Sequential Integration of Object Locations in a Spatial Updating and Reasoning Task

Sven Bertel, Hyunkyu Lee and Wai-Tat Fu ({bertel, hyunklee, wfu}@illinois.edu)

Applied Cognitive Science Lab, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Abstract

We present results from an experiment studying how people mentally integrated partial configurations of objects shown across a sequence of displays with varying matches between frames of reference. Consistent with previous research on spatial updating, performance was better when the frame of reference in the final display aligned with the main display axes (up/down, left/right) than when it aligned with the diagonal axes. However, we also found that spatial updating was more efficient when the sequence of presentation of objects was consistent with the final frame of reference from which objects were integrated. Results suggested that spatial updating depended on the sequence of spatial operations required to integrate new spatial information into existing ones. Implications to theories of spatial updating in reasoning tasks are discussed.

Keywords: Spatial reasoning; spatial updating; frames of reference; sequential integration; spatial abilities.

Introduction

Many spatial tasks involve situations in which spatial information will only become available over time and in parts, and in which the person will need to mentally integrate the parts to reason with the distributed spatial information. Think, for example, of how a smart phone may allow you to access a large map on a small display by moving the display's focus across the map. In such cases, people not only have to mentally integrate a sequence of partial configurations, but also align the potentially different frames of reference to infer the spatial relations among objects in multiple representations.

There are two main challenges in this kind of a spatial updating task. First, spatial relations among objects in each partial configuration need to be encoded into memory using some forms of spatial representations. These representations are often sensitive to the intrinsic frames of reference that emerge from the object locations (e.g., a chair is on the left of the table, or a building is above the subway station on a map), or the allocentric frames of reference with respect to a global reference axis (e.g., North/South). Second, given that the spatial frame of reference between different configurations may not be identical, mental rotation and integration is needed to align and combine partial configurations to infer the relative locations of objects in a specific frame of reference. The current study explores factors that influence the representations and processes underlying such spatial updating in reasoning tasks, and the extent to which they are related to general spatial abilities.

Main and Diagonal Axes

Previous research has examined a number of aspects of human spatial reasoning (Barkowsky, 2007; Bertel, 2007; Knauff et al., 1994; Ligozat, 1998; Ragni et al., 2005), and spatial updating (Klatzky et al, 1998; McNamara, 2003; Mou & McNamara; Wang & Spelke, 2000; Wang, Sun, Johnson, & Yuan, 2005). One robust finding from the latter group is that certain types of alignments between a reference system in spatial memory and an externally available (e.g., environmental) one are easier to cognitively process than others, particularly when body orientation or movement play a role. In general, orientations that align with salient axes, such as *up/down*, *left/right*, *North/South*, or *front/back*, are processed faster than "derived" orientations (e.g., diagonal axes), such as *up right/bottom left* or *North-east/Southwest*. In "you-are-here" maps, for instance, Levine (1982) provided arguments for a mapping in which *ahead* in the environment should correspond to *up* on the map to facilitate orientation. Other research has looked into more dynamic tradeoffs in navigation between *track-up* and consistent *north-up* alignments for maps (e.g., Aretz, 1991).

Spatial Representations and Spatial Updating

When humans learn object locations in a new environment, the initial egocentric experience (e.g., provided by the perspective of the first view), is found to regularly dominate the mental reference system during navigation (Wang & Spelke, 2000). Often, the initial reference system will be kept also for subsequent views, unless these are better aligned with prominent natural features or axes; in that case, an updating of the mental reference system may occur (McNamara, 2003). Related memory models elaborate, among other aspects, relationships between choice of reference systems and viewpoint dependency or viewpoint invariance (Huff et al., 2007). The degree of alignment of a spatial relation between two objects with a represented intrinsic reference direction is then thought to influence how well that relation will get represented in spatial memory (Rump & McNamara, 2007). However, it is still not sufficiently clear how a sequential presentation of partial configurations of objects will be represented, integrated, and updated in spatial reasoning tasks. The current study focuses on investigating for a directional reasoning task how (a) the constructed mental representations will be influenced by changes in the alignment between display orientations and orientation of content in a global reference system, and (b), whether the updating process will be influenced by the order of presentation of objects.

General Study Design

The current study addresses issues of reference systems in a directional reasoning task. Specifically, we target spatial reasoning about relative locations between objects with

respect to an allocentric frame of reference. Human subjects were presented with a series of two premise screens, each offering a partial view of an overall 2D spatial configuration with four colored, rotationally symmetric objects. The partial views differed from one another in two ways: first, in which part of the overall configuration they displayed, and second, in their respective rotational alignment with an underlying cardinal orientation system, which provided the allocentric frame of reference (made explicit by the *North* arrows in the displays shown in Fig. 1). In a third screen, we tested participants' spatial updating and spatial reasoning abilities by asking them to name cardinal direction relations for selected pairs of presented objects (see bottom screen in Fig. 1). As the task required a spatial updating of the sequentially presented displays, we refer to it as the *Sequential Spatial Updating and Reasoning Task (SSURT)*.

In the current study, we were chiefly interested in three aspects: First, whether SSURT performance can be explained by individual differences on spatial abilities. If so, which aspects of spatial ability explain SSURT performance best? Does SSURT depend more on the ability to reorient one's imagined self or to spatially manipulate imagined objects in mind (Kozhevnikov & Hegarty, 2001)? Will self-reported spatial measures also be predictive (e.g., as found for certain non-reasoning, spatial updating tasks; Kozlowski & Bryant, 1977; Hegarty et al., 2002)?

Second, we were interested in effects induced by how the display and allocentric reference frames aligned on the first premise display and the test display. We used an 8-sector cardinal direction system and created two alignment classes, *"main"* and *"diagonal"*, depending on whether the depicted north arrow was parallel with a main display axis (0º, 90º, 180º, 270º; 0º = *up*) or with a diagonal display axis (45º, 135º, 225º, 315º). In line with some of the basic assumptions about spatial updating discussed above, we postulated that more use of *main* displays would lead to higher performance on SSURT than diagonal displays.

Third, we were interested in whether there was an effect of sequence on the spatial updating process. In order to examine this, we analyzed the data based on the temporal sequence on the premise screens of the *reference object* (the object presented at the center in the test display) and the *test object* (the object presented at the question in the test display). We expected that, if spatial updating depended on the temporal sequence of object presentation, a pair of a reference objects from the first premise display and a test object from the second premise display would lead to better performance than a pair of a reference object from the second premise display and a test object from the first premise display.

Method

Participants. Twenty-eight participants (13 females) from the University of Illinois at Urbana-Champaign were paid to participate in one and a half hour session. All participants had normal or corrected-to-normal near and far visual acuity and were unaware of the purpose of the experiment.

Stimuli. All stimulus images were presented on a white background. As with the examples provided in Fig. 1, premise screens consisted of 3 colored circles and a black arrow indicating the direction of North. The objects were each measuring ~2° of visual angle in diameter and were positioned on a grand circle (see Fig. 1). The distance between adjacent objects was ~3°. The black arrow was measuring ~1.7°. The first premise screen was centered 6° left of the center of display, and the second premise screen was centered 6° right of the center of display. The color of the objects was randomly chosen from a set of four colors ('red', 'green', 'blue', 'yellow'). The object at the center of the grand circle was the same on both premise screens, while north arrows on premise screens pointed to different directions. The displayed configuration in the second premise display was rotated to match the offset between the arrows. The test screen consisted of a reference object at the center and numbers 1 to 8 around the reference color towards the eight possible test object locations. It also contained a black arrow indicating the direction of *North*. A question was presented at the bottom of the test screen that read "Type in the direction of <*object*>" The reference and test objects were chosen from among the non-center objects from either first or second premise screens.

Procedure details. Participants were administered a battery of assessment tasks in order to measure their spatial abilities: *Paper-folding test* (*PFT*; French et al., 1963). In each trial, a square piece of paper which was being folded was presented, and a figure with a circle drawn on it to show where the paper has been punched. Participants were instructed to select the correct one among five drawings of the fully unfolded paper. PFT has been found to be indicative of one's ability to spatially manipulate objects in mind (Kozhevnikov & Hegarty, 2001).

Mental rotation test (*MRT*; Vandenberg & Kuse, 1978). In each question, participants were given an object and were asked to find the object when it was presented at different rotational offsets within a set of dissimilar objects.

Spatial orientation test (*SOT*; Hegarty & Waller, 2004). On each question, participants saw a picture of an array of objects. Participants imagined that they were standing at an object in the array facing another object, and indicated the direction to a third object from this position and orientation. Object perspective tests such as the SOT have been found to strongly predict the ability to spatially reorient one's imagined self (Kozhevnikov & Hegarty, 2001).

Santa Barbara Sense of Direction (*SBSOD*; Hegarty et al., 2002). Participants self-reported about spatial and navigational abilities.

Sequential Spatial Updating and Reasoning Task (SSURT). The procedure of a single trial is shown in Fig. 1. The first premise screen was presented with three objects. Participants were instructed to remember their relative locations and to proceed to the next premise screen by pressing a key. On the second premise screen, one of the previously displayed objects was removed and a new object was presented. This display's orientation changed with

respect to the direction of the *North* arrow, while the central object remained fixed. Participants were instructed to remember all presented objects and then to proceed to the next screen. On the following screen, the *test screen*, participants were asked to determine the cardinal direction of the object shown in the question (*test object*) as seen from the object presented at the center (*reference object*), and to respond through keys 1 to 8 from the upper row of a standard keyboard. The keyboard's number pad was not accessible, nor visible to reduce potential interferences of its main orientation axes with those in subjects' mental representations of the configuration, or with their choices of answers. For the same reasons, we selected a circular mapping of numbers to directions; it was learned, and learning was tested, before the main experiment.
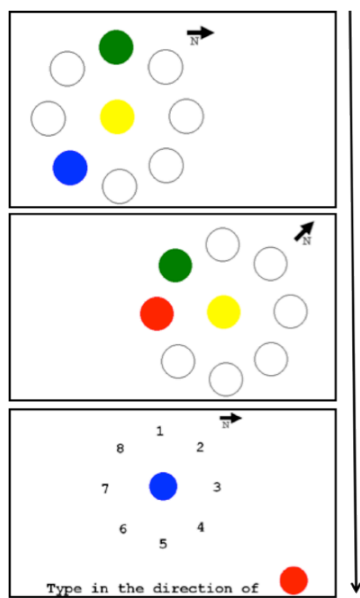

Figure 1. The procedure of the experiment. First premise screen with class *"main" North* direction (top), second premise display with *"diagonal" North* direction (middle) and test display (bottom) with *"main" North* direction.

Conditions. (a) There were two conditions each (*main* and *diagonal*) of the directions of the first premise screen and of the direction of the test screen. The directions of the first and second premise displays were always different. (b) An additional condition coded whether the display orientation in the test screen repeated that of one of premise screens, or whether it was different from that of both (*repeat* and *no-repeat*). (c) There were three conditions each on the origins of the reference and test objects (*first screen only*, which is an object only shown on the first screen, *both screens*, which is an object presented on both premise screens, and *second screen only*, which is an object newly added on the second premise screen). The trial in Figure 1 depicts a direction *repeat* condition between premise and test screens with the reference color from the *first screen only* and the test color from the *second screen only*. Participants were

given 8 training trials that were different from the main experiment. There were 44 trials in the main experiment.

Results

The general accuracy of the SSURT in this study was 54% (ranged from 9%-98%). Five participants' data were excluded from the analysis because of low general accuracy (less than 20%). The exclusion of these five participants' data did not change the general pattern of the results.

Spatial ability measures. First, in order to examine how well the current experiment can be explained by the participants' general spatial ability, we ran a multiple-regression analysis with the general accuracy of the SSURT as a dependent variable and scores from the selected spatial ability measures (MRT, PFT, SOT, SBSOD) as predictor variables. A linear regression analysis revealed a good fit (R^2=.485), and an Analysis of Variance (ANOVA) confirmed that the overall model was significant (R=.697) (F (4,22)=4.24, p<0.05), sugesting that our SSURT task is tapping into spatial abilities. Performance on SSURT was especially predicted by paper-folding (see Table 1, step 2), indicating that spatially manipulating objects in mind may be is especially important for SSURT. The result might stem from procedural and representational similarities between our task and the PFT: Both require temporal and spatial updating of a mental spatial representation either by the sequence of premise screens or by a folding sequence, as well as the representation's manipulation by adding an object (either a colored object or a hole). Table 2 shows the correlations between accuracy in our SSURT and the adminstered spatial ability tests.

Table 1. Regression results with general accuracy of experiment as a dependant measure and scores of spatial ability battery as predictors. [Note: For step 1, R^2 = 289, for step 2, R^2 = .485. ΔR^2 = .196, ^ p<.10* p<.05, two tailed.]

Category	B	SE B	B
Step 1			
MRT	.022	.012	.454^
SOT	.008	.011	.172
SBSOD	.001	.004	.061
Step 2			
PFT	.735	.281	.554*
MRT	.004	.012	.088
SOT	.007	.010	.158
SBSOD	-.003	.004	-198

Pearson's correlations revealed that some of the predictor variables were significantly correlated. Although, for our model the Variance Inflation Factor (VIF) values were all well below 10 (Myers, 1990) and the tolerance statistics were all well above 0.2 (Menard, 1995), suggesting that there was no colinearity within our data, there is still the possiblity that the effects from MRT, SOT and SBSOD were muffled because of high correlation between those

predictor variables and the paper folding task. In order to test if PFT accounted for SSURT performance independently of other predictors, we ran another two-step multiple regressions analysis, step one with the general accuracy of the SSURT as a dependant variable and scores for MRT, SOT, and SBSOD as predictors, and step two with PFT scores included on the previous model. We were interested in seeing if the inclusion of PFT on the previous model could produce a significant contribution of PFT on the score of SSURT independent of the other ability tasks. Results showed that step 1 was marginally significant (R=.538, $\underline{F}$ (3,22)=2.57, p=.084), and step 2 was significant (R=.697, $\underline{F}$ (4,22)=4.24, p<0.05). Most importantly, the change in F values from step 1 to step 2 was significant, p=.017, showing that adding paper-folding as a predictor accounts for a significant amount of variation in SSURT.

Table 2. Correlations between experiment (SSURT) accuracy and scores of MRT, PFT, SOT, and SBSOD [Note: * p<.05, ** p<.01, one tailed].

	MRT	PFT	SOT	SBSOD
SSURT	.518**	.596**	.396*	-.23
MRT	1	.446*	.552**	-491**
PFT		1	.189	.132
SOT			1	-428**
SBSOD				1

What are the plausible explanations for this finding? First, as suggested above, we believe that our SSURT task and PFT may share more properties with regard to mental processing than other spatial tasks, namely MRT and SOT. For both the SSURT and paper folding, an initial spatial representation of the configuration needs to be encoded that, subsequently, not only gets updated (i.e., transformed to reflect the results of folding operations or rotations), but its content is also directly manipulated through additional operations (e.g., adding new objects to the mentally represented set of objects in a specific frame of reference). In contrast, MRT and SOT do not require this latter kind of operation, participants are asked to perform operations on a static display continuously visible during the task. Secondly, and perhaps more importantly, it seems likely that performance in both SSURT and PFT can benefit from being well able to maintain the temporal sequence of past spatial transformations in memory. In the PFT, this corresponds to memory about the folding sequence, which should help in mentally performing the unfolding operations and infer the location of the hole; in the SSURT, it corresponds to memory about the presented configuration from previous partial displays. We will see further down in this analysis that our data did indeed point to differences in the ways in which objects from the first and the second display were encoded. Such differences can be well explained by a mental representation that encodes more details than just the current state after mental transformations, notably details regarding the relative locations of objects in the sequence of partial configurations and the sequence of operations required to update the integrated representation. Neither MRT nor SOT would benefit from such memory traces. Third, it may be that the SSURT and PFT are simply cognitively more demanding than the other spatial tasks. Whereas, in the MRT, it is sufficient to compare a current mental model to an external representation, SSURT also requires querying the mental representation for a relation and – following our previous discussion – may potentially benefit from keeping track of how the current configuration came about. Measuring the correlation between performance and working memory capacities, especially for visuo-spatial working memory capacities, should help to shed more light on this issue.

Main vs. diagonal axes. Next, we tested the effect of axis alignment type of the first premise and test screens. Mean accuracies were analyzed via a within-subject analysis of variance, with the orientation of the first premise and the question screens (*main axes* and *diagonal axes*) as independent variables. The main effect of the orientation of the first premise screen was not significant, $\underline{F}$(1,22)=.20, p>.1, while the main effect of the orientation of the question screen was significant, $\underline{F}$(1,22)=9.04, p<.01. In other words, participants showed better performance with *main* than with *diagonal axis* orientation on the question screen. The interaction between the first premise and question screens was not significant, $\underline{F}$(1,22)=1,67, p>.1 (see Table 3).

Clearly, an alignment of main axes in the configuration's cardinal direction system with main display axes was beneficial to performance. This seems consistent with previous findings about spatial updating on advantages to cognitive processing of those mental representations that are aligned with salient external axes over those that are not. However, as neither an interaction of display orientations, nor the orientation of the first display significantly predicted performance, our data did not support the hypothesis that the initially constructed mental representation was based on the orientation of the first display. Rather, it supports the notion that mental representations (and processes) were preferred when the main axes of the two (intrinsic and allocentric) reference systems matched, independent of whether the first display also had this property.

Table 3. The accuracy results based on the first premise and test screen orientations (standard deviations in parentheses).

	First Screen main	First Screen diagonal
Test Screen main	66 (31.0)	68 (25.9)
Test Screen, diagonal	59 (25.5)	55 (31.3)

Considering the high variation of participants' performance on our SSURT task, there might be a possibility that participants recruited different strategies to perform the SSURT and led to differences in performance. In order to examine this possibility, we formed two groups based on the performance level (*high* and *low*) by way of a

median split. Mean accuracies were analyzed with the orientation of the first premise and the question screens (*main* and *diagonal* axes) as within-subject factors and with performance level as between-subject factor. The result showed that there was no significant interaction between orientation of the first and the question screen, and performance group, $F(1,21)=2.19$, $p>.1$. We thus found no qualitative differences between the performance groups in terms of the orientation of the first and question screens.

Repetition of orientations. In order to further assess the influence that rotational updating had on our SSURT task, we tested if the repetition of an orientation from one of premises displays in the question display would help performance. Mean accuracies were analyzed with within-subject analysis of variance, with the repetition of orientation as a factor (accuracies with standard deviations in parentheses: *repeat*, 63 (26.8); *no-repeat*, 61 (25.3)). The main effect of the repetition of orientation was not significant, $F(1,22)=.786$, $p>.1$, showing that pure repetition of the orientation of any of the premise displays in the test display did not improve task performance. Processes of rotational updating of the mentally held representation thus did not seem to play an overly important role in determining the accuracies in the SSURT. This further supported the findings that the mental rotation and SOT tasks were only relatively poor predictors of task performance. Second, this result suggests that rote memory retrievals of displays (retrieval without mental rotation) may not have played as strong a role in this task as speculated above, even though memory processes were necessary to maintain the representations. If it had been, we should have found increased task performance when the questions displays repeated a previous orientation. It was therefore possible that the operations performed on the encoded representations in memory were more critical for performance in the current SSURT than memory retrievals, possibly because the number of objects was small.

Cross-display integration. In order to further examine the role of being able to keep track of how the configuration changes across the display sequence, the origin of the two objects that were queried on the question display was examined (*first screen only*, *both screens* and *second screen only*, see Table 4). Again, the reference object is the one presented in the center of the question screen; the test object is given in the question at the bottom. Results showed that the condition in which the reference object stemmed from the first display and the test object from the second showed higher performance (66.7%) than the condition in which the reference object came from the second display and the test object from the first display (57.6%), $F(1,22)=6.66$, $p<.01$. These results again suggest that the process of spatial updating and reasoning might depend on the temporal presentation sequence of reference and testing objects.

The main effect of the origin of the reference object was significant, $F(2,44)=3.28$, $p<.05$. Participants showed highest accuracy on the task with the reference object presented on both the first and the second screens (67%),

followed by the reference object from the first screen only (65%). Performance was worst when the reference object was from the second screen only (58%). Planned comparisons showed that the differences between the second screen only and the first screen only, and that between the second screen only and both screens were significant ($p=.028$, $p=.055$, respectively). The main effect of the origin of test color was not significant, $F(2,44)=1.44$, $p>.1$.

These findings do point to an asymmetry of how objects from the first and second screens are represented in memory. Presenting the task on the question screen in a way that mimics the temporal sequence of the premise screens' presentation (e.g., such that the first-screen object is used as the reference object from which the second-screen object is seen) helps performance. We interpret this as evidence that when subjects reached a question screen, their memory of the configuration did not just reflect the spatial properties of the most recently updated configuration – it also structurally reflected the sequence of information presentation. Retrieving contents from working memory in congruence with that the sequence of presentation was believed to be easier as it led to higher accuracy than retrieving content against the flow of events (Anderson, Bothell, Lebiere, Matessa, 1998). Memory for spatial transformation sequences was also important for performance in the paper-folding task, which explains why PFT was such a good predictor for task performance in our task. Given that it was not as important for performance in MRT and SOT, performance in these tasks therefore did not predict performance in the SSURT as well.

Table 4. Accuracy results based on the origin of reference and test objects. Standard deviations in parentheses.

	First only	Both	Second only
Reference color	65 (24.8)	67 (26.5)	58 (29.6)
Test color	60 (28.3)	59 (34.3)	65 (23.3)

Conclusions and Discussion

This paper presented original work on spatial reasoning in a sequential updating task, which required the integration of information across partial spatial representations and the drawing of spatial inferences. Consistent with previous work, we found that an alignment of the main display axes in the test screen of our experiment with the main axis of an underlying allocentric frame of reference in the displayed content improved performance. In addition, the order of object presentation was found to influence performance on the spatial inference task: Performance was improved if the reference object came from the first premise display. We believe that this effect is, to a large part, due to a mimicking of the original presentation order by the inference task, to the extent that the object presented first is used as the reference object from which the relative direction of a later presented object needs to be determined. Such sequence-

based interpretation also offers a plausible explanation of the third finding that PFT performance predicts SSURT performance much better than performance on the other included spatial ability measures. Performance in both PFT and SSURT seems to highly depend on subjects' abilities to spatially manipulate objects in mind.

This paper introduced the SSURT task and it shed new light on spatial updating for sequences of partial spatial representations that are used for spatial reasoning. Particularly, our finding on object presentation order for reasoning performance has interesting implications to our understanding of how objects are mentally represented depending on when they are introduced in a display sequence. The finding that objects which were introduced earlier are better suited for the role of a reference object than later ones suggests that not all steps in a sequential directional reasoning task lead to object representations that can be used equally well in the different roles of a spatial query. We argue that, by the time participants see a test screen, the processes involved in reading off a directional relation from the integrated mental representation fit the structure of that representation better when the reference object in the query had been introduced early on.

Assuming that future research on SSURT tasks will expand the validity of the current findings (e.g., by investigating other spatial inference tasks, longer sequences, more objects, or different spatial perspectives of the displays), our current findings have additional implications for the design of multimodal system in human-computer interaction for spatial tasks: the order of the original object presentation and the orientation of displays may, for instance, be of relevance for effectiveness and efficiency of interactions with map-based navigation systems which also verbally communicate spatial relations among objects that it reads off from the map.

Acknowledgments

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