

# Tests and Models of Non-compositional Concepts

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## Abstract

The question of under what conditions conceptual representation is compositional remains debatable within cognitive science. This paper proposes a well developed mathematical apparatus for a probabilistic representation of concepts, drawing upon methods developed in quantum theory to propose a formal test that can determine whether a specific conceptual combination is compositional, or not. This test examines a joint probability distribution modeling the combination, asking whether or not it is factorizable. Empirical studies indicate that some combinations should be considered non-compositionally.

**Keywords:** conceptual representation; compositionality; context; probabilistic tests

## Conceptual Representation

Within cognitive science, the question of how to represent concepts is still being debated. Different positions have been put forward (e.g. the prototype view, the exemplar view, theory theory view), and Murphy (2002) contrasts some of these positions. He asks which is most supported by the various aspects of cognition related to conceptual processing, but concludes somewhat disappointingly, that “there is no clear, dominant winner”. Here, we take the position that it is possible to progress by asking a broader question about the nature of concepts; can they *always* be modeled compositionally? Or do they sometimes take a non-compositional form?

Some arguments for compositionality center around the systematicity and productivity of language; there are infinitely many expressions in natural language and yet our cognitive resources are finite. Compositionality ensures that this infinity of expressions can be processed, as it allows an arbitrary expression to be understood in terms of its constituent parts. Since compositionality is what explains systematicity and productivity, Fodor (1998) claimed that concepts *must be* compositional, however, this is at odds with prototypicality effects (Frixione & Lieto, [In Press]; Fodor, 1998). For example, consider the by now well known conceptual combination PET FISH. A “guppy” is not prototypical PET, nor a prototypical FISH, and yet a “guppy” is a very prototypical PET FISH (Hampton, 1997). Therefore, the prototype of PET FISH cannot result from the composition of the prototypes of PET and FISH, and so the characterization of concepts in prototypical terms is difficult to reconcile with compositionality (Hampton, 1997; Fodor, 1998). This supports a view put forward by Weiskopf (2007) when he observed that conceptual

combinations are “highly recalcitrant to compositional semantic analysis”.

Here, we take a novel approach to this debate, by providing a mathematical test which determines whether a conceptual combination can be considered compositionally, or not. We start with a consideration of what compositionality might mean probabilistically.

## Probabilistic Models of Compositionality

Figure 1 represents a basic probabilistic scenario involving a ‘black box’ composed of two proposed subsystems,  $A$  and  $B$ . What would it mean if this system was declared to be compositional? Acknowledging that it is the experiments which can be performed upon this system (and their likely outcomes) that will define this notion allows us to move beyond philosophy and into the realms of a mathematical definition.

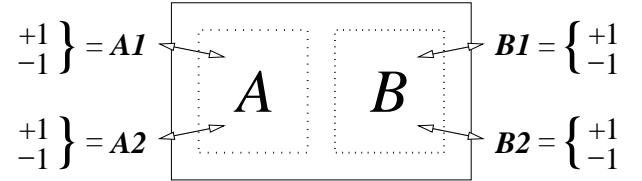


Figure 1: A potentially compositional system, consisting of two identifiable sub-components  $A$  and  $B$ . The system can perhaps be understood in terms of a mutually exclusive choice of experiments upon those sub-components, one represented by the random variables  $A1, A2$  (pertaining to an interaction between the experimenter and component  $A$ ), and the other by  $B1, B2$  (pertaining to an interaction between the experimenter and component  $B$ ). Each of these experiments can return a value of  $\pm 1$ , representing yes and no.

We define a compositional system as one which can be validly decomposed in such a manner that different experiments can be carried out upon each of its sub-systems, and that these will answer a set of ‘questions’ regardless of the experimental behavior of any other sub-systems. For the sake of simplicity, we shall assume that the answers to these questions are binary, they might be termed ‘yes’ and ‘no’, but are for generality labeled as  $+1$  and  $-1$ .<sup>1</sup> Standard probabilistic reasoning suggests that

<sup>1</sup>This assumption is more reasonable than it might at first appear: it is always possible to break a complex question into a set of simple binary questions, as the popular game

it is possible to describe this behavior in terms of four random variables representing the bivalent outcomes:  $\{\mathbf{A1}, \mathbf{A2}, \mathbf{B1}, \mathbf{B2}\}$ . What analysis can be brought to bear upon such a situation? As with many systems, the outcomes of our experiments will have a statistical distribution over all available outcomes, and it is possible to develop a set of probabilistic arguments about this scenario. For example, it is possible to consider the joint probability  $\Pr(\mathbf{A1}, \mathbf{A2}, \mathbf{B1}, \mathbf{B2})$  describing the likely behavior of our experimental black box, however, this very formulation forces us to consider what exactly a non-compositional probability distribution *would look like*. This paper is devoted to answering this question, but in order to approach the answer, we must first provide a model of non-compositional behavior, and this is not an easy task; almost all of our mathematical formalisms are based upon a notion of compositionality (Kitto, 2008). However, one mathematical model is widely accepted as non-compositional, Quantum Theory (QT), and so we take this formalism as the basis of our formulation of non-compositional behavior. Our reasons for this choice will become clearer from a psychological perspective as our argument progresses.

## Senses and Concepts

Our model takes Gärdenfors (2000) conceptual space as its starting point, extending this notion through the use of a vector space representation of concepts. For the purposes of this paper, we shall construct this representation through reference to the word association networks and vocabulary of the human mental lexicon, although this is not a necessary step for the formalism proposed; any sensible vector space construct would suffice if it has a similar structure to that discussed below. The Univer-

Associate	Probability	Associate	Probability
<b>ball</b>	<b>0.25</b>	<b>fighter</b>	<b>0.14</b>
cave	0.13	<b>gloves</b>	<b>0.14</b>
vampire	0.07	<b>fight</b>	<b>0.09</b>
fly	0.06	dog	0.08
night	0.06	<b>shorts</b>	<b>0.07</b>
<b>baseball</b>	<b>0.05</b>	<b>punch</b>	<b>0.05</b>
bird	0.04	<b>Tyson</b>	<b>0.05</b>
blind	0.04	...	...
animal	0.02		
...	...		

(a)

(b)

Figure 2: The free association data for two words, (a) bat, and (b) boxer. Both cases show a clear division of each concept into a sport sense (highlighted in bold), and an animal sense.

sity of South Florida (USF) word association data maps the strength of word associations displayed by a large sample of psychology students over a period of 30 years

of 20 questions illustrates. Quantum theory has provided a more sophisticated proof of this result using the Spectral Decomposition Theorem (Isham, 1995).

(Nelson et al., 2004). In Figure 2 we see a set of association strengths for two words, “boxer” and “bat”. Note the manner in which both words can be attributed a meaning that belongs to one of two *senses*; an animal sense and a sporting sense. Thus, we claim that the concepts BOXER and BAT are both ambiguous. Despite this ambiguity, humans are adept at recognizing the sense that is intended for an ambiguous word. They do this through reference to the context in which the word is being used, and this context might depend upon a wide range of factors (e.g. the co-occurrence of other words spoken before and after, the history of a conversation, the social context of the speaker). We note at this point that even our simple scenario has far more ambiguity than has appeared in the USF data (e.g. some people would interpret boxer as a pair of shorts, and someone could bat their eyes etc.), indeed, there are a wide range of very fine gradations in meaning that might be attributed to even these simple concepts. This added complexity can be dealt with in our model through an extension of the state space to higher dimensions, and through the use of a more sophisticated set of data<sup>2</sup> to construct the vector space model that we shall present.

In the next section we shall show that it is possible to construct a simple model of this ambiguity and its contextual dependency through use of the quantum formalism.

## A Quantum-Like Model of Word Associations

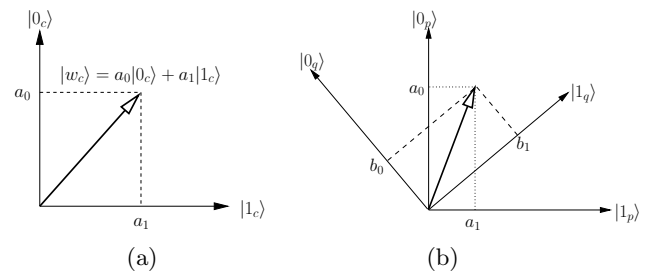


Figure 3: (a) A concept  $w$ , for example *bat*, is represented in some context  $c$  which takes the form of a basis  $\{|0\rangle, |1\rangle\}$ . (b) Changing the cue might change the chance of recall.

A simple model of the manner in which context might affect the interpretation that a subject ascribes to an ambiguous word can be constructed through the use of a *superposition state*, which is a novel concept of a state arising in Quantum Theory (QT). In Figure 3(a), an ambiguous word  $w$  is represented in some context  $c$ , as a superposition of recalled,  $|1\rangle$  and not recalled  $|0\rangle$  within the mind of a subject. When presented with a cue (rep-

<sup>2</sup>Such as the one being collected here: <http://www.smallworldofwords.com>

resented by the context  $c$ ) the subject might return word  $w$ , or not, with some probability. These probabilities can be estimated through reference to the online USF data,<sup>3</sup> which, in the context of a cue word “ball” suggests that a subject will recall the concept BAT with a probability  $Pr(BAT|ball) = .19$ , or they might recall something else ( $Pr(\overline{BAT}|ball) = .81$ ). We put this data into the quantum superposition state of Fig. 3(a) and so represent the cognitive state of our subject in the context of being presented the cue word “ball” as

$$|BAT\rangle_{ball} = \sqrt{0.81}|0\rangle_{ball} + \sqrt{0.19}|1\rangle_{ball}. \quad (1)$$

Figure 3(a) represents these probabilities *geometrically* using the measurement postulate of QT (Laloë, 2001; Isham, 1995), but this same state can be easily obtained through use of the Pythagorean theorem.

This simple model is made more interesting in Figure 3(b), where we have represented the fact that a different context (in this case a cue, but context could be a more complex semantic component) might result in a different set of recall probabilities. Thus, we could represent BAT as the superposition in the cognitive state of a subject when presented with the cue “cave”:  $\sqrt{0.94}|0\rangle + \sqrt{0.06}|1\rangle$  so giving a 6% probability that the word “bat” will be recalled by a subject who is presented with this different cue (or context). We see that the word “bat” is more likely to be retrieved from memory when a subject is presented with the cue “ball” than the cue word “cave”, and this change in probability can be obtained from the *same initial cognitive state* through a shift (i.e. a rotation) in the basis vectors representing the context in figure 3(b).

How should we consider the combination of two words in this model? While it is possible that a simple tensor multiplication of the two superposition vectors might suffice, this is not necessarily the correct mechanism (Bruza et al., 2009). Indeed, it seems possible that not all senses of a word remain accessible during conceptual combination. Thus, it might prove to be the case that a BOXER BAT is only ever interpreted by human subjects as “a small furry mammal with boxing gloves on”, or “a toy bat that a boxer dog chews on”, which would imply a case of perfect anti-correlation in the senses attributed by a subject to the combination. That is, considering the interpretation of the novel (i.e. non-lexicalized) conceptual combination BOXER BAT in the context of two priming conditions, one applied to each of the concepts in the combination (e.g. BOXER primed by “dog” and BAT by “ball”) we denote a concept which is recalled with the same sense as that for which it was primed as 1 and a failure to return in this sense by 0. For this scenario we might find that not all possible combinations of

the two senses in the combination can be realized. For example, we might find that a subject’s cognitive representation of BOXER BAT should be represented as

$$|BOXER\rangle \oplus |BAT\rangle = a|01\rangle + b|10\rangle, \text{ where } |a|^2 + |b|^2 = 1. \quad (2)$$

denoting a scenario where either BOXER has a sporting sense and BAT an animal sense ( $|01\rangle$ ) with probability  $|a|^2$ , or BOXER an animal sense and BAT a sporting sense ( $|10\rangle$ ) with probability  $|b|^2$ .

Such a cognitive state has profound consequences for the notion of compositionality. Indeed, QT has consistently shown that similar states *cannot* be interpreted compositionally (Isham, 1995; Laloë, 2001). Thus, if a similar set of experiments can be found that apply to human language processing, then this would give strong support for the claim that language cannot always be considered compositionally. The remainder of this paper will briefly sketch out recent work which attempts to test for such non-compositional conceptual behavior (Kitto et al., 2010, 2011; Bruza et al., 2012).

## Tests of (Non-)Compositionality

QT has a well developed suite of tests that can be applied to systems of the form shown in Figure 1, and these can be quickly adapted towards the the analysis of compositionality in language. For example, it is possible to construct a variation of the Clauser–Horne–Shimony–Holt (CHSH) inequality (Isham, 1995; Laloë, 2001), using an analysis derived from Cereceda (2000), which tells us that a system of this form can only be described as a combination of it’s subcomponents if:

$$\begin{aligned} 2 \geq \Delta = & |2(\Pr(\mathbf{A1} = +1, \mathbf{B1} = +1) \\ & + \Pr(\mathbf{A1} = -1, \mathbf{B1} = -1) + \Pr(\mathbf{A1} = +1, \mathbf{B2} = +1) \\ & + \Pr(\mathbf{A1} = -1, \mathbf{B2} = -1) + \Pr(\mathbf{A2} = +1, \mathbf{B1} = +1) \\ & + \Pr(\mathbf{A2} = -1, \mathbf{B1} = -1) + \Pr(\mathbf{A2} = +1, \mathbf{B2} = -1) \\ & + \Pr(\mathbf{A2} = -1, \mathbf{B2} = +1) - 2)| \quad (3) \end{aligned}$$

This formula (and a number of variations of it) has a substantial history in the physics and philosophy literature (Laloë, 2001; Shimony, 1984), and lack of space prevents a detailed explanation, however, we can briefly motivate its usage through a discussion of figure 1 and of the potentially compositional nature of the system it describes. Each subsystem  $A$  and  $B$  is represented by random variables:  $\{\mathbf{A1}, \mathbf{A2}\}$  and  $\{\mathbf{B1}, \mathbf{B2}\}$ , denoting whether a particular sense was observed (+1), or not (−1) under a given experimental arrangement. For this system, compositionality is expressed in terms of a *factorizable probability distribution*:  $\Pr(A, B) = \Pr(A) \Pr(B)$ . The syntax of this equation clearly shows how the model of the combined system is assumed to be expressed a product of the distributions corresponding to the individual subsystems  $A$  and  $B$ . When the inequality in (3) is violated, such a

<sup>3</sup>These numbers are obtained by finding the value for “bat” in the “cave” matrix that is depicted at <http://web.usf.edu/FreeAssociation/AppendixC/>.

compositionality assumption does not hold. Thus, the  $\Delta$  value in (3) gives us a clear criterion for deciding whether a given concept combination should be considered compositional or not. In a set of recent work (Kitto et al., 2010, 2011; Bruza et al., 2012), we have performed a number of experiments aimed at testing our formulation of the compositional hypothesis, and we shall now briefly discuss these results.

## Empirical Evaluation

We utilized four different priming regimes in order to generate the four different experimental scenarios required by Fig. 1. These experiments start by biasing subjects towards a particular interpretation of a non-lexicalized conceptual combination through exposure to words that have a particular *sense* representing the underlying concept. They then ask subjects to interpret the conceptual combination, and to designate the senses that they used in that interpretation. If conceptual combinations such as BOXER BAT are genuinely compositional, then it seems reasonable to assume that “vampire” primes BAT but has no priming effect on BOXER. A probabilistic analysis using (3) was performed upon the data obtained to test this assumption.

Table 1 lists the set of ambiguous conceptual combinations chosen, as well as the primes used to bias subjects towards each of two senses for the respective concepts. Primes were selected from the USF norms (Nelson et al., 2004) and the trials were composed of six phases.

**Phases 1-2:** Two consecutive double lexical decision tasks were carried out, where participants were asked to decide as quickly as possible whether two strings, a prime and the concept to be presented as a part of the compound given in Phase 3, were legitimate words, or if one of the strings was a non-word. Participants responded by pushing a button on the keyboard, labeled ‘word’ or a button labeled ‘non-word’ (left arrow and right arrow keys respectively). For instance, if given the strings “coil” and “spring”, then participants were expected to decide that both strings are words and so push the ‘word’ key, whereas if given “grod” and “church” then participants were expected to decide that they had been shown a non-word combination and to push the ‘non-word’ key. Each lexical decision consisted of the the two letter strings presented in the center of screen, one below the other. They were presented in this arrangement to discourage participants from interpreting the two words as a phrase. As soon as the participant responded, the screen was replaced by a blank screen for 800 ms, which was then immediately followed by the second lexical decision phase. The participant’s second lexical decision was followed by a 800 ms blank screen, and then immediately followed by phase 3. For example, one lexical decision task exhibited “coil” and “spring”, and was designed to prime the mechanical sense of the concept SPRING in

the conceptual combination SPRING PLANT. The order of the two double lexical decision tasks was counter-balanced, so that half were presented in the same order as the compound words (e.g., “coil” and “spring” are first presented, then “factory” and “plant”) and half were presented in the reverse order (e.g., first “factory” and “plant” are presented for lexical decision, followed by “coil” and “spring”). **Phase 3:** A bi-ambiguous conceptual combination was presented in the center of the screen (e.g. “spring plant”). Participants were asked to push the space bar as soon as they thought of an interpretation for the compound. Filler compounds were included for the filler (i.e. non-word) trials so as not to disrupt the participant’s rhythm in making two lexical decisions followed by an interpretation. **Phase 4:** Participants were asked to type in a description of their interpretation. **Phases 5-6:** Two disambiguation tasks were carried out, where participants choose what sense they gave to each word from a list (e.g., plant = A. ‘a living thing’; B. ‘a factory’; C. ‘other’). The order of test and filler trials were randomized. Participants completed 24 test trials and 24 filler trials, and the full procedure took 20-30 minutes. Experimental subcomponents utilizing non-words were discarded during the analysis presented here.

## Results

Table 1 lists a number of  $\Delta$  values, some of which violate equation (3). Confidence intervals around the CHSH value  $\Delta$  were computed using the bootstrap method that both removed and added data points that corresponded to interpretations that were either not present or added, and for each iteration, a pseudo  $\Delta$  was computed. Confidence intervals were computed using:  $\text{mean}(\text{pseudo}) \pm t_{0.975, n-1} \sqrt{\text{var}(\text{pseudo})/n}$ .

These results imply that there is good reason to believe that some conceptual combinations must be analyzed in a non-compositional framework. However, it is still possible to provide further details about *how* exactly the joint probability behaves during such a violation. In what follows, we shall analyze three specific examples from the three different categories of result: BOXER BAT (where  $\Delta < 2$ ); APPLE CHIP (where  $\Delta = 2$ ); and BANK LOG (where  $\Delta > 2$ ).

## Further Analysis

It is possible to write the joint probability in a form that starts to explain how violations of (3) occur. To do this we represent the four different random variables  $\{\mathbf{A1}, \mathbf{A2}, \mathbf{B1}, \mathbf{B2}\}$  in a matrix where each random variable contribution is split into a set of possible outcomes. This allows us to break down the results from Table 1 into a form that allows for a consideration of the underlying structure required for violations (or not) of (3). In this representation we can write the data gathered from the above experiments out as a set of joint distributions,

	Concept A		Concept B		Results	
Combination	Prime 1 ( <b>A1</b> )	Prime 2 ( <b>A2</b> )	Prime 3 ( <b>B1</b> )	Prime 4 ( <b>B2</b> )	$\Delta$	n
boxer bat	<i>dog</i>	<i>fighter</i>	<i>ball</i>	<i>vampire</i>	0.91 [0.74,1.09]	64
<b>bank log</b>	<i>money</i>	<i>river</i>	<i>journal</i>	<i>tree</i>	<b>2.13</b> [2.01,2.32]	65
apple chip	<i>banana</i>	<i>computer</i>	<i>potato</i>	<i>circuit</i>	2 [1.82,2.06]	65
stock tick	<i>shares</i>	<i>cow</i>	<i>mark</i>	<i>flea</i>	2.15 [1.98,2.41]	64
<b>seal pack</b>	<i>walrus</i>	<i>envelop</i>	<i>leader</i>	<i>suitcase</i>	<b>2.14</b> [2.01,2.32]	64
<b>spring plant</b>	<i>summer</i>	<i>coil</i>	<i>leaf</i>	<i>factory</i>	<b>2.29</b> [2.18,2.48]	64
<b>poker spade</b>	<i>card</i>	<i>fire</i>	<i>ace</i>	<i>shovel</i>	<b>2.15</b> [2.05,2.33]	65
slug duck	<i>snail</i>	<i>punch</i>	<i>quack</i>	<i>dodge</i>	1.41 [1.20,1.55]	63
<b>club bar</b>	<i>member</i>	<i>golf</i>	<i>pub</i>	<i>handle</i>	<b>2.28</b> [2.17,2.46]	64
web bug	<i>spider</i>	<i>internet</i>	<i>beetle</i>	<i>computer</i>	2 [1.82,2.06]	63
table file	<i>chair</i>	<i>chart</i>	<i>nail</i>	<i>folder</i>	0.38 [0.24,0.50]	63
<b>match bowl</b>	<i>flame</i>	<i>contest</i>	<i>disk</i>	<i>throw</i>	<b>2.21</b> [2.06,2.43]	64
<b>net cap</b>	<i>gain</i>	<i>volleyball</i>	<i>limit</i>	<i>hat</i>	<b>2.17</b> [2.04,2.39]	65
<b>stag yarn</b>	<i>party</i>	<i>deer</i>	<i>story</i>	<i>wool</i>	<b>2.24</b> [2.08,2.36]	61
mole pen	<i>dig</i>	<i>face</i>	<i>pig</i>	<i>ink</i>	1.44 [1.29,1.60]	63
battery charge	<i>car</i>	<i>assault</i>	<i>volt</i>	<i>prosecute</i>	2 [1.81,2.07]	63
count watch	<i>number</i>	<i>dracula</i>	<i>time</i>	<i>look</i>	1.54 [1.39,1.64]	65
bill scale	<i>phone</i>	<i>pelican</i>	<i>weight</i>	<i>fish</i>	1.77 [1.56,1.97]	64
rock strike	<i>stone</i>	<i>music</i>	<i>hit</i>	<i>union</i>	2.01 [1.84,2.18]	64
port vessel	<i>harbour</i>	<i>wine</i>	<i>ship</i>	<i>bottle</i>	1.53 [1.38,1.61]	65
crane hatch	<i>lift</i>	<i>bird</i>	<i>door</i>	<i>egg</i>	2.05 [1.89,2.24]	63
toast gag	<i>jam</i>	<i>speech</i>	<i>choke</i>	<i>joke</i>	1.23 [1.08,1.36]	63
star suit	<i>moon</i>	<i>movie</i>	<i>vest</i>	<i>law</i>	1.68 [1.50,1.84]	62
<b>fan post</b>	<i>football</i>	<i>cool</i>	<i>mail</i>	<i>light</i>	<b>2.13</b> [2.02,2.32]	63

Table 1: Results of the CHSH analysis:  $\Delta$  denotes the CHSH value with an associated confidence interval ( $\alpha = 0.05$ ),  $n$  the number of subjects. Conceptual combinations that significantly violate the CHSH inequality are bolded.

which allows for a further understanding of the resulting behavior.

For example, under this analysis, the joint probability of BOXER BAT can be written as (Bruza et al., 2012):

$$\begin{array}{c} \text{boxer} \end{array} \begin{array}{cc} & \text{bat} \\ & \begin{array}{cc} \mathbf{B1}(ball) & \mathbf{B2}(vampire) \\ +1 & -1 \\ -1 & +1 \end{array} \\ \begin{array}{cc} \mathbf{A1} & +1 \\ (dog) & -1 \end{array} & \begin{pmatrix} 0.43 & 0 & 0.1 & 0.2 \\ 0.28 & 0.28 & 0.1 & 0.6 \end{pmatrix} \\ \begin{array}{cc} \mathbf{A2} & +1 \\ (fighter) & -1 \end{array} & \begin{pmatrix} 0.13 & 0 & 0.31 & 0.06 \\ 0.33 & 0.53 & 0.31 & 0.31 \end{pmatrix} \end{pmatrix} \quad (4)$$

Here, we see no particular ordering or patterns when we compare equations (3) and (4). We can see that the data gathered does not center the distribution in such a way that it can violate the CHSH inequality.

In contrast, APPLE CHIP leads to a joint distribution that has a far more interesting structure:

$$\begin{array}{c} \text{apple} \end{array} \begin{array}{cc} & \text{chip} \\ & \begin{array}{cc} \mathbf{B1}(potato) & \mathbf{B2}(circuit) \\ +1 & -1 \\ -1 & +1 \end{array} \\ \begin{array}{cc} \mathbf{A1} & +1 \\ (banana) & -1 \end{array} & \begin{pmatrix} 1 & 0 & 0.73 & 0 \\ 0 & 0 & 0 & 0.27 \end{pmatrix} \\ \begin{array}{cc} \mathbf{A2} & +1 \\ (computer) & -1 \end{array} & \begin{pmatrix} 0.69 & 0 & 0.53 & 0 \\ 0 & 0.31 & 0 & 0.47 \end{pmatrix} \end{pmatrix} \quad (5)$$

In this case, we see a complete correlation between the subject responses. Thus, whenever a subject interprets APPLE as a fruit they decide that CHIP is a food, and when APPLE is interpreted as a computer then CHIP is

interpreted as an electronic device. This complete correlation of the senses attributed to the bi-ambiguous words leads to a value of  $\Delta = 2$ . This is still a compositional concept combination.

Finally, if we consider conceptual combination BANK LOG then we can see how a non-compositional value of  $\Delta > 2$  is obtained:

$$\begin{array}{c} \text{bank} \end{array} \begin{array}{cc} & \text{log} \\ & \begin{array}{cc} \mathbf{B1}(journal) & \mathbf{B2}(tree) \\ +1 & -1 \\ -1 & +1 \end{array} \\ \begin{array}{cc} \mathbf{A1} & +1 \\ (money) & -1 \end{array} & \begin{pmatrix} 0.94 & 0 & 0.53 & 0 \\ 0 & 0.06 & 0.07 & 0.4 \end{pmatrix} \\ \begin{array}{cc} \mathbf{A2} & +1 \\ (river) & -1 \end{array} & \begin{pmatrix} 0.21 & 0 & 0.8 & 0.13 \\ 0.03 & 0.76 & 0 & 0.07 \end{pmatrix} \end{pmatrix} \quad (6)$$

While this case is similar to the one illustrated in (5), it exhibits a key difference; a non-zero value has been returned by the ensemble of subjects for the off-diagonal case where  $\Pr(\mathbf{A2} = +1, \mathbf{B1} = -1) = 0.13$ , which corresponds to the case where the subjects interpret “bank log” as e.g. a financial institution made of wood. The off-diagonal term in (3) means that there is enough probability ‘mass’ for a violation. Comparing (4–6) with the set of equations typified by (3) we can understand that while it is necessary to for a system violating such inequalities to have some correlation in the random variables, it is just as important to have an anti-correlation.

## Conclusions

There is nothing in equation (3) that restricts its domain of application to quantum theory. Indeed, there are many systems that appear to be separated in a similar way, and so should adhere to the probabilistic behavior that it requires. Indeed, an early work by Aerts et al. (2000) proposed that the formalism of quantum theory could be widely applied to the description of a broad class of nonseparable systems, and this paper further contributes to this stream of work. More recently Bussemeyer et al. (2011) have applied the formalism of quantum theory to obtain a unified description of human decision making and the way in which it violates many of the axioms of standard probability theory. Together with the work presented here, these and many other results suggest that the formalism of QT is widely applicable to the analysis of psychological problems. We suggest that this is due to the ability of the formalism to incorporate a complex notion of *context* into its models, a significant advantage in cognitive modeling.

More specifically, the work presented here has considered only two possible senses for each word, and a very simple priming procedure, but we claim that this is not a limitation of the model *per se*. Firstly, the *spectral decomposition theorem* (Isham, 1995) implies that any measurement can be decomposed into a sum of projection operators. Secondly, more complex primes and cues can possibly be modeled through the use of a vector space approach that extracts the meaning of proceeding sentences, phrases and part word cues.

In summary, it seems likely that a broad class of systems which exhibit strong contextual dependencies among their subcomponents can be well modeled in this approach, and future work will seek to further clarify the conditions under which such systems become non-compositional.

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