The Effects of Feedback During Exploration Depend on Prior Knowledge

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Abstract

Providing exploratory activities prior to instruction has been shown to facilitate learning. However, questions remain regarding the provision of guidance during the exploration phase. In this study, we replicated and extended a previous experiment by examining the effects of feedback during exploratory problem solving for children with varying levels of prior knowledge. Ninety-five children (M age ≅ 8 yrs) solved 12 novel math problems and then received brief conceptual instruction. After solving each problem, they received (a) no-feedback, (b) outcome-feedback, or (c) strategy-feedback. Consistent with the previous experiment, the results resembled an aptitude by treatment interaction. Feedback during exploration prior to instruction improved children's procedural knowledge, but only for those with low prior knowledge. For children with higher prior knowledge, no feedback resulted in better procedural knowledge. Results suggest that providing feedback may not always be optimal.

Keywords: Guided Discovery Learning; Feedback; Aptitude by Treatment Interaction; Math Equivalence.

Guided Discovery Learning

An emerging consensus suggests that people learn best through some form of *guided discovery*, which is defined as exploratory learning with supplemental instructional guidance. Learning tasks are exploratory if learners have not received instruction on how to complete them and instructional guidance encompasses a variety of tools, from in-depth instruction manuals to minimal feedback or hints. For example, Mayer's review (2004) suggests a "mixture of guidance and exploration is needed" (p. 17). Additionally, Alfieri et al.'s (2011) recent meta-analysis revealed the superiority of guided discovery over both pure discovery learning and pure direct instruction.

Providing exploratory activities prior to instruction is one form of guided discovery that has been recommended by researchers in education and cognitive psychology alike (e.g., Hiebert & Grouws, 2007; Schwartz & Bransford, 1998), and it is the form we focus on in this study. For example, several mathematics education researchers suggest, "each person must struggle with a situation or problem first in order to make sense of the information he or she hears later" (Stigler & Hiebert, 1998, p. 3). Similarly, Schwartz and Bransford (1998) suggest that exploratory activities facilitate the development of differentiated knowledge of the target problem space, which prepares learners for subsequent instruction.

There is a growing body of evidence to support the claim that exploration prior to instruction is beneficial (e.g., DeCaro & Rittle-Johnson, 2011; Schwartz & Bransford, 1998; Schwartz & Martin, 2004; Schwartz, Chase, Oppezzo, & Chin, 2011). For example, college students who explored novel examples learned more from a subsequent lecture than students who merely summarized a relevant text prior to the lecture (Schwartz & Bransford, 1998). Further, the timing of exploration and instruction matters. For example, children in elementary school benefited more from solving unfamiliar math problems before receiving instruction rather than vice versa (DeCaro & Rittle-Johnson, 2011).

However, questions remain regarding how and for whom this form of guided discovery is effective. First, should any guidance be provided during the exploratory activity? Mayer's review (2004) indicates that it is the guidance provided *during exploratory problem solving* that is crucial. Second, for whom is this guidance during exploration most advantageous? As noted by Cronbach and Snow (1977), often "the instructional approach that is best on the average is not best for all persons" (p. 1).

Feedback and Prior Knowledge

Feedback is touted as one form of guidance that may be particularly beneficial during exploration. *Feedback* is any information about performance or understanding that the learner can use to confirm, reject, or modify prior knowledge. For example, Alfieri et al. (2011) specifically recommend "providing timely feedback" as an optimal approach to learning (p. 13). Similarly, Mayer (2004) cites feedback as an effective tool (among others) for keeping learners on track. Further, past research indicates that feedback's primary function is to identify errors and encourage the adoption of correct alternatives (e.g., Kulhavy, 1977), which may be particularly helpful when exploring a novel problem space. Given these positive effects, it seems likely that providing feedback during exploration would be universally beneficial.

However, a growing body of research indicates that the effects of feedback depend on learners' prior domain knowledge (e.g., Fyfe, Rittle-Johnson, & DeCaro, 2011; Kraise, Stark, & Mandl, 2009; Luwel et al., 2011). For example, college students with low prior knowledge learned more about statistics if they received feedback during problem solving than if they did not. However, students with higher prior knowledge did not benefit from such feedback (Kraise et al., 2009). Similarly, Luwel et al. (2011) examined children's performance on a numerosity judgment task that could be solved using one of two correct strategies. Children who knew neither strategy at pretest benefited greatly from feedback in terms of correct strategy selection,

but feedback had a much weaker effect for children who already knew the strategies at pretest. Together, these studies suggest that learners with low prior knowledge should benefit from feedback during exploration, but learners with higher prior knowledge may not.

This idea is consistent with past work on aptitude by treatment interactions (Cronbach & Snow, 1977), which occur when instructional treatments have positive effects for one kind of person, but neutral or even negative effects for another. Importantly, these interactions often occur in the context of differing levels of external guidance. For example, Snow and Swanson (1992) suggest tutors "should provide more scaffolding for less able learners and less scaffolding for more able learners" (p. 610). A large number of aptitude by treatment interactions involve interactions between instructional guidance and learners' prior knowledge in the target domain (Kalyuga, 2007). For example, learners with low prior knowledge learn more from studying structured worked examples than from solving problems on their own. However, as knowledge increases, independent problem solving becomes the superior learning activity (e.g., Kalyuga & Sweller, 2004). In general, this work supports the notion that providing guidance (i.e., feedback) during exploration prior to instruction may help learners with low prior knowledge, but learners with higher prior knowledge may not need it.

Previous Experiment

Thus, we compared the effects of feedback (i.e., guidance during exploration) to no feedback (i.e., no guidance during exploration) prior to instruction. We hypothesized that feedback during exploration would result in higher learning than no feedback. However, we expected the effect to be stronger for children with low prior knowledge.

We also explored whether the type of feedback mattered. *Outcome feedback* provides a judgment about the accuracy of the learner's response, whereas *strategy feedback* provides a judgment about how the learner obtained that response. Outcome feedback has been studied extensively and is generally related to positive outcomes (e.g., Kluger & DeNisi, 1996). In contrast, few empirical studies have examined the effects of strategy feedback (e.g., Luwel et al., 2011). The limited evidence suggests strategy feedback can benefit strategy selection, but more research is needed to examine its effects across tasks and outcome measures.

We examined the effects of feedback in the context of children exploring math equivalence problems (problems with operations on both sides of the equal sign, such as 3 + 4 + 5 = 3 + __). These problems are not typically included in elementary mathematics curricula (Rittle-Johnson et al., 2011), and research shows that U.S. children exhibit poor performance on math equivalence problems (e.g., Alibali, 1999; McNeil, 2008). Thus, these problems are novel and difficult for elementary school children, providing an apt domain to investigate exploratory problem solving.

In an initial experiment, children received a tutoring session that included exploratory problem solving followed by brief conceptual instruction (Fyfe et al., 2011). The session was identical for all children with the exception that the feedback provided after each problem differed by condition. In the strategy-feedback condition, children received feedback on how they solved each problem. In the outcome-feedback condition, children received feedback on their answer to each problem. In the no-feedback condition, children did not receive feedback and were simply told to go on to the next problem. After the tutoring session, children completed a posttest (immediately and after a 2-week delay) that assessed conceptual and procedural knowledge of math equivalence. *Conceptual knowledge* is an understanding of the principles governing a domain and *procedural knowledge* is the ability to execute action sequences to correctly solve problems (e.g., Rittle-Johnson et al. 2011).

In line with our hypothesis, the effects of feedback on procedural knowledge depended upon prior knowledge. For low-knowledge children, feedback during exploration improved their procedural knowledge relative to no feedback. In contrast, for children with higher prior knowledge, no feedback resulted in superior performance than feedback, though this effect was slightly stronger for strategy-feedback than outcome-feedback. There were few effects on children's conceptual knowledge. Thus, the results resembled an aptitude by treatment interaction. Children with low knowledge benefitted from receiving feedback, but children with higher knowledge benefitted more from exploring independently without feedback.

Although we predicted that prior knowledge would moderate the impact of feedback, we did not have a prior reason to expect a reversal such that feedback would actually harm learning for children with higher prior knowledge. Also, several limitations in the design constrained the strength of the conclusions. First, the manipulation was not as clean or as strong as it could have been. For example, all children were asked to report how they solved each problem, which inevitably guided all children's attention to their strategies. The strategy-feedback manipulation would be stronger if only children in the strategy-feedback condition were encouraged to attend to their strategy use. Also, the feedback provided in both feedback conditions was relatively vague and not specific to the child's response. For example, in the strategy-feedback condition, incorrect strategies were referred to as "not a correct way," which may have been unclear. Further, children in both the strategy-feedback and outcome-feedback conditions were told if their target response (strategy or answer, respectively) was correct, but only children in the outcome-feedback were given additional information (i.e., the correct answer). The contrast between the two feedback conditions could be improved.

Second, we sought to clarify the influences of feedback type during exploration prior to instruction. Given the paucity of research comparing outcome-feedback to strategy-feedback, we wanted to confirm that feedback type is not central to children's learning during exploration. To address these concerns, we conducted a second experiment

similar to Experiment 1, but with several modifications intended to strengthen the design.

The Current Experiment

The current experiment was designed to strengthen the condition manipulation in Fyfe et al. (2011) and verify the results with an independent sample. Specifically, we attempted to replicate the finding that low-knowledge children benefit from feedback during exploration prior to instruction, whereas children with higher prior knowledge benefit from no feedback. Additionally, we sought to clarify the influences of outcome-feedback and strategy-feedback to confirm that feedback type did not impact children's learning during exploratory problem solving.

We strengthened the condition manipulation in three ways. First, to differentiate the conditions, we only had children in the strategy-feedback condition report how they solved each problem. Children in the other conditions were asked to report other information to mimic the interaction with the experimenter (i.e., their answer in the outcome-feedback condition and their completion of the problem in the no-feedback condition). Second, we made the feedback more specific by re-voicing the child's response. In the strategy-feedback condition we restated the child's strategy and in the outcome-feedback condition we restated the child's answer. Finally, we did not provide the correct answer in the outcome-feedback condition. In Fyfe et al. (2011), only children in the outcome-feedback condition received additional information (i.e., the correct answer). An alternative solution was to provide children in the strategy-feedback condition with additional information (i.e., a correct strategy). But, telling people how to solve a problem is a form of direct instruction, and we were interested in the guidance provided *prior* to direct instruction. So we eliminated the correct answer in the outcome-feedback condition to enhance parallelism across conditions.

Consistent with Fyfe et al. (2011), we predicted that children who received feedback during exploratory problem solving prior to instruction would exhibit better procedural knowledge of math equivalence than children who did not. However, we expected this effect to be larger for children with lower prior knowledge and to reverse for children with higher prior knowledge Further, we did not expect any differences in children's conceptual knowledge.

Method

Elementary school children received a tutoring session that included exploratory problem solving followed by brief conceptual instruction about math equivalence. The presence and type of feedback was manipulated during the exploratory problem solving.

Participants

Participants were 111 second- and third-grade children. Ten were excluded from participation because they scored above 80% on pretest measures designed to assess children's prior knowledge of math equivalence. Six additional children were excluded from analysis for not completing all activities. The final sample contained 95 children (*M* age = 7 yrs, 11 mo; 60 girls, 35 boys; 97% Black, 3% White).

Design and Procedure

We used a pretest – intervention – posttest design with a two-week retention test. For the intervention, children were randomly assigned to one of three conditions: strategy-feedback (*n* = 31), outcome-feedback (*n* = 33), or no-feedback (*n* = 31). Children completed the pretest in their classrooms in a 20-minute session. Within 1 week they completed a one-on-one tutoring intervention and posttest in a single session lasting approximately 45 minutes. Approximately two weeks after the intervention session, children completed the retention test in their classrooms.

The intervention began with exploratory problem solving. Children solved 12 novel math equivalence problems (e.g., $9 + 7 + 6 = \_\_ + 6$). In Fyfe et al. (2011), the problem were presented one at a time on a computer screen. In this study, we presented the problems in paper/pencil format to simulate a more typical classroom activity.

In the *strategy-feedback condition*, children reported how they solved each problem and received feedback on the strategy, which included a re-voicing of their report ("Good job! That is one correct way to solve that problem. [Child's strategy] was a correct way to solve it. / "Good try, but that is not a correct way to solve the problem. [Child's strategy] is not a correct way to solve it."). The experimenter re-voiced the strategy just as the child reported it to ensure no added information was provided. In the *outcome-feedback condition*, children reported their numerical answer and received feedback on it, which included a re-voicing of their report, but not the correct answer ("Good job! You got the right answer, [child's answer] is the correct answer." / "Good try, but you did not get the right answer, [child's answer] is not the correct answer."). In the *no-feedback condition*, children reported when they completed each problem and were then told to move on.

After exploratory problem solving all children received brief conceptual instruction on the relational function of the equal sign. The experimenter provided a definition of the equal sign and explained how the left and right side of a problem were equal, using number sentences as examples (e.g., $3 + 4 = 3 + 4$). Between the exploratory problem solving and instruction, children completed a brief form of the assessment (midtest) to gauge the immediate effects of exploration prior to instruction.

Math Equivalence Assessment

The math equivalence assessment, adapted from past work (Rittle-Johnson et al., 2011) was administered at pretest, posttest, and retention test. It included both conceptual (10 items) and procedural (8 items) knowledge subscales. Conceptual items assessed knowledge of the meaning of the equal sign and the structure of equations. Procedural items consisted of math equivalence problems,

and scores were based on children's use of a correct strategy to solve the problem. Example items and scoring are presented in Table 1. A brief version of the assessment (5 more difficult items) was used as the midtest.

Table 1: Example items on the assessment.

Task	Scoring
Procedural Knowledge	
Solve 8 = 6 + □ (operation on right side)	Use correct strategy (if unclear, response must be ±1 of correct answer)
Solve 3 + 4 = □ + 5 (operations on both sides)	Same as above
Solve □ + 6 = 8 + 6 + 5 (blank on left)	Same as above
Conceptual Knowledge	
Define equal sign	Provide relational definition (same amount)
Judge equations such as 3 = 3 as true or false	Correctly judge equations
Select choice that shows 10¢ is same as 1 dime	Select equal sign

Analysis and Results

We used a planned contrast analysis of variance model. Because our condition variable had three groups (no-feedback, outcome-feedback, strategy-feedback), we created two coded variables. The first variable (feedback) compared no-feedback to the two feedback conditions combined. This allowed us to address our primary hypothesis regarding the presence or absence of guidance during exploration. The second variable (feedback type) compared outcome-feedback to strategy-feedback and allowed us to explore differences in the type of guidance provided. To evaluate whether condition effects depended on prior knowledge, we included two interaction terms: feedback by prior knowledge and feedback type by prior knowledge. We used procedural knowledge pretest scores as the prior knowledge measure as it is the most relevant domain knowledge for learning during exploratory problem solving. Finally, we included three covariates (children's age as well as procedural and conceptual knowledge pretest scores).

To evaluate children's performance on the assessment we conducted repeated measures ANCOVAs with feedback (feedback vs. none) and feedback type (outcome vs. strategy) as between-subject variables and time (midtest, posttest, retention test) as the within-subject variable. The two interactions and three covariates were also included. We examined procedural and conceptual knowledge separately.

Pretest

On the pretest, children answered few procedural ($M = 20\%$, $SD = 18\%$) and conceptual ($M = 19\%$, $SD = 18\%$) items correctly. Importantly, there were no differences between conditions on either scale at pretest, F's < 1.

Procedural Knowledge

Children's procedural knowledge increased from midtest ($M = 26\%$, $SE = 3\%$) to posttest ($M = 37\%$, $SE = 3\%$), and stayed similar two weeks later ($M = 32\%$, $SE = 3\%$).

There were no main effects of feedback or feedback type, nor did feedback type interact with prior knowledge, F's<1. However, consistent with Fyfe et al. (2011), there was a feedback by prior knowledge interaction, $F(1, 87) = 4.67$, $p = .03$, $\eta_p^2 = .05$. As prior knowledge increased, the benefits of feedback decreased ($B = -1.06$, $SE = 0.49$). To help interpret the interaction, we categorized children as having higher prior knowledge (scored above the median on the procedural knowledge pretest measure) or low prior knowledge and examined the main effects of feedback for each group (see Figure 1). For the low-knowledge group, children who received feedback exhibited higher procedural knowledge ($M = 33\%$, $SE = 4\%$) than children who did not receive feedback ($M = 20\%$, $SE = 5\%$), $F(1, 87) = 4.00$, $p = .05$, $\eta_p^2 = .04$. For the higher-knowledge group, children who received feedback exhibited lower procedural knowledge (M = 28%, SE = 5%) than children who did not receive feedback (M = 50%, SE = 6%), $F(1, 87) = 7.54$, $p = .007$, $\eta_p^2 = .08$. Feedback during exploration was more beneficial than no feedback for children with low prior knowledge, but for children with higher prior knowledge, the reverse was true. Feedback type did not matter, suggesting that both types of feedback were beneficial for low-knowledge children, and both types of feedback were detrimental for higher-knowledge children.
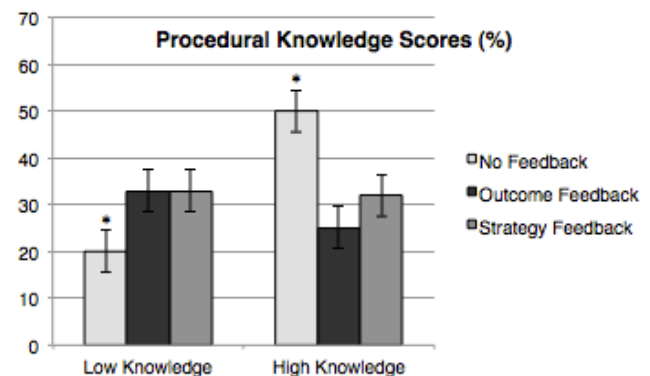

Figure 1: Percent correct on procedural knowledge assessment by condition and prior knowledge. Scores are estimated marginal means based on midtest, posttest, and retention test scores.

Conceptual Knowledge

Children's conceptual knowledge increased from midtest ($M = 21\%$, $SE = 2\%$) to posttest ($M = 50\%$, $SE = 2\%$) and stayed similar at retention test ($M = 43\%$, $SE = 2\%$).

There were no main effects of feedback or feedback type, nor did feedback type interact with prior knowledge, F's<1. However, there was a marginal feedback by prior knowledge interaction, $F(1, 87) = 3.63$, $p = .06$, $\eta_p^2 = .05$.

As prior knowledge increased, the benefits of feedback tended to decrease ($B = -0.70$, $SE = 0.37$). To help interpret the marginal interaction, we examined the effect of feedback for low- and higher-knowledge children separately (based on a median split of procedural knowledge pretest scores; see Figure 2). For the low-knowledge group, children who received feedback exhibited somewhat higher conceptual knowledge ($M = 44\%$, $SE = 3\%$) than children who did not receive feedback ($M = 37\%$, $SE = 4\%$), $F(1, 87) = 2.56$, $p = .11$, $\eta_p^2 = .03$. For the higher-knowledge group, children who received feedback exhibited somewhat lower conceptual knowledge ($M = 29\%$, $SE = 3\%$) than children who did not receive feedback ($M = 39\%$, $SE = 5\%$), $F(1, 87) = 2.60$, $p = .11$, $\eta_p^2 = .03$. Although not reliable, particularly when dichotomizing prior knowledge, these results resemble the pattern of findings found for procedural knowledge.
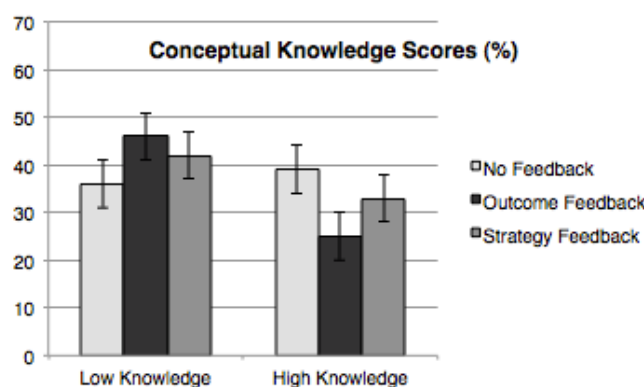

Figure 2: Percentage correct on conceptual knowledge assessment by condition and prior knowledge. Scores are estimated marginal means based on midtest, posttest, and retention test scores.

Discussion

Guided discovery generally facilitates deeper learning than discovery or instruction alone (e.g., Alfieri et al., 2011; Mayer, 2004). For example, providing exploratory activities with subsequent instruction can be beneficial (e.g., DeCaro & Rittle-Johnson, 2011; Schwartz & Bransford, 1998). However, the amount of guidance provided during the exploratory activities has largely gone unstudied, leaving questions as to how and for whom the guidance can work. In a previous experiment, we attempted to address these questions by examining the effects of feedback during exploratory problem solving prior to instruction. Some children received feedback (on their answer or on their strategy) after solving each problem, while others did not. In this study, we strengthened the condition manipulation and verified the results with an independent sample of children. Our results were consistent with those in Fyfe et al.'s original report. For children with low prior knowledge, feedback led to higher procedural knowledge than no-feedback. But for children with higher prior knowledge, feedback hindered performance relative to no-feedback.

There was a similar, but weaker effect for conceptual knowledge. Feedback type had little effect in general. Overall, we replicated the previous findings and provided evidence for the reliability of the results.

The results are consistent with prior work demonstrating aptitude by treatment interactions, which demonstrate that a single instructional method is often not best for learners with varying levels of prior knowledge (Cronbach & Snow, 1977; Kalyuga, 2007). In particular, a common conclusion is that low-knowledge learners benefit from more guidance, while high-knowledge learners benefit from less guidance (Snow & Swanson, 1992). Aptitude by treatment interactions have been found in a variety of domains including math, science, and problem solving (see Kalyuga et al., 2003). The current study (coupled with Fyfe et al., 2011) extends the aptitude by treatment interaction work to the presentation of feedback during exploratory problem solving. Children who enter the situation with low knowledge of the domain need feedback to improve their knowledge of correct procedures. Children with higher domain knowledge, on the other hand, do not need this feedback and actually perform better without it. This occurred even though higher knowledge children in our study were far from experts and still had a lot to learn.

Despite the growing evidence that prior knowledge moderates the impact of feedback during problem solving (e.g., Fyfe et al., 2011; Kraise et al., 2009; Luwel et al., 2011), the reasons underlying this effect remain unclear. One potential explanation relies on the learner's experience of cognitive load (Paas, Renkl, & Sweller, 2003). For low-knowledge learners, novel tasks can easily overload their working memory; thus, they often need some form of external guidance to reduce cognitive load. In contrast, higher-knowledge learners can use their existing, relevant schemas to help them complete the task without cognitive overload; thus, they often do not need external guidance. This may explain why low-knowledge learners benefited from feedback, but high-knowledge learners did not. It is also possible that differences in motivation would help explain the findings. Children who are more knowledgeable may also be more motivated to learn. In turn, those who are more motivated may thrive in less structured, challenging environments whereas children who are less motivated may not (Schnotz, 2010). Finally, changes in children's strategy knowledge may also play a role. For low-knowledge children, the constraining effects of feedback may have sped up the process of strategy acquisition, which in turn could jumpstart subsequent strategy changes including the strengthening of correct strategies (Siegler, 1996). However, for higher-knowledge children, the constraining effects of feedback may not have been necessary since these children already knew a correct strategy. More work is needed to tease apart these alternative explanations.

Our results also have important implications for research on guided discovery learning. They suggest that prior knowledge (and other learner characteristics) should be considered when evaluating the efficacy of guided discovery

methods (Cronbach & Snow, 1977). They also highlight the need to evaluate and optimize different aspects of guided discovery techniques. We examined the amount of guidance provided during exploration prior to instruction and found that more was not always better. Unfortunately, even when researchers recognize the benefits of combining exploration and instruction, the recommendation is usually to include more guidance (e.g., Alfieri et al. 2011).

Despite the positive contributions of the current study, future research is needed. For example, researchers should continue investigating the effects of feedback type. We did not detect many differences between outcome feedback and strategy feedback, but past research suggests strategy-feedback can be more beneficial, at least in terms of strategy selection (Luwel et al. 2011). Further, research should more carefully address what counts as sufficient prior knowledge. As more research finds that the effectiveness of instruction depends on prior knowledge, instructors will need guidance on how to choose instructional techniques for particular children with particular levels of prior knowledge.

This study extends research on guided discovery learning in which exploration is provided prior to direct instruction. Providing feedback during the initial exploration facilitates learning for low- but *not* higher-knowledge children. Thus, providing feedback may not always be optimal.

Acknowledgments

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