Negating compound sentences

Sangeet Khemlani (khemlani@aic.nrl.navy.mil)
Navy Center for Applied Research in Artificial Intelligence
Naval Research Laboratory, Washington, DC 20375 USA

Isabel Orenes (iorenes@ull.es)
Departamento de Psicología Cognitiva
Universidad de la Laguna, Tenerife, Spain

P.N. Johnson-Laird (phil@princeton.edu)
Department of Psychology
Princeton University, Princeton, NJ, USA

Abstract

How do reasoners negate compound sentences, such as conjunctions of the form *A and B* and disjunctions of the form *A or B or both*? A theory based on mental models posits that reasoners negate each clause independently, and enumerate the various possibilities consistent with the negation. It makes a novel prediction: negations of conjunctions should be more difficult to comprehend than negations of disjunctions. Two experiments corroborate the prediction. Experiment 1 tested participants' ability to comprehend sentential negations by giving them assertions of the form: *Bob denied that he wore a yellow shirt and he wore blue pants on Tuesday*. Participants selected the clothing options that Bob possibly wore on Tuesday. Experiment 2 gave participants descriptions such as *Bob loves Mary or Mary loves John or both*, and they were required to formulate a denial by completing a sentence that started with "No, …". In both studies, participants' responses were more accurate for denials of disjunctions than denials of conjunctions.

Keywords: enumerative negation, sentential negation, conjunctions, disjunctions, and mental models

Introduction

Consider the following two sentences:

1a. It's not the case that *both* Pat loves Viv *and* Viv loves Pat.
1b. It's not the case that Pat loves Viv *or* Viv loves Pat, *or both*.

How do people understand and reason about negated compound sentences like the negated conjunction (1a) and the negated disjunction (1b) above? Which one of the sentences is easier to process? Naïve individuals, i.e., those with no background in logic, syntax, or semantics, should have difficulty understanding the logical negation of multiple clause assertions, particularly when those assertions are complex (Clark, 1974; Clark & Chase, 1972; Hoosain, 1973). Nevertheless, people use negations frequently in everyday reasoning. Indeed, many of the world's languages contain a linguistic construction geared towards negating a disjunction, *neither A nor B*, where *A* and *B* stand for any clauses. Similar constructions exist in many other languages (Gazdar & Pullum, 1976) including German, Malay, and Portuguese. And in logic, the negations of conjunctions and disjunctions, i.e., the NAND and the NOR connectives, can be used to derive every other logical connective. Negations therefore have powerful semantic implications, and they're used often in daily life, but individuals probably do not comprehend the full logical implications of a complex negated assertion. So, how do reasoners cope with sentential negations? In the present paper, we show that the theory of mental models can account for how individuals interpret such negations.

Mental models and enumerative negation

The theory of mental models – the "model" theory for short – posits that individuals use the meaning of an assertion and any relevant knowledge to envisage what is possible (Johnson-Laird, 1983), and that they interpret negations by considering the various possibilities to which the negations refer (Khemlani, Orenes, & Johnson-Laird, 2012). Consider the examples above. When individuals represent the sentential negation of a conjunction, such as (1a), they need to consider the three separate possibilities that render it true. That is, the negation is true when a) neither loves the other; b) Pat doesn't love Viv but Viv loves Pat; or c) Pat loves Viv but Viv doesn't love Pat. In contrast, the sentential negation of the disjunction is consistent with only one possibility: neither loves the other. Assertion 1a above has the grammatical form:

2. It's not the case that both P and V.

where *P* stands for *Pat loves Viv*, and *V* stands for *Viv loves Pat*. According to the model theory, the core interpretation of negation and conjunction, (1a) refers to the following mental models:

¬ P ¬ V
¬ P V
P ¬ V

where '¬' denotes the symbol for negation. And (1b) refers to only the first of these models.

How do individuals construct the models for the assertions above? If individuals had prior knowledge of De

Morgan's laws for interrelating conjunctions and disjunctions, then they would not need to build models, and could simply apply the laws to infer the correct negation. Naïve individuals are unlikely to have mastered De Morgan's laws, however, and so the model theory postulates a more plausible hypothesis. The theory assumes that individuals think about discrete possibilities, where a possibility consists of a conjunction of individuals, their properties, and the relations among them. In the diagram above, the three rows refer to models of three separate possibilities consistent with the negation of the conjunction. To interpret the negation of a multiple-clause assertion, such as (1a), individuals envisage these models separately: they make a series of independent negations of individual clauses *P* and *V*. Hence, with *It is not the case that both P and V*, individuals begin with the possibility in which the negation is applied to each clause: *not-P and not-V*. This possibility is not consistent with the original affirmative assertion, *P and V,* and so they realize that it is one possibility in which the negation holds. At this point, some reasoners may stop and consider only this initial possibility in which both clauses are negated. However, if individuals go further, they apply the negation to only one of the clauses, e.g., *not-P and V*. Once they do, they can detect that it too is inconsistent with the original affirmative and accordingly a possibility consistent with the negation. Likewise, they may grasp that *P and not-V* is also a possibility that renders the negation true. Finally, reasoners neer to consider the case, *P and V*. The possibility is consistent with the unnegated conjunction, and it is therefore inconsistent with the negation of the conjunction.

The general procedure, which we refer to as *enumerative negation*, is to construct a series of models of conjunctive possibilities for any sort of complex compound assertion. It starts with negations of both clauses, and checks whether the resulting possibility is consistent with the unnegated assertion. It then negates each clause, and accepts only those possibilities that are not consistent with the unnegated assertion. Finally, it affirms both main clauses. In each case, if a model is consistent with the unnegated assertion, it is rejected; otherwise, it is accepted as consistent with the negation. This hypothesis applies to all connectives between main clauses, but it is recursive so that it can cope with clauses within clauses. To be right for the right reasons depends on completing the full sequence of all possible conjunctions based on the two clauses.

There is an important rider to enumerative negation: individuals are likely to fail to construct the full sequence of models, which is difficult and time-consuming to envisage. Hence, they should be more likely to respond correctly if they are asked to evaluate given possibilities. In sum, naïve individuals should formulate the denial of compound assertions with multiple main clauses by envisaging, one at a time, the various sorts of possibility in which the denial holds. The order of constructing the models is unlikely to be constant, but it should usually begin with the negations of both clauses.

The model theory of negation makes a novel, and perhaps counterintuitive prediction. In the case, of affirmative assertions, conjunctions are easier to understand than disjunctions, but this difference should switch in the case of their negations. A conjunction has a single model; an inclusive disjunction has multiple models. But, the negation of a conjunction has multiple models; and the negation of an inclusive disjunction has one model. The relation is complementary. The prediction presupposes that the greater the number of mental models of various sorts of compound assertions, the harder it should be to understand them. The effect is easy to understand in the case of compound assertions such as conjunctions and disjunctions. Two atomic propositions and their respective negations yield four possible models:

A	B
A	¬ B
¬ A	B
¬ A	¬ B

A conjunction of the form, *A and B,* refers to only one of these models, but an inclusive disjunction of the form, *A or B or both*, refers to the first three of them. Hence, the conjunction should be easier to understand than the disjunction. In contrast, the negation of the conjunction, *not both A and B*, refers to the three models that are the complement of the model of the original conjunction, *A and B*, whereas the negation of the disjunction, *not (A or B)*, refers to the one model that is the complement of the three models of the original disjunction, *A or B or both*. This predicted interaction hinges, of course, on the theory that individuals construct mental models of assertions, and on the core meaning of negation. Theories in which models of possibilities play no part are unlikely to make the prediction (cf., e.g., Braine & O'Brien, 1998; Rips, 1994).

To test this prediction, we carried out two experiments examining the negation of conjunctions and disjunctions. In both studies, the participants had to deal with denials instead of negations, because pilot studies showed that naïve reasoners don't understand what it means to "negate" a sentence. The studies also examined the denials of conditional *if-then* assertions. The theory predicts that conditionals should be complicated to deny. On the one hand, denials of conditionals should be easier to comprehend than denials of conjunctions because individuals are likely to reduce the scope of the negation to the subordinate *then*-clause (the consequent). On the other hand, the correct negation of the conditional, *A and not-B*, is unlikely to be the first model that reasoners enumerate, so it should be difficult. Thus, the theory predicts that denials of conditionals should be an intermediate case, i.e., not as difficult to understand as denials of conjunctions but more difficult to understand than denials of disjunctions. The results of both studies corroborated the predictions of the model theory.

Experiment 1:
Understanding sentential negations

Experiment 1 tested the enumerative negation hypothesis for the task of listing what is possible given affirmations and denials of three sorts of statement: *A and B*, *A or B or both*, and *if A then B*. Conditionals are complicated. Their affirmations should yield two or three possibilities depending on whether participants make a biconditional, (e.g., If and only if A then B) or a regular conditional interpretation. Their negations, however, should either reduce the scope of the negation to the main clause, *If A then not-B*, or else be the correct response, *A and not-B*.

We carried out various preliminary studies, both online and face-to-face, which showed that the task was difficult. For example, when we asked participants to list what was impossible given a sentential negation, their performance was almost at chance. As a result of these initial studies, we settled on a task in which participants judged whichever of four cases: *A and B*, *A and not-B*, *not-A and B*, *not-A and not-B*, was "possible" given a statement. The statements, in turn, were either affirmations or denials of the three sorts of assertion.

Method

Participants. 22 adult native-English speaking participants were recruited through an online system, Mechanical Turk, hosted by Amazon.com that allows people to volunteer for experiments for monetary compensation.

Design and materials. Participants acted as their own controls and selected the possible instances of three affirmations (based on *and*, *or*, and *if*) and the possible instances of their three denials. The sentences were presented as a block of affirmations and a block of denials in a counterbalanced order. The actual sentences concerned the color of the clothes of various individuals, who affirmed or denied what they wore on a particular day, e.g.,

Bob [asserted/denied] that he wore a yellow shirt [and/or] he wore blue pants on [Monday/Tuesday/...].
Bob [asserted/denied] that if he wore a red shirt then he wore pink pants on [Monday/Tuesday/...].

We used adverbial phrases, such as "on Tuesday", to convey that the statement was about what a person wore on a particular occasion. For the preceding example, the participants indicated whichever of the following cases they judged to be possible given the statement:

Bob wore a yellow shirt and he wore blue pants.
Bob wore a yellow shirt and he wore non-blue pants.
Bob wore a non-yellow shirt and he wore blue pants.
Bob wore a non-yellow shirt and he wore non-blue pants.

The participants were told to select all the cases that they judged to be possible for each sentence. The order of presentation of the four cases was counterbalanced over the trials.

Results and discussion

No reliable difference occurred in the accuracy of the responses in the two blocks, and so we pooled the data for subsequent analyses. The predicted interaction between polarity and the connectives (conjunctions and disjunctions) was reliable. For affirmations, conjunctions yielded 86% correct responses and disjunctions yielded 68% correct responses; whereas for denials, conjunctions yielded 18% correct responses and disjunctions yielded 89% correct responses (Wilcoxon test, $z = 3.47$, $p < .0005$). Denials of conjunctions were very difficult: the participants' mainly judged *not-A and not-B* alone as possible (45%), and 14 out of the 22 participants thought of only one possibility, whether right or wrong (Binomial $p < .005$, given a prior probability of .33).

The data for the conditionals also corroborated the model theory. Their affirmations yielded 45% conditional interpretations, 18% biconditional interpretations, and 27% interpretations equivalent to conjunctions – a phenomenon that occurs in judgments of probability (Girotto & Johnson-Laird, 2004; Johnson-Laird, Byrne, & Girotto, 2009), and which suggests a regression to a more child-like interpretation in a difficult task (see Barrouillet, Grosset, & Lecas, 2000). The denials of conditionals fell mainly into the two predicted categories: an interpretation that reduced the scope of the negation, *if A then not-B* (59%, see Khemlani et al., 2012, for an elaboration of this effect) or the correct response, *A and not-B* (14%). No one selected the correct possibilities for the denial of a biconditional despite the fact that this interpretation occurred in the affirmation.

The task called for the participants to understand affirmative and negative statements and to evaluate explicit possibilities in relation to them. When connectives interrelate main clauses, the model theory predicts the interaction with polarity: conjunctions are easier than disjunctions when they are affirmed, but their relative difficulty is reversed when they are denied. Conditionals also yield the predicted but unusual pattern of judgments: many individuals take the denial of a conditional, *if A then not-B*, to hold in some of the same possibilities as its affirmation, *if A then B*. Since this interpretation yields only a contrary to the affirmed conditional, such "small scope" interpretations are predictable, but erroneous.

When individuals have to formulate a denial of an assertion, their task is to map their models of the possibilities into a conclusion. Hence, the task should be easier in case their starting point is only one model as in the case of a denial of a disjunction than in case it is several models as in the case of a denial of a conjunction. In this way, the enumerative negation hypothesis yields predictions about the formulation of negative statements. Experiment 2 tested these predictions.

Experiment 2: Formulating sentential negations

The previous study examined participants' understanding of denials; Experiment 2 examined their formulation of denials. A preliminary study showed that when individuals are asked to "negate" a conditional, they tended to negate both of its clauses: they did so on 69% of trials. This result suggests that the task of "negating" a compound sentence is unfamiliar to naïve individuals. The present experiment, like the one before it, was accordingly framed in terms of the semantic task of "denial". The participants had to formulate denials of three sorts of sentence:

conjunctions, *A and B*.
inclusive disjunctions, *A or B or both*;
conditionals, *If A then B*;

The enumerative negation hypothesis predicts that individuals should construct a set of conjunctive models and retain those that are inconsistent with the statement. It follows that the participants should tend to be most accurate in denying inclusive disjunctions, because the first conjunction that they are likely to consider, *not-A and not-B*, is the one and only correct denial. They should be less accurate with conditionals, because they are likely to have to construct more than one conjunction before they encounter the correct denial: *A and not-B*. And another factor of greater importance may intervene. Individuals may reduce the scope of the negation, and this process is likely to apply to conditionals too. Hence, some individuals should assert *if A then not-B* as the denial of the affirmative conditional. Finally, the participants should tend to be least accurate with conjunctions, because their correct denial depends on enumerating three models of possibilities: *not-A and not-B*, *not-A and B*, and *A and not-B*. These possibilities are equivalent to the inclusive disjunction: *not-A or not-B*, but this realization is likely to be beyond anyone who does not know De Morgan's laws.

Method

Participants, design, and procedure. 21 native English-speaking participants were recruited though the same participant pool as in Experiment 1. They acted as their own controls and had to formulate denials of six conjunctions, six disjunctions, and six conditionals, all of which were expressed in English, and which were presented to each participant in a different random order. They were instructed to deny the statements by formulating a complete sentence that began with the word, *No*, as a preface to their denial, and the sentence could be of any length. Each clause in the statements to be denied contained two noun phrases based on proper nouns, a transitive verb, and one co-reference, e.g., "If Bob loves Mary then Mary hates Julie." The materials were constructed so that no proper name or transitive verb occurred more than once in the experiment.

Results and discussion

Table 1 presents the percentages of the various sorts of denial. The participants corroborated the predicted trend: they made correct denials for 67% of inclusive disjunctions (*not-A and not-B*), 28% of conditionals (*A and not-B*), and 0% of conjunctions (*not-A or not-B,* or the list of three conjunctive possibilities). The predicted trend was highly reliable (Page's $L = 281.5$, $z = 4.55$, $p < .00001$). The conditionals also elicited 34% of denials of the form: *If A then not-B*, which is consistent with the hypothesis that reasoners reduce the scope of the negation to make it easier to comprehend. The participants making this response tended to be different from those who made the correct denials: 7 out of the 21 participants responded *if A then not B* on half or more of the trials, and 10 out of the 21 participants responded *A and not B* on half or more of the trials. The difference between these two post-hoc groups in the frequency with which they responded *if A then not B* was highly reliable (Mann-Whitney test, $z = 3.50$, $p < .0001$). In accord with the enumerative negation hypothesis, when participants had to deny statements, they were most accurate in denying inclusive disjunctions and least accurate in denying conjunctions. The model theory predicts this result, but it is contrary to Rips's PSYCOP theory (1994, p. 113), which makes the opposite prediction based on its formal rules for De Morgan's laws: $\neg(A \,\&\, B)$, therefore, $\neg A \vee \neg B$; and $\neg(A \vee B)$, therefore, $\neg A \,\&\, \neg B$. For rules that work forwards from premise to conclusion, a single step yields the inference from the negation of a conjunction, whereas three steps based on different rules are needed for the inference from the negation of a disjunction.

Table 1: The percentages of the different denials of disjunctions, conditionals, and conjunctions in Experiment 2, where the balances of responses in each column were different miscellaneous errors that occurred on less than 10% of trials.

	Type of assertion to be denied		
The structure of the participants' denials	Disjunctions: *A or B or both.*	Conditionals: *If A then B.*	Conjunctions: *A and B.*
No, not A and not B.	67	9	66
No, A and not B.	2	28	9
No, if A then not B.	0	34	0
No, not A.	11	3	8
No, not B.	2	21	6

In sum, the model theory may be unique in its prediction that negated conjunctions should be more difficult than negated disjunctions, and the data from Experiment 2 corroborate the hypothesis.

General Discussion

Two experiments showed that participants find negated conjunctions more difficult to understand and to formulate than negated disjunctions, whereas previous research has established that affirmative conjunctions are easier to understand than affirmative disjunctions (García-Madruga, Moreno, Carriedo, Gutiérrez, & Johnson-Laird, 2001). The data therefore revealed a novel interaction between the grammatical form of a sentence and its polarity, and they corroborated a theory of negation based on mental models (Khemlani et al., 2012). The theory posits that individuals do not know the negations corresponding to the different sentential connectives. They have to construct them on an ad hoc basis, so they consider a sequence of conjunctive models of possibilities, checking that they render the corresponding affirmative assertion false. This *enumerative negation* hypothesis predicts that individuals should find it easy to comprehend and formulate denials of inclusive disjunctions of the form *A or B or both*, because the first model that individuals should consider is the only true negation of the disjunction: *not-A and not-B*. In contrast, the hypothesis predicts that a conjunction, *A and B*, should be difficult to deny, because its denial is equivalent to *not-A or not-B or neither*, and so individuals need to envisage fully explicit models of three separate possibilities.

Denials of conditionals with the structure *if A then B* are an intermediate case. They should be easier to comprehend than denials of conjunctions but harder to comprehend than denials of disjunctions. The correct negation of the conditional, *A and not-B*, should be more difficult to envisage because, according to the enumerative negation hypothesis, this model is unlikely to be the first one that comes to mind. And their denials should also be susceptible to a reduction of scope, because *if* introduces a subordinate clause, whereas neither of the other sorts of compound contains a subordinate clause. Hence, some individuals should deny a conditional by using another conditional: *if A then not-B*.

When individuals had to understand affirmations and denials in Experiment 1, their evaluations of what was possible corroborated the model theory's predicted interaction. For affirmations, they found it easier to understand conjunctions than disjunctions, but for denials, they found it easier to understand disjunctions than conjunctions. The affirmation of a conjunction yields one possibility, and the affirmation of a disjunction yields three possibilities. In contrast, the denial of a conjunction requires an inference of three possibilities, and the denial of a disjunction requires an inference of only one possibility. The inferential aspect of this task for negatives may explain why it is so much harder than merely listing the three possibilities corresponding to an inclusive disjunction. Experiment 2 corroborated the interaction. Both experiments also revealed the occurrence of two sorts of negation of conditionals, as did a study by Handley and colleagues (Handley, Evans, & Thompson, 2006). These authors argue that the negation of a conditional, *if A then B*, should be *if A then not-B*. This view, however, has a major drawback: it no longer treats negations as contradicting corresponding affirmatives. Likewise, it offers no principled explanation of why some individuals do take *A and not-B* to be the denial of a conditional, or why most people take it to falsify a conditional too (Espino & Byrne, 2011; Evans, Newstead, & Byrne, 1993; Johnson-Laird & Tridgell, 1972).

Acknowledgements

This research was supported by a National Science Foundation Graduate Research Fellowship to the first author, and by National Science Foundation Grant No. SES 0844851 to the third author to study deductive and probabilistic reasoning. We are grateful to Jay Atlas, Jeremy Boyd, Herb Clark, Alan Garnham, Sam Glucksberg, Adele Goldberg, Geoff Goodwin, Jennifer Heil, Olivia Kang, Philipp Koralus, Mark Liberman, Max Lotstein, Anna Liu, Paula Rubio, Carlos Santamaría, and Elizabeth Sucuyan for their helpful suggestions and criticisms.

References

Barrouillet, P., Grosset, N., & Lecas, J.F. (2000). Conditional reasoning by mental models: chronometric and developmental evidence. *Cognition*, 75, 237-266.

Braine, M.D.S., & O'Brien, D.P., Eds. (1998). *Mental logic*. Mahwah, NJ: Lawrence Erlbaum Associates.

Clark, H.H. (1974). Semantics and comprehension. In T.A. Sebeok (Ed.) *Current trends in linguistics, Vol. 12: Linguistics and adjacent arts* (pp. 1291-1428). The Hague: Mouton.

Clark, H.H., & Chase, W.G. (1972). On the process of comparing sentences against pictures. *Cognitive Psychology*, 3, 472-517.

Espino, O., & Byrne, R.M.J. (2011). It is not the case that if you understand a conditional you know how to negate it. *Journal of Cognitive Psychology,* in press.

Evans, J.St.B.T., Newstead, S.E., & Byrne, R.M.J. (1993). *Human reasoning: The psychology of deduction*. Hillsdale, NJ: Erlbaum.

García-Madruga, J.A., Moreno, S., Carriedo, N., Gutiérrez, F., & Johnson-Laird, P.N. (2001). Are conjunctive inferences easier than disjunctive inferences? A comparison of rules and models. *Quarterly Journal of Experimental Psychology A: Human Experimental Psychology, 54,* 613-632.

Gazdar, G., & Pullum, G. K. (1976). Truth-functional connectives in natural language. *Papers from the Twelfth Regional Meeting of the Chicago Linguistic Society,* 220-234.

Girotto, V., & Johnson-Laird, P.N. (2004). The probability of conditionals. *Psychologia*, 47, 207-225.

Handley, S.J., Evans, J. St. B.T., & Thompson, V.A. (2006). The negated conditional: A litmus test for the suppositional conditional? *Journal of Experimental Psychology: Learning, Memory, Cognition,* 32, 559-569.

Hoosain, R. (1973). The processing of negation. *Journal of Verbal Learning and Verbal Behavior,* 12, 618-626.

Johnson-Laird, P. N. (1983). *Mental models: Towards a cognitive science of language, inference, and consciousness*. Cambridge, MA: Harvard University Press.

Johnson-Laird, P.N., Byrne, R.M.J., & Girotto, V. (2009). The mental model theory of conditionals: A reply to Guy Politzer. *Topoi*, 28, 75-80.

Johnson-Laird, P. N., & Tridgell, J. (1972). When negation is easier than affirmation. *Quarterly Journal of Experimental Psychology,* 24, 87-91.

Khemlani, S., Orenes, I., & Johnson-Laird, P.N. (2012). Negation: A theory of its meaning, representation, and use. *Journal of Cognitive Psychology,* in press.

Rips, L.J. (1994). *The psychology of proof*. Cambridge, MA: MIT Press.