

Rational Search of Associative Memory

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Abstract

An important component of many, if not all, real-world retrieval tasks is the decision to terminate memory search. Despite its importance, systematic evaluations of the potential rules for terminating search are scarce. Recent work has focused on two variables: the total time spent in memory search before search is terminated and the exit latency (the time between the last retrieved item and the time of search termination). These variables have been shown to limit the number of plausible rules for terminating memory search. Here, we introduce an alternative stopping rule based on a rational moment-to-moment cost-benefit analysis. We show its ability to capture critical latency data and make testable predictions about the influence of changing the relative costs and benefits of memory search. Results from an experiment are presented that support the model's predictions. We conclude that the decision to terminate memory search is based on moment-to-moment changes in subjective value.

Keywords: Stopping rule; memory retrieval; free recall.

Introduction

One of the most influential developments in cognitive psychology and cognitive science is that of a detailed theoretical framework of memory processes. In the late 1960s, Murdock (1967) summarized a view held by many theorists into the "modal model", a model in which information (memoranda) transfers from sensory memory to short-term memory and then to long-term memory, with each subsequent system having greater memory persistence. The modal model was mainly a framework of memory encoding and the details of memory retrieval were left less-specified. Later theories explicated the retrieval processes in more detail (Anderson, 1972; Metcalfe & Murdock, 1981; Raaijmakers & Shiffrin, 1980, 1981). A common aspect in these theories is the assumption that retrieval from memory can be seen as a search process (Yntema & Trask, 1963) which takes time to complete. Importantly, in order to characterize this search process, models of memory were endowed with stopping rules that prevent the models from continuing search indefinitely. Despite the fact that theoreticians have been quick to incorporate stopping rules into models of memory, research evaluating the class of

stopping rules that might characterize people's decision to terminate memory search is limited.

The evaluation of stopping rules in models of recall is of both theoretical and practical interest. From a theoretical perspective, the goal of developing a comprehensive model of memory retrieval necessitates that we specify the control systems that operate on the memory representations (Newell, 1973). Any particular memory model might yield qualitatively different predictions depending on the specification of the control structures. This is particularly true for stopping rules, since the particular stopping rule employed will affect how long the model will persist in search, which can potentially affect the output of the model (number of items retrieved) and retrieval latencies.

From a practical perspective, understanding stopping rules in the domain of memory retrieval can be informative for the development of artificial intelligence and decision support systems, as well as for cognitive models of diagnostic hypothesis generation and judgment (Thomas, Dougherty, Sprenger, & Harbison, 2008). Within these systems, different stopping rules may yield qualitatively different solutions to diagnostic problems, with optimal solutions requiring different stopping rules depending on the task requirements.

In this paper, we extend the analytical work by Harbison et al., (2009) and implemented a stopping rule that is motivated by a rational analysis of memory (Anderson & Milson, 1989). The resulting rational model is tested against new data.

Stopping rules

Atkinson and Shiffrin (1968, page 121) suggested a number of stopping rules, which have been implemented in models by a number of authors. These different stopping rules are: an internal time limit (Davelaar, et al., 2005; Davelaar, 2007; Diller, Nobel & Shiffrin, 2001; Farrell & Lewandowsky, 2002; Metcalfe & Murdock, 1981), a strength threshold (Anderson, et al. 1998; Diller, Nobel & Shiffrin, 2001), and an event-counter that would terminate search after a prespecified number of events (Raaijmakers & Shiffrin, 1980; Shiffrin, 1970).

Given the various stopping rules employed in the literature, it is clear that little heed has been paid to how a chosen stopping rule might affect the model's retrieval dynamics. Furthermore, the empirical research on which to test candidate stopping rules has been missing. The presence of self-terminating stopping rules in models of memory is in recognition of the fact that human observers are often required to self-terminate retrieval. Yet, most empirical studies of free recall have masked the contribution of stopping rules by providing participants with a pre-set retrieval interval. The use of pre-set retrieval intervals eliminates the need for the participant to utilize a stopping rule and even if participants were to use such a rule there would be no method of measuring it.

In order to address stopping rules in recall, one needs to allow participants to terminate their own retrieval episode. Consequently, the procedure of interest here is one in which the participant is given all the time they need for retrieval, but allowed to terminate retrieval whenever they wish (Dougherty & Harbison, 2007; Harbison, et al., 2009). This paradigm yields two temporal variables anticipated by models of memory that are important for understanding search termination, but which have received relatively little attention in the literature. The first of these reaction time measures is total time. Total time indexes the elapsed time between the onset of a retrieval cue (i.e., the initiation of the retrieval episode) and the decision to terminate retrieval (i.e., termination of the retrieval episode). The fact that models of memory incorporate stopping rules suggests that these models yield total time predictions. Obviously, different stopping rules will yield different total time predictions, but on an intuitive level one would expect total time to be monotonically related to total number of items retrieved: Total time should increase with the number of items retrieved.

The second reaction time measure is what Dougherty and Harbison (2007) called the exit latency. Exit latencies index the amount of time between the final successful retrieval and the decision to terminate of search. In contrast to total time, there is no obvious, intuitive prediction regarding how long participants will persist in retrieval as a function of number of successful retrieval attempts. Thus, exit latencies provide a potentially diagnostic source of data for evaluating stopping rules, particularly when considered in conjunction with the total time measure.

Few published studies report data on the two temporal variables relevant for measuring termination decisions (Dougherty & Harbison, 2007; Harbison, et al., 2009; Unsworth, Brewer & Spiller, 2011). In the study by Dougherty and Harbison (2007), participants were visually presented with a cue word and 10 target words (A-X₁, A-X₂, ..., A-X₁₀). They were told to remember the target words that were presented with each cue word. Each list of 10 target words had a unique cue word. Twelve such lists were presented in blocks of three. After each block of lists were presented, participants were given a cue word and had to report verbally as many words studied with that cue word

(A-?) as they could. Responses were recorded and participants pressed the space-bar to indicate that they could not generate additional words. The total time participants spent in search was measured as the time between presentation onset of the cue for retrieval and the time of pressing the space-bar. The exit latency was measured as the time interval between the last retrieved item and the time of pressing the space-bar.

The pattern of results regarding the stopping and exit latencies as a function of the number of words retrieved in that trial has been shown to be consistent across experimental manipulations (Harbison, et al., 2009). Typically, total time is an increasing function of the number of words retrieved in that trial, whereas exit latency is a negatively decelerating function of the number of words retrieved in that trial.

Evaluating Stopping Rules

Harbison et al. (2009) conducted a simulation study to compare several of stopping rules suggested by Atkinson and Shiffrin (1968). They used the Search of Associative Memory (SAM; Raaijmakers & Shiffrin, 1981) and implemented the different stopping rules. The models were evaluated on their fit to data. Of the rules tested, only the total number of failures rule fitted the data both qualitatively and quantitatively. This is the rule that was used in the original SAM paper. The total number of failures rule is a special case of an iterative rule that is only concerned with the current sample from memory and the total accumulated number of failures. This lends itself to a rational analysis of the same rule which can make novel predictions.

We see memory retrieval as a form of information sampling for which a cost is incurred with every sampling attempt and a benefit is obtained for successful retrievals. We define the memory value function in which the total net value during the retrieval phase is a function of the total number of items retrieved at the elapsed retrieval time.

Elsewhere (Davelaar, Yu, Harbison, Hussey & Dougherty, submitted) we have derived a closed-form expression for the exit latency, where the decision to terminate search depends only on the information of the last time-step. We converged on the following rule (cf. Anderson & Milson, 1989):

Terminate search when the additional cost of retrieving the next item starts to outweigh the relative or marginal benefit of having retrieved that item.

We assume that a cost, a , is incurred with every sampling attempt, t , and a benefit, b , is obtained with every successful retrieval. We define the memory value function as:

$$V_t = Q + bN(t) - at \quad (1)$$

where b and a are the benefit and cost parameters. $N(t)$ is the total number of items retrieved at time t . The net value, V_t ,

has a constant, Q , which is interpreted as the starting value that is related to factors such as motivation or time-pressure.

This stopping rule is based on the additional cost of retrieving the next item compared to the relative benefit of having retrieved that item. In other words:

$$\text{cost}(t + 1) - \text{cost}(t) > b/V_t \quad (2)$$

This equation states that when the difference in cost at time t and time $t + 1$ is greater than the relative benefit, the memory search will be terminated.

We implemented this rule in SAM, replacing the retrieval failures rule. Figure 1 (top panel) shows the latency functions for the original SAM model. The retrieval failures rule captures both the increase in total time with total number of words recalled and the convex exit latency function. The bottom panel of Figure 1 shows the latency functions of SAM with relative benefit stopping rule. This model also captures the typical data patterns. In addition, when the relative cost is increased, the model predicts that both latency functions are lowered. That is, increased cost decreases the total time spent in memory search and decreases the time spent after the last item before deciding that further retrieval is futile. Importantly, these changes are independent of the total number of items retrieved.

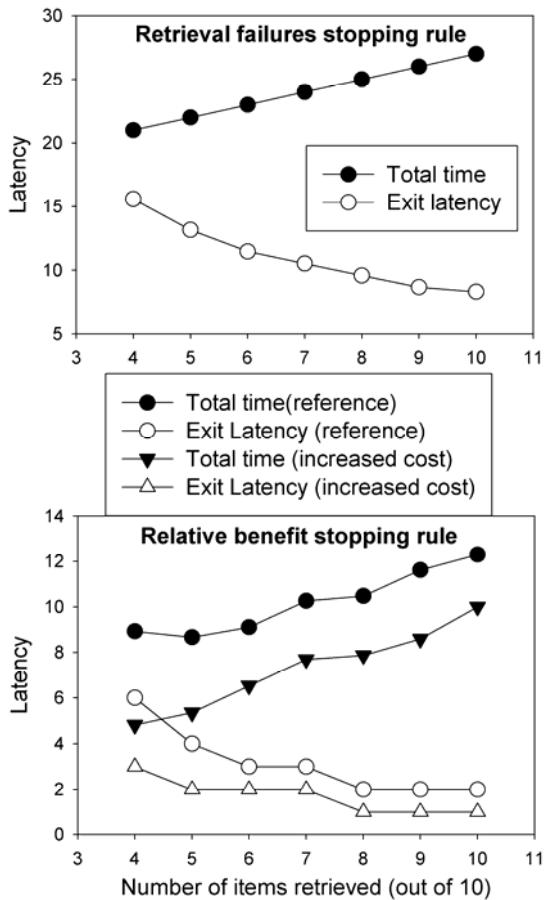


Figure 1. Simulation results of SAM with a retrieval-failures (top) and a relative-benefit stopping rule (bottom).

To summarize, a SAM implementation in which the decision to stop memory search is based on a moment-to-moment cost/benefit analysis predicts that when the cost increases (or benefit decreases) the search will terminate sooner. The next experiment tested these predictions.

Experiment

Methods

Participants

Forty-five college-aged participants were recruited from the University of Maryland subject pool and received performance-based compensation (\$15 or \$20) for participation in the study. Two participants were removed from analysis due to data collection errors.

Design and materials

The design used a delayed free recall paradigm whereby participants studied word lists, completed distractor math problems, and verbally recalled words from the most recent list using a PC-based microphone. The session was presented in two blocks. The first was a baseline block of 16 trials with the same payoff structure across participants (+100 for a correct recall, -100 for each second spent and incorrect recall). In the second block, cost and reward were varied between participants: one group was given an increase in reward (+150) for a correct recall and a simultaneous decrease (-50) in each second spent and each incorrect recall; the other group was given the inverse (+50 rewards, -150 cost). Retrieval protocol followed the self-terminated search paradigm used by Dougherty and Harbison (2007): participants were instructed that they had unlimited time to recall words and could end the recall period at any time by pressing the spacebar. The experimenter monitored the participant's recall and updated the participant's score in real-time, providing feedback to the participant on screen. Thirty-two lists of monosyllabic words were randomly created for each participant. List length was varied between 5, 7, 9, and 11 words and presentation order was randomized to prevent strategy use.

Procedure

Participants were informed they would complete a verbal recall task. The study words were sequentially presented in the center of the computer monitor for 2 s each. Following each study list, a distractor task was presented, which consisted of two simple, timed math problems. Problems contained three digits and two operands (e.g., $3 * 2 + 1 = ?$) and always resulted in a single-digit answer (digits 0-9). A question mark prompted the participant to enter an answer. Components of the math problem were presented sequentially for 1 s each. After two math problems, participants were prompted to begin verbally recalling words from the most recent study list and press the spacebar when they were finished retrieving. After the spacebar press, participants were prompted to press the spacebar again to begin the next study list when they were ready.

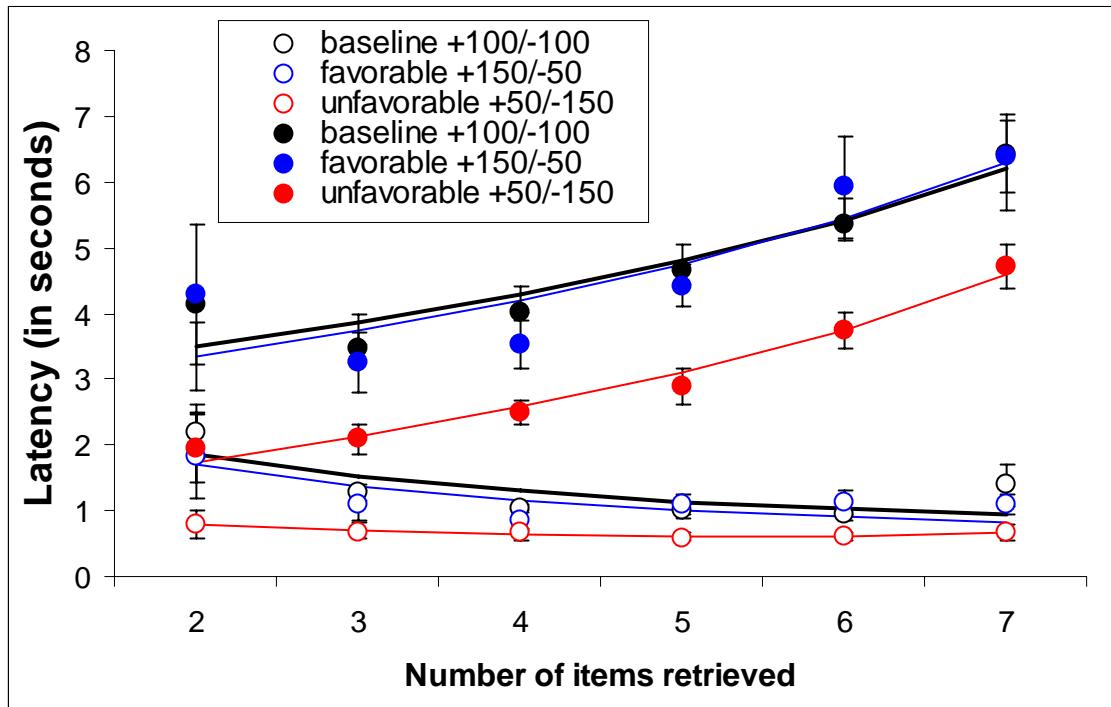


Figure 2. Total time and exit latency functions for the baseline block (both groups combined) and the second block (favorable and unfavorable condition). Error bars are standard errors of the mean. Only the last 8 trials of each block were used. Lines represent the best-fitting regression equation (Davelaar, et al., 2012).

Coding

Using PennTotalRecall audio-analysis software, verbal retrieval data were retrospectively analyzed with millisecond accuracy. Two coders independently coded: 1) all words that were produced by each participant on each trial, 2) the time stamps of the verbal onset of all generated words, and 3) the time stamps of retrieval termination (i.e., times associated with spacebar presses). From these data, number of items retrieved, number of intrusions including repetitions and intra- and extra-list false alarms, inter-retrieval times, and exit latencies (i.e., time between end of final word retrieved and retrieval termination) were calculated. Each subject's trials were averaged before summarizing across subjects.

Results

A 2x2 mixed design included an initial baseline control environment (+100 correct recall, -100 second spent or incorrect recall) and a second payoff environment varied between subjects (favorable: +150, -50; unfavorable: +50, -150). Due to steep learning curves in each new environment, only the last 8 of the 16 trials in each block were included in the following repeated measures ANOVA analyses.

The net points (rewards for correct recalls less the penalties for incorrect recalls and time spent) were updated in real-time for participants to use as feedback to monitor their own retrieval performance. As predicted, net points earned in each block increased over time [$F(1,41) = 6.77$, $p < .013$, $\eta_p^2 = .14$] and the participants for whom the rewards

increased and costs decreased earned more points overall [$F(1,41) = 15.23$, $p < .001$, $\eta_p^2 = .27$]; while net points in the baseline block were equivalent across conditions (favorable: $M = -23.21$, $SE = 41.04$; unfavorable: $M = -35.80$, $SE = 40.10$), performance splits drastically in the second block (favorable: $M = 281.85$, $SE = 54.97$; unfavorable: -161.08 , $SE = 53.71$; condition x time: 38.80 , $p < .001$, $\eta_p^2 = .49$), showing that the manipulation worked.

Total number recalled, including intrusions and repetitions, did not vary due to time, payoff environment, or an interaction of the two [conditions: $F(1,41) = 1.61$, ns, $\eta_p^2 = .04$; time: $F(1,41) = 3.36$, ns, $\eta_p^2 = .08$; condition x time: $F(1,41) = 3.84$, ns, $\eta_p^2 = .09$]. Overall, the rate of intrusions was low (0.3 intrusions per list).

Temporal measures were sensitive to learning across the experiment: total time and exit latency both improved significantly for all participants [total time: $F(1, 41) = 22.19$, $p < .001$, $\eta_p^2 = .35$; exit latency: $F(1,41) = 12.95$, $p < .001$, $\eta_p^2 = .24$]. This performance improvement came primarily from the participants for whom the rewards decreased and the costs increased: the interaction between time and payoff structure was significant for both measures [total time: $F(1,41) = 29.01$, $p < .001$, $\eta_p^2 = .41$; exit latency: $F(1,41) = 9.98$, $p < .003$, $\eta_p^2 = .20$], but the main effects of condition were not significant [total time: $F(1,41) = 1.14$, ns, $\eta_p^2 = .03$; exit latency: $F(1,41) = 2.54$, ns, $\eta_p^2 = .06$].

Figure 2 shows the data on the retrieval latencies broken down by block and condition. Only those trials for which there were sufficient datapoints were included for the model

fit. The solid lines are the best-fitting regression equation derived in Davelaar et al. (2012). This regression model is based on the rational analysis and is a closed-form expression of the simulation rule in equation 2. The use of a closed-form expression facilitates identification of misfits that are due to theoretical misfits instead of sampling noise. The regression model also speeds up the simplex fitting procedure, which requires extremely large samples to fit the latencies at very low and very high total recall. The prediction was that increase in cost or decrease in benefit would lower the latencies. Compared to the baseline condition, making the test hard by increasing the cost and decreasing the benefit did indeed lower all retrieval latencies. Nevertheless, the opposite manipulation, decreasing the cost while simultaneously increasing the benefit, did not change the latencies compared to baseline. We address this asymmetry in the general discussion.

General Discussion

The purpose of this paper was to extend our earlier work on stopping rules by proposing a stopping mechanism that is motivated by a rational analysis of decisions made on a moment-to-moment basis. The resulting rational SAM model produces the typical latency functions that several commonly used stopping rules failed to capture. The model makes testable predictions about the influence of monetary payoff structure on retrieval latencies and the decision to stop memory search.

The prediction was that making it harder to gain points would lower the retrieval latencies due to higher probability of stopping, whereas the reversed would be the case when it was easier to gain points. Interestingly, only the former prediction was borne out by the data and model fits. The results might be seen as an instance of loss aversion by suggesting what could be called an “it-ain’t-broke” hypothesis. Loosely put, when it is harder to obtain points, the cognitive system readjusts itself to avoid losing too much. However, when the environment changes to such an extent that it becomes easier to gain points, the system will not calibrate itself to then minimize the gains. Hence, if the cognitive system is not losing by what it does (i.e., it-ain’t-broke) then there is no reason for adjusting the cognitive parameter (i.e., don’t-fix-it).

Anderson and colleagues provided a rational analysis of the free recall task (Anderson & Milson, 1989; Anderson & Schooler, 1991), in which each item has a need probability associated with it. Only those items are retrieved whose need probability is larger than a certain criterion, which increases with the time spent inspecting an item before accepting or rejecting it. Anderson and Milson (1989) were able to capture a number of basic memory phenomena using their adaptive perspective. However, their analysis only provided the time of the last retrieved item and not of the exact time of terminating memory search. A possibility would be to use the criterion to estimate the termination time, but this would require knowing the functional form of how the criterion changes during item inspection.

Nevertheless, the success of Anderson’s rational analysis and our current results warrants investigating how these can be combined and would allow analyzing the consequences of different retrieval processes on stopping rules. This also applies to research based on the animal foraging literature, such as problem solving (Payne & Duggan, 2011) and information foraging (Pirolli & Card, 199). We leave such an endeavor for the future.

Our analysis suggests that stopping rules should play a more central role in the development and testing of models of memory. The choice of stopping rule has major impact on the overall model behavior. Obviously, one of the ultimate goals of memory theory is to characterize memory retrieval in general, both in and out of the lab. By focusing more on how people terminate memory search, we can bring our models more in line with the type of retrieval tasks that characterize retrieval tasks outside of the free-recall paradigm.

Investigating stopping rules has important implications for understanding tasks other than free-recall. For example, within the medical decision making literature, it is clear that physicians entertain costs when determining when to terminate their retrieval of diagnostic hypotheses from memory (Weber et al., 1993). More recently, Dougherty and Hunter (2003a; 2003b) showed that the perceived probability of any particular event (a hypothesis) is partially dependent on the number of alternatives retrieved from memory, which was affected by time pressure. This suggests that the decision to terminate memory search will affect his or her perceived probability of a particular hypothesis. Within the frequency judgment literature, Brown and colleagues (Brown, 1995; 1997; Brown & Sinclair, 1999; Conrad, Brown, & Cashman, 1998) have shown that participants’ responses to survey questions often are a monotonically increasing function of total time spent searching memory. Thus, the magnitude of participants’ frequency judgments on behavioral survey questionnaires should be affected by when they terminate search of long-term memory. Although the above tasks are all quite distinct, they serve to underscore the ubiquity of stopping rules in real-world retrieval tasks. Therefore, understanding how people terminate memory search, and the psychological variables that affect search termination, is paramount to the development of comprehensive models of memory retrieval and to understanding the dynamics of memory retrieval outside the lab.

In summary, in this paper we obtained further evidence for the view that participants are making adaptive choices to search termination that are based on a cost-benefit analysis.

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