they nor Dunbar systematically account for them. Also, it remains an interesting question as to what extent the difference in definitions contributes to the difference in estimates.

Ecology (defined here as an actor's physical and social environment) influences cognition not only by presenting a set of opportunities and resources but also by moderating our perceptions of those opportunities (Brantingham & Brantingham, 1993). We also know that humans are sensitive to environment when recalling sets of relations, using different approaches in different settings (Metz & Shultz, 2010). There has been far less work, however, examining to what extent ecology reliably influences tie formation in memory. For instance, one criticism of Dunbar's estimate is that it does not include the significance of environmental complexity. In other words, would the number of social ties, as Dunbar defines them, ever emerge in a distributed social structure like Suburbia, or could it? Does neocortex size impose a maximum, or is the relationship more complex?

More generally, how does memory and environment mediate and constrain social networks?

We begin to examine these questions by modeling the effects of memory on the formation of generative networks (networks arising from an initially empty set). We use McCarty's et al.'s (2001) definition of ties, that is, identification is sufficient to constitute a tie in this experiment. We believe that understanding the rate of network formation is a necessary precondition for reconciling Dunbar's and McCarty et al.'s estimates because this rate, in itself, constrains the opportunities available in the social set. In other words, the rate of network formation influences the emergence of in-and-out group dynamics which in turn mediates the formation of all subsequent groups (Festinger, Schachter, & Back, 1950).

Agent-based simulations have been used for social network studies for many years now. Carley and Newell (1994) were, to our knowledge, the first to use a cognitive architecture (Plural Soar) to study organizations. More recently, some studies have applied cognitive architectures to model human decision making in collaborative tasks (Lebiere, Gonzalez, Dutt, & Warwick, 2009; Morgan, Morgan, & Ritter, 2010; Prietula & Carley, 2001). These authors have, however, primarily focused on small group collaborations and interactions with less than 20 agents.

In this paper, we use a cognitive architecture based socio-cognitive simulation to examine the effect that memory activation thresholds, navigation strategies, and map-configurations have on the rate of network formation. Examining different memory activation thresholds for links between agents enables us to model not only the effects of memory retention on network formation but also provides us a means of representing differences in the modeled social ties' *quality*, as Dunbar defines this term (Dunbar, 1998, pp. 76-77). In other words, higher quality relationships are associated with greater cognitive investment and higher memory strength. Comparing navigation strategies and map configurations allows us to represent the social opportunities associated with activity spaces in the environment.

This study draws from previous work (Kaulakis et al., 2012) and (Zhao, Kaulakis, Morgan, Hiam, & Ritter, 2012). In Kaulakis et al.'s, we introduced an earlier version of an ecological model and modeling environment (VIPER). Kaulakis et al.'s presents a structural analysis, examining how the agents' declarative representation of their social ties reliably differed from the experiment's ground truth network, or the network formed from all the agents' room co-occurrences. Kaulakis et al. found population size had the greatest influence on network construction in memory, but that the similarity results were tentative. Zhao et al. (2012) elaborated on the model by adding navigation strategies. Zhao et al.'s primary contribution, however, showed that parameters in the simulation, world size, length of interactions, and navigation strategies, led to changes in the agents' average activation values in their social networks. While promising, these studies provided no insight as to the rate of network formation and did not examine Dunbar's number in detail.

To reconcile Dunbar's (1998) and McCarty's et al.'s (2001) estimates, we need to understand time not only as defining all the possible social opportunities available to the network but also how previous tie formation constrains future choices. To do this, we first need some notion of a simulated network's formation baseline, when in other words does the network reach equilibrium and its members are primarily maintaining in memory as opposed to making ties? We examine this question here.

Experiment Environment

To model multi-agent social behavior, we constructed a simulation environment, VIPER. All of our experiments were conducted on a 2GHz eight-core Linux 2.6.31 machine with Ubuntu 11.04 with 8GB of RAM, with SBCL 1.0.52 as our Lisp. We use ACT-R 6 in Anderson et al. (2004).

ACT-R

ACT-R (Anderson et al., 2004) is a cognitive architecture and unified theory of cognition. It tries to provide a fully functional system that produces all aspects of human behavior at the cognitive level. We use ACT-R because its memory mechanisms enable us to fully implement the cognitive capacities and constraints we believe necessary to model the emergence of networks.

VIPER

VIPER, a text-based multi-agent simulation, models physically embodied social networks(Kaulakis et al., 2012). It is designed to support multi-agent simulations used to study network science. It is lightweight in that it is text based, but is extensible and records agent behaviors over time to support studies on how networks form. VIPER represents these constraints in several ways, the chief being a strong separation between the agents and their environment. VIPER is dynamic, agent-based, and designed to be a part of a distributed model that resolves events in either real or accelerated time.

To handle large amount of agents simulation, we utilized file imaging techniques in Linux system to reduce the memory cost of ACT-R. This reduces the cost of a single ACT-R thread from 50 Mb to less than 20MB, which allows us to run 1,000 agents on one machine.

Experiment

To explore the effects of environmental connectivity, navigation strategy, and memory activation thresholds on the pace of network formation, we ran a simulated study that examined each of these three factors.

Map Configuration

Drawing from work in environmental psychology and crime mapping, we know environmental complexity influences network formation; we represent environmental complexity

with three room configurations. We measure the relative connectivity of our three map configurations by defining its *grid ratio*, the ratio of the number of edges over the total number of edges possible for a rectangular grid containing the same number of rooms.

We used three map configurations, shown in Figure 1. The first configuration (Figure 1a) is a two-hallway configuration with *grid ratio* 0.6. This configuration should lead to low connectivity due to the large distances between the agents. The second configuration (Figure 1b) has a central area with *grid ratio* 0.75. We predict that Figure 1b's central meeting point will lead to network connectivity that is less than that found in Figure 1c but greater than that found in Figure 1a. The last configuration (Figure 1c) is a full 5x5 grid with *grid ratio* 1.0.

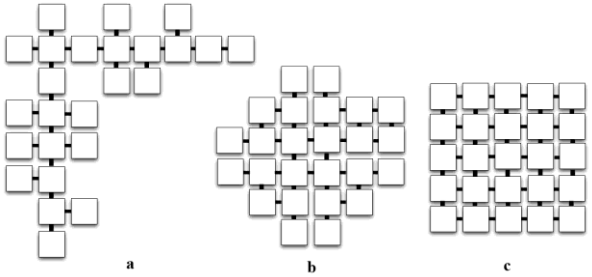

Figure 1: Maps (hallway, central, full grid) used in the simulation study.

Navigation Strategies

In a social network the agents' movement patterns will influence the social network's topology by again influencing any one's agent's interaction opportunities. For example, a policeman walking beat will likely have a larger number of acquaintances than a person who spends most of his or her time at home because the policeman has more opportunities to meet people. To replicate human navigation behavior, we implemented two navigation strategies: random-walk and fixed-path.

1) *The Random-walk strategy* implements a random walk.
2) *The Fixed-path strategy* follows a specific path to navigation in a small area. This strategy simulates the routine navigation behavior, such as going work or going to school.

Experiment Parameters

Zhao et al. (2012) found that the map configurations and navigation strategies influence network measures. In this experiment, we will examine the two navigation strategies for each of the map configurations in 4 runs. The total agent size is currently an arbitrary choice, 40; forty provides a populated but not crowded environment to study. The parameters of the 4 runs are shown in Table 1.

Table 1: Setting of experiment parameters

Runs	Agent size	Map configuration	Navigation strategy
Run 1	40	Hallway	Fixed
Run 2	40	Hallway	Random
Run 3	40	Central	Random
Run 4	40	Grid	Random

To examine the growth curve of the network, we captured the network growth over 18 time slices between 10 and 500 s. Those sample running times were selected by running a pilot experiment, from which we found that the curve changes significantly from 80 to 150 s. We present more sample times here, resulting in a more interesting and precise curve.

Results and Analysis

We examine the effect of the three parameters (map connectivity, navigation strategy, and memory threshold) in order. Each run took approximately 500 seconds in real-time, with the analysis logs being analyzed by hand using ORA (Carley, Reminga, Storrick, & Columbus, 2011).

Memory Networks

With 4 runs and 18 sampling times, we created 2,880 egocentric memory networks (one for each agent, noting who that agent thought they knew, as shown in Figure 2a), and 72 merged memory networks across a run of 40 agents (merging memories across agents in a run, as shown in Figure 2b). Both networks in Figure 1 consist of agents where no memory threshold was applied.

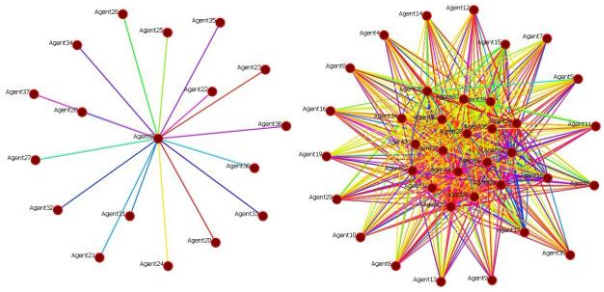

Figure 2: Example egocentric network (left) and merged memory network (right) for agents without a memory threshold applied.

Curves of the Growth of Merged Networks

In this section, we show the effects of the model's three parameters on the rate of tie formation. Based on these figures, we will discuss how memory thresholds, map configurations, and navigation strategies influence the formation rates of simulated networks.

Figure 3 shows the growth curve of a network consisting of agents using a fixed-path navigation strategy in the Hallway map. The lower line represents the network formation rate of a network where no memory threshold was applied—if an agent met an agent, they formed a permanent tie. We find that the lower curve increase rapidly and then

flattens when it reaches 1,336 ties (the maximum is 40*39, or 1,560, if the agents' paths completely overlap, which they do not). This flattening occurs once the network has achieved equilibirum and is fully connected.

In Figure 3, the top solid line represents the network formation rate of a network where an activation threshold of 0.0 was applied. According to the ACT-R theory, the activation threshold represents a memory limitation, meaning that memory chunks with an activition value lower than the threshold cannot be retrieved. The top curve's more gradual progression illustrates the influence of memory on the formation rate, multiple exposures are required to remember another agent. While the difference in total number of links (800 versus 1,336) illustrates memory's effect on the network's topology. In addition, this network never achieves a fully connected state, in the sense that the agent's declarative representation at no point includes the total set of possible interactions. In other words, these agents must continue to maintain their relationships because they continue to forget. Nevertheless, this networks does eventually acheives equilibirum at 150 seconds with a network size of 800 links.

Comparing the two solid curves in the Figure 3, we noticed another difference, the time at which the rate of growth begins to increase. For the thresholded network, this time happens later than for the un-thresholded network. This is because the agents tie formation requires multiple exposures. Initially, agents are busy simply encountering other agents and building their friends list. As they, however, begin to meet more "old friends", the activation values of friendships start to increase. The dash curve in Figure 3 shows the number of relations that could not be retrieved. The curve grows fast at the beginning because most of new ties are weak and un-retrievable. It decrease after 200seconds as the network acheives equilibirum.

The x-axis of the Figure 3 represents the simulation running time in real seconds. In our experiment, we set the traval interval between rooms at 16 seconds to make the effect of memory decay more prominent. Nevertheless, this interval is still not long enough to be realistic because people might take minutes or hours to find another person. As this work only focuses on the growth pattern of the social network, we would argue that the measurement of time is a secondary factor of our study because over 80 percent of the decay happens in the first 16 seconds acorrding to the ACT-R decay equation, with little additional decay occouring at greater time scales. Consequently, we believe total running time of 500 seconds and a short travel intervel of 16 seconds are acceptable for simulating the growth pattern.

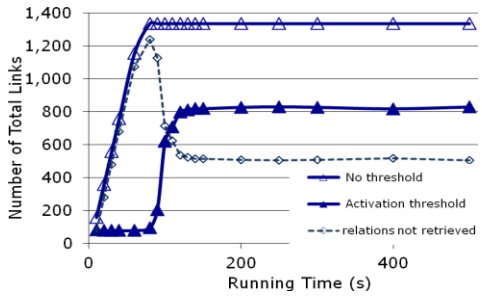

Figure 3: The effect of memory threshold on network formation over time for the fixed path navigation strategy in the hallway map.

Figure 4 shows the growth curve of a network of agents using the random navigation strategy in the Hallway map. Comparing Figure 4 with Figure 3, the non-threshold curves have the same growth pattern, but the threshold curves appear to be different. Memory appears to have different effects based on the setting in which the agents operate.

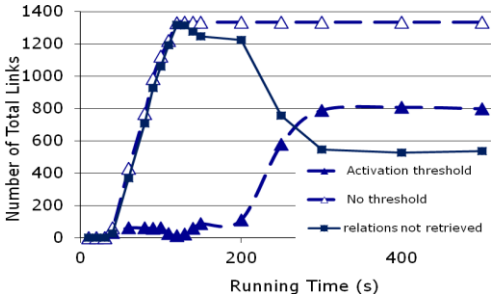

Figure 4: The effect of memory threshold on network formation over time for the random walk strategy in the hallway map.

Figure 5 compares the growth curves of two networks where a memory retrieval threshold of 0.0 was applied; these networks differ with respect to the navigation strategy used by their members. The fixed path-strategy (dash line) forms ties more quickly than the random-path strategy. We suspect that the fixed-path strategy achieves equilibrium sooner because it is more localized, and thus provides more chances for agents to meet their "old friends". On the other hand, both networks achieve equilibrium at about 800 links, suggesting that the navigation strateies in this simulation do not constrain the number of relations an agent can maintain in memory.

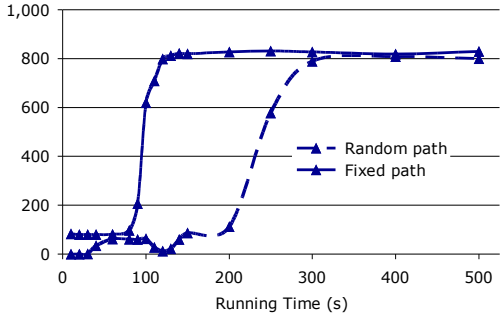

Figure 5: The effect of navigation strategy on network formation over time in the hallway map with threshold.

Figure 6 compares the network formation rates of networks occurring in each of the three map configurations (full grid, central, and hallway); all these networks consist of agents with a memory activation threshold of 0.0. We find that the map configurations have a similar influence on the networks' growth curves as the navigation strategies. Again, the map configurations influence the rate of formation but not the network's size at equilibrium.

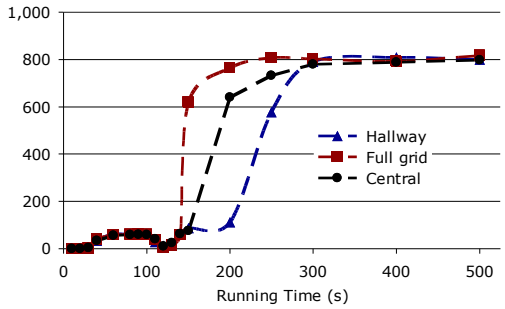

Figure 6: The effect of map configuration on network formation over time.

Comparing the three curves, we find the Hallway map (*grid ratio*=60%) is associated with the longest delay in network formation and the lowest rate of increase; the full grid map (*grid ratio*=100%) has the shortest delay and the fastest rate of link formation. These results show that delay in the network's growth rate is negatively correlated with *grid ratio*, while the network's growth rate during its growth spurt is positively correlated.

Activation Normalization: Semantic Challenges

One of the main issues we face in the analysis and interpretation of our results is the need to assign semantic meanings to the activation values associated with our ACT-R agents' memory chunks. Because raw activation values may grow or shrink indefinitely, we see normalization as a process by which the data can be made more regular, and to help scale between time scales used in our simulation and those occurring in the real world.

In the introduction to this paper, we cited Dunbar's (1998) concerns about tie "quality". In this work, we used raw activation values in our measures, which is fine for our purposes, but is insufficient for many other questions. Activations are not portable or easily interpretable in social terms. To make sensible translations between activation levels and Dunbar's notion of tie quality, we suggest that the ties be normalized as we describe in this section. This normalization recasts activations as statements about the "probability of recall" within a particular timeframe. Using activations in this way supports the measurement of environmental parameters, and the prediction of environmental distractions that are likely to prevent tie consolidation by limiting the time available for tie maintenance. This grounding provides metrics that are empirically measurable and come closer to Dunbar's "quality" concept. Derived from the ACT-R Probability of Recall Equations (Anderson et al., 2004), where the normalized activation value is a function of three variables internal to the agent, such that *i* is the current chunk, *Tau* is the threshold for recall, and *s* is noise, then the normalized value is a probability that a particular chunk will be recalled:

$$A_P(i,\tau,s) = \frac{1}{1+e^{\frac{-(A(i)-\tau)}{s}}} \qquad \text{(eq. 1)}$$

This method fulfills all of the requirements above and provides a concrete interpretation of activation levels as Probabilities of Recall. Additionally, it also ties the threshold to the time of recall in seconds, like this:

$$T_i = Fe^{-A_i} \qquad \text{(eq. 2)}$$

These properties will make the analysis of normalized activation values able to generate much stronger statements about the settings in which the agents live.

Conclusion and Discussion

This study simulated an effect like Dunbar's number on networks of cognitive architecture-based agents. The first analysis examined to what degree **cognitive limitations** (represented by a memory activation threshold) influenced the generative process of a network. *The results suggest that cognitive limitations influence both the rate of network formation and the size of the network at equilibrium.* These findings roughly mirror what is found in empirical studies (Brantingham & Brantingham, 1993).

We can view the progression of the curves in Figures 3-6 as corresponding to three stages in network formation, though at abbreviated time scales. Between 0 and 100 seconds, the size of the network does not grow significantly, and the average number of relations stays constant at 60. This represents the tendency of people to initially remain in localized relations with a few people. During this period of the simulation, the non-thresholded network grows very fast (becoming fully connected at 100 seconds) because most the agents wander around to meet new friends and initialize new relationships. For the thresholded networks, there is greater period of latency because the agents have not yet had the time to consolidate their friendships, but rather are primarily building their friends list. Between approximately 100 to 150 seconds, we see that thresholded networks begin to rapidly increase in size as the agents become more familiar with the activity space. Finally, between 150 to 500 s, the thresholded networks stop growing because the agents have shifted from primarily establishing to maintaining their friends network. In the stable state, the number of total links remains around 800, meaning that the average number of relations per agent in these networks is about 20.

In the second analysis, we examined the influences of two **navigation strategies**. The results suggest that navigation strategies have little influence on the non-thresholded network but for growth time, and it does change the growth speed and pattern of the thresholded network. In Figure 5, we see that the network using the fixed-path strategy grows much faster. This is because the fixed-path strategy is a more focused strategy that provides more chances for

people to meet their "old friends". In this case, people more easily form small groups associated with their starting location, such as people living on the same street or attending the same school. *We see that the fixed-path strategy facilitates the rapid creation of smaller tighter groups than the random-walk strategy*.

The third analysis focused on examining the influences of **spatial configurations** on generative networks. We defined *grid ratio* as the ratio of the number of edges over the total number of possible edges to quantify the connectivity of the map configurations. We find that delay increment is negatively correlated with *grid ratio*, while the formation rate during the growth phase is positively correlated with the *grid ratio*. This result validates our definition of *grid ratio*, because it shows the *grid ratio* does have influence on network formation; it also proves that lower *grid ratio* maps with more gaps and obstacles decrease the network's growth rate. *We found, however, that our map configurations did not influence the final size of the network*.

Summarizing our results, we see that navigation strategies and room configurations only seem to significantly influence our networks' delay increment and growth rate, while the final size of our thresholded networks remains at 800 links. We suspect that one possible way to adjust the final size of the network is by changing the cognitive parameters in ACT-R, for instance adjusting memory decay speed or base level learning activation. Moreover, our results also imply, at least for our world, that navigation strategies and environmental complexity do not significantly influence the number of friends that a person can maintain in memory (Dunbar's number), as the average number of relations were same for both networks. They do, however, suggest the ecological factors significantly contribute to the degree of localization, and perhaps in a more complex world the total size and evolution of the network as defined by the number of total environmental possibilities.

Future Work

Future work will build upon both our insights regarding the effect of cognitive resources on network topology, as well as rate of growth. First, we would like to extend our analysis of the normalized thresholds to see if there are regularities in their effects on network topology. Second, we will run more agents, because our test systems were kept deliberately small. Finally, we will extend our analysis on the effects of cognition on network measures analogous to Dunbar's Number, such as information and knowledge diffusion.

Acknowledgments

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References

Anderson, J. R., Bothell, D., Byrne, M. D., Douglass, S., Lebiere, C., & Qin, Y. (2004). An integrated theory of the mind. *Psychological Review, 111*(4), 1036-1060.

Brantingham, P. L., & Brantingham, P. J. (1993). Nodes, paths and edges: Considerations on the complexity of crime and the physical environment. *Journal of Environmental Psychology, 13*(1), 3-18.

Carley, K. M., & Newell, A. (1994). The nature of the social agent. *Journal of Mathematical Sociology, 19*(4), 221-262.

Carley, K., Reminga, J., Storrick, J., & Columbus, D. (2011). ORA User's Guide, Carnagie Mellon University.

Dunbar, R. I. M. (1998). *Grooming, gossip, and the evolution of language.* Cambridge, MA: Harvard .

Festinger, L., Schachter, S., & Back, K. (1950). The Spatial Ecology of Group Formation. In S. S. Festinger, & K. Back (Eds.), *Social Pressure in Informal Groups*.

Kaulakis, R., Zhao, C., Morgan, J. H., Hiam, J. W., Sanford, J. P., & Ritter, F. E. Defining factors of nterest for large-scale socio-cognitive simulations. In the proceedings of ICCM 2012, Germany.

Lebiere, C., Gonzalez, C., Dutt, V., & Warwick, W. (2009). Predicting cognitive performance in open-ended dynamic tasks: A modeling comparison challenge. In *Proceedings of the 9th International Conference on Cognitive Modeling*, Manchester, UK.

McCarty, C., Killworth, P. D., Bernard, H. R., Johnsen, E. C., & Shelley, G. A. (2001). Comparing two methods for estimating network size. *Human Organization, 60*(1), 28-39.

Metz, A., & Shultz, T. R. (2010). Spatial factors in social and asocial learning. Paper presented at the The Annual Meeting of The Cognitive Science Society, Portland. OR.

Milgram, S. (1967). The small world problem. *Psychology Today, 1*(1), 61-67.

Morgan, J. H., Morgan, G. P., & Ritter, F. E. (2010). A preliminary model of participation for small groups. *Computational & Mathematical Organization Theory, 16*(3), 246-270.

Prietula, M. J., & Carley, K. M. (2001). Boundedly rational and emotional agent cooperation, trust, and rumor. In T. C. Castelfranchi, Y. H. (Eds.), *Trust and Deception in Virtual Societies*. Norwell, MA: Kluwer.

Simon, H. A. (1991). *Models of bounded rationality: Economic analysis and public policy*. MIT Press.

Zhao, C., Kaulakis, R., Morgan, J. H., Hiam, J. W., & Ritter, F. E. Socio-cognitive networks: modeling the effects of space and memory on generative social structures. In the Proceedings of *BRIMS 2012*, Amelia Island, FL.