

# Time-interval statistics adaptively modulate decision makers' willingness to wait for delayed outcomes

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## Abstract

The present work examines persistence in situations where delays are open-ended. From a normative standpoint, appropriate behavior in such situations depends on the statistical distribution of possible delay lengths. Depending on this distribution it may be appropriate either to persist indefinitely or to give up after a short period of time. In a behavioral experiment, human participants experienced reward timing statistics that implied it was productive to adopt either a high or low level of persistence. Human decision makers were highly responsive to these statistical cues. In a condition where timing statistics implied patience was productive, participants performed exceptionally well, and had little difficulty in waiting for delayed outcomes. In contrast, participants showed substantially lower willingness to wait when temporal statistics implied patience was an inappropriate strategy. The results demonstrate that seemingly impatient behavior can arise as an adaptive response to the perceived statistics of the environment.

**Keywords:** decision making; intertemporal choice; delay of gratification; dynamic inconsistency

## Introduction

Persistence in pursuing delayed rewards is widely regarded as an important self-control skill (Ainslie, 1975; Mischel, Shoda, & Rodriguez, 1989). Likewise, the inability to tolerate delay of gratification has been viewed as a source of maladaptive inconsistencies in choice.

Theoretical analyses of intertemporal choice have focused primarily on situations where decision makers know in advance how long a delay is scheduled to last (e.g., Ainslie, 1975). However, real-life decision makers routinely find themselves waiting through uncertain, open-ended delay intervals. When decision makers do not know how soon a reward will arrive, they face a nontrivial task of deciding how long to continue waiting.

Open-ended delays occur when commuters wait for buses, when job-seekers wait for offers, and when sit-and-wait predators wait for prey. In each of these cases, a decision maker must continuously choose whether to keep waiting or move on to new opportunities. Several of the most famous and compelling empirical examinations of delay-of-gratification behavior have involved delays that were open-ended from the decision maker's point of view (Mischel & Ebbesen, 1970; Mischel, Ebbesen, & Zeiss, 1972; Mischel et al., 1989).

Little is known about how decision makers cope with timing uncertainty during intertemporal choice. From a normative standpoint, as we show in detail below, the appropriate

behavior depends on the statistical distribution of possible delay durations. The shape of this distribution determines how a decision maker's expectation should change as time passes. For some distributions the anticipated delay grows steadily shorter over time, presumably increasing the reward's present subjective value. For others, however, time-passage can actually increase the expected remaining delay time. In this case a delayed outcome loses value over time. Giving up on the delayed outcome would be inappropriate in the former case, but is potentially justified in the latter.

Here we present a behavioral experiment in which decision makers had the opportunity to wait for rewards in environments that differed in terms of their timing statistics. Results suggest that individuals successfully learned about these timing statistics through experience, and responded by making appropriate adjustments in their willingness to tolerate delay.

Impulsivity and inability to tolerate delay often have a detrimental influence on human decision making. Nevertheless, a potentially productive route to understanding these aspects of behavior is to examine situations in which they are adaptive. It may turn out that mechanisms that support appropriate responding in some situations are also responsible for maladaptive delay-of-gratification failure in others. As a general point, it is important to recognize that a decision maker's computational-level objective is to *calibrate* behavioral persistence, not merely to maximize persistence in all cases.

The present work relies on the assumption that an individual's temporal expectations take the form of a probability distribution, not merely a point estimate. We therefore begin by reviewing evidence that supports this contention.

## Experience-based learning of time-interval distributions

Considerable evidence suggests that decision-making organisms can encode and use information about full distributions of time intervals.

A first category of evidence involves instrumental behavior under interval schedules of reinforcement. Fluctuations in response frequency are highly sensitive to the specific reward schedule in effect. If rewards are made available at fixed temporal intervals, response rates show a "fixed-interval scallop" (Gibbon, 1977), rising and falling as if to reflect subjective reward probability. Catania and Reynolds (1968) observed that pigeons' response rates tracked reward hazard rates that

were increasing, constant, or decreasing as a function of time. More recent work confirms that when animals experience bimodal interval schedules, response rates show a corresponding non-monotonic pattern (Bateson & Kacelnik, 1995).

Other evidence comes from “variable foreperiod” paradigms (Nickerson, 1965). Here, a preparatory cue precedes an imperative stimulus by a random “foreperiod” interval. The hazard rate for the imperative stimulus rises within the foreperiod. Reaction times are faster if the stimulus appears later, suggesting preparation is based on veridical instantaneous expectancy.

Distribution knowledge can influence the perceived duration of individual time intervals (Jazayeri & Shadlen, 2010). Distribution knowledge can also support explicit inferences. Griffiths and Tenenbaum (2006) asked survey respondents to estimate total durations for familiar types of events *given* given that the duration had already exceeded some minimum (for example, predicting the total length of a movie that has played for 110 min already). Participants’ responses were largely consistent with valid inference based on objectively accurate prior distributions.

### A willingness-to-wait task

We developed a behavioral task in which participants acquired experience with a statistical distribution of time intervals and decided how long they were willing to wait for delayed rewards. Participants’ objective was to maximize their total monetary reward in a fixed 10-min period. In essence they faced a rate-based optimization task, similar to a foraging problem. Participants could wait for one reward at a time, with each reward being delivered at the end of a random delay. At any time (and as often as they wished), participants could give up waiting, receive a much smaller immediate reward, and begin a new trial.

Two participant groups each experienced a different distribution of delay intervals. The two distributions were selected so that they implied qualitatively different optimal strategies. Delays in one condition were drawn from a uniform distribution spanning (0,12) sec (UD group). Delays in the second condition were drawn from a truncated heavy-tailed distribution with quartile upper boundaries at 0.8, 3.6, 15.9, and 90 sec (HTD group). Figure 1A shows the two distributions.

Timing statistics in the UD group were such that waiting was productive. The hazard rate for reward increased as a function of time already waited. Subjective reward expectancy should increase over time, and the expected remaining delay should correspondingly decrease. As a result, the rate-maximizing strategy was always to continue waiting. Intuitively, it would be unwise to quit after waiting 8 sec, because at that point a reward in next 4 sec is guaranteed (a better prospect than starting a new trial).

For the HTD group, conversely, waiting was counterproductive. This distribution was characterized by a falling hazard rate, meaning that reward became less likely in successive temporal intervals, and the passage of time *increased* the expected length of the remaining delay. A rate-maximizing

### Time-interval distributions

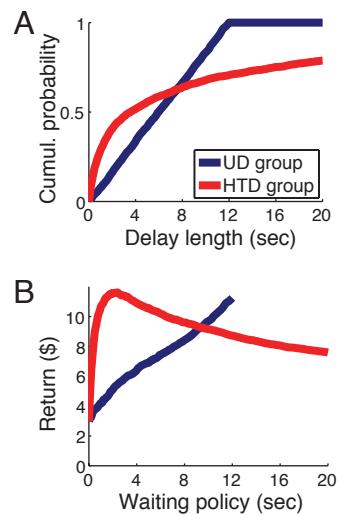


Figure 1: Panel A shows the distribution of delay intervals in each condition. Delays in the UD group were drawn from a uniform distribution ( $a = 0$ ,  $b = 12$ ). Delays in the HTD group were drawn from a generalized Pareto distribution ( $k = 8$ ,  $\sigma = 3.4$ ,  $\theta = 0$ ) truncated at a maximum value of 90 sec. Panel B shows the total monetary return expected under a range of waiting policies (see text for details).

strategy would at some point call for giving up and moving on to a new trial. The heavy-tailed distribution can be intuitively understood as involving a mixture of short and long delays. As more time passes it becomes more likely that the current trial falls in the long tail of the distribution and is best abandoned. (See Griffiths & Tenenbaum, 2006 for further discussion of prediction updating as a function of elapsed time.)

These two distributions were selected because they broadly represent two categories of situations that real-world decision makers are likely to encounter. The UD condition reflects a simple form of uncertainty: the delay’s precise length is unknown, but the decision maker expects it will lie within a delimited range of values. A Gaussian probability distribution over delay lengths would have the same essential properties as the uniform distribution: estimates of the time remaining would decrease monotonically (though not linearly) as time passed. This form of uncertainty could arise not just from variability in external events, but also from internal noise associated with time-interval estimation (Gibbon, 1977).

The HTD condition, in contrast, was designed to represent situations where delays are open-ended. There is growing evidence that time intervals associated with many human activities are well characterized by a heavy-tailed form (e.g., a power-function distribution). Such distributions characterize activities such as email reply latencies, where most delays are short but some are very long (Barabási, 2005). Heavy-tailed distributions tend to arise when fast and slow processes are in-

termixed. From a decision maker's point of view, such mixing implies that a given event's past duration is a direct predictor of its duration in the future (for an application of the same principle to the longevity of memory traces, see Anderson, 2001). In addition, when specific predictive information is unavailable, a heavy-tailed distribution may be a reasonable uninformative prior (Gott, 1993). As such, this type of distribution might characterize individuals' expectations in uncertain situations such as waiting on hold (Griffiths & Tenenbaum, 2006) or waiting for unreliable buses (Rachlin, 2000).

In principle, an exponential distribution occupies a middle ground between these two categories, implying a constant probability of reward arrival per unit time. Waiting would be neither productive nor counterproductive. However, uncertainty about the value of the exponential rate parameter would produce, in effect, a mixture of exponentials, which would take on a heavy-tailed form (Sozou, 1998).

It would clearly be beneficial for decision makers to be capable of distinguishing situations where waiting is productive from those where it is not. We predicted that decision makers would adaptively calibrate their level of behavioral persistence on the basis of the timing statistics they experienced.

### Normative analysis

We will define a decision maker's *waiting policy* as the time at which he or she would quit a trial if the reward had not yet arrived. Figure 1B shows the total earnings expected under a range of policies in each condition (if each policy were applied consistently over the course of the entire experiment). These curves are based on the following parameters: a 10-minute session, a 15¢ reward, a 1¢ gain upon quitting, a 2 sec inter-trial interval (ITI), and the delay-length distributions shown in Figure 1A.

Expected total earnings were calculated in the following way. Suppose a given policy calls for quitting at time  $t$ . Let  $p_t$  be the proportion of trials that will be rewarded because they have a delay shorter than  $t$  (i.e., the cumulative probability at  $t$ ). Let  $\tau_t$  be the mean duration of these rewarded trials. The expected return for a single trial, in dollars, is  $R_t = 0.15(p_t) + 0.01(1 - p_t)$ . One trial's expected cost, in seconds, is  $C_t = \tau_t(p_t) + t(1 - p_t) + 2$ , including the 2-sec ITI. The expected return over the 600-sec experiment is  $600 \times R_t / C_t$ . This is the quantity that participants should seek to maximize.

At one extreme, quitting immediately on every trial would yield 1¢ every 2 sec, or \$3.00 total. At the opposite extreme, a perfectly patient participant in the UD condition could obtain a 15¢ reward every 8 sec on average, for \$11.25 total. The maximum possible rate of return in the HTD condition is comparable, but is achieved under a very different waiting policy. Here, a return of \$11.25 or greater could be obtained with waiting policies between 1.4 sec and 3.4 sec. Persistence beyond this point would be counterproductive.

Of course, individual decision makers enter the task with no advance knowledge. Lacking direct access to the distributional information in Figure 1A, they are not equipped to undertake a complete optimality analysis at the outset. It is

nonetheless possible that participants will be capable of responding adaptively to the timing statistics they experience over the course of the task. If this is the case, individuals in the UD condition may come to show greater persistence than those in the HTD condition.

A result of this kind would indicate that decision makers are capable of calibrating persistence in an adaptive manner through temporal learning. This finding would open the way for an examination of the specific information-processing steps that enable time-interval experience to impact subsequent behavioral persistence.

### Empirical investigation

Participants in a behavioral experiment were given opportunities to wait for delayed outcomes under temporal uncertainty. We tested whether the form of uncertainty—specifically, the statistical distribution of delay lengths—would impact participants' willingness to wait.

### Methods

**Participants** The experiment was run in a New Jersey shopping mall. There were  $n = 40$  participants, 23 female, ranging in age from 18–64 (mean = 32). Years of education ranged from 11–20 (mean = 15). Each participant was randomly assigned to one of two conditions ( $n = 20$  each).

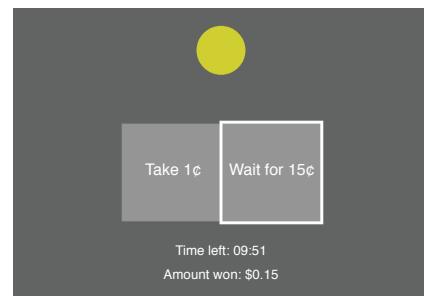


Figure 2: Behavioral task interface.

**Materials and procedure** Participants were tested individually at a laptop computer. The waiting task was programmed using the Psychophysics Toolbox extensions for Matlab (Brainard, 1997; Pelli, 1997); the interface is shown in Figure 2. The task proceeded as follows. A yellow light appeared near the top of the screen. Instructions stated that the light would remain illuminated for a random length of time, eventually going out and delivering a 15¢ reward. Below the light were two boxes representing possible responses. The right-hand box was labeled, "Wait for 15¢." Participants could wait for the reward by leaving the cursor in this box. The left-hand box was labeled, "Take 1¢." By moving the cursor to this box, participants could extinguish the light, receive 1¢ immediately, and move on to a new trial. If participants did not wish to wait at all, they could simply leave the cursor in the left-hand box across multiple trials.

Each monetary outcome (either 15¢ or 1¢) involved a 2-sec ITI before a new trial began. The bottom of the screen displayed the participant's total earnings so far, as well as the amount of time left in the 10-min session.

A delay duration was selected on each trial according to the relevant distribution (see Figure 1A). The reward was delivered at the end of this delay if the participant did not choose to end the trial earlier.

We wished to ensure that even short spans of experience would reflect the statistics of the underlying distribution. To accomplish this, delays were not drawn fully randomly. Instead, successive samples were balanced over the four quartiles of the distribution (i.e., a sample was drawn from all quartiles in random order before a quartile was repeated). While this approach has the disadvantage of introducing subtle sequential structure, it has the important advantage of reducing within-condition variability in the timing statistics participants experienced.

## Results

We hypothesized that both groups would shift toward a waiting policy that was productive given the timing statistics in place. Willingness to wait should increase in the UD group and decrease in the HTD group.

**Monetary earnings** Total monetary earnings serve as a rough gauge of task success. The maximum possible return was approximately \$11.25 for each group. The median amounts actually earned were \$10.69 in the UD group and \$7.29 in the HTD group (see Figure 3). These values differed significantly (rank-sum  $p < 0.001$ ). Performance was strikingly successful in the UD group, with 12 of 20 participants obtaining an amount within \$1 of the theoretical optimum.

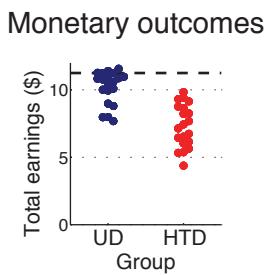


Figure 3: Total monetary earnings in each group. Each point represents a participant. The dashed line shows the approximate earnings expected under the best waiting policy (see Fig. 1B for details).

**Survival analysis** Each group's willingness to wait was summarized by plotting a conditional survival curve (see Figure 4A). For each time  $t$ , the survival curve considers only trials that were experimentally scheduled to last longer than  $t$ . It plots the proportion of these trials that were still in progress at  $t$  (i.e., that went on either to be rewarded or to be quit at a

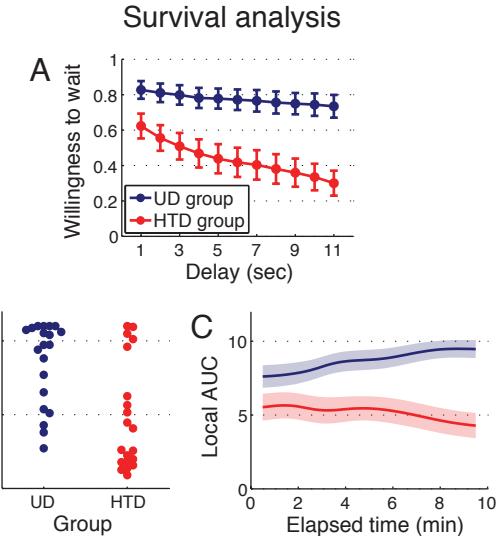


Figure 4: Survival analysis results. Panel A shows the mean conditional survival curve in each group (with standard error). These curves represent participants' proportional willingness to wait a given length delay. Panel B summarizes the survival curves by showing each participant's area under the curve (AUC). Panel C shows the mean local AUC in each group (shaded region shows standard error) for a sliding Gaussian-weighted epoch across the testing session.

later time). The survival curve is more informative than a plot of raw frequencies because it is not directly influenced by the experimentally imposed pattern of delay lengths. Instead, it depicts participants' *willingness* to wait various durations.

It is important to note that individual trials reveal different amounts of information about participants' waiting policies. Quit trials are maximally informative, as they provide a point estimate for the longest time a participant is willing to wait. In contrast, rewarded trials only signify a willingness to wait *at least* the length of the trial.<sup>1</sup> Rewards that follow short delays are relatively uninformative, whereas a reward after a long delay conveys considerable information about an individual's willingness to wait.<sup>2</sup>

In general, the survival curves in Figure 4A suggest that for any given delay from 1 to 11 sec, UD participants showed greater willingness to wait than HTD participants. The area under the curve (AUC) is a useful summary statistic. The AUC values in Figure 4B were obtained by summing the 11 points in each participant's survival curve. The AUC represents an individual's average willingness to wait during the

<sup>1</sup>In the terminology of survival analysis, these observations are "right-censored," analogous to a patient in a clinical trial who was still alive at the end of data collection.

<sup>2</sup>A participant in the task faces an inverted version of the same problem. For an individual trying to estimate the typical duration of delays, a rewarded trial is the most informative because it reveals the delay's exact length. In contrast, quit trial reveals only a lower bound on the delay's duration.

11-sec interval evaluated. An individual who always waited at least 11 sec would have an AUC of 11, whereas someone who was willing to wait 4 sec but not 5 sec would have an AUC of 4.

A comparison of the AUC values in Figure 4B supports the study's central prediction. Individuals in the UD group (median AUC = 9.73) showed greater willingness to wait than individuals in the HTD group (median AUC = 3.34; rank-sum  $p = 0.002$ ).

The results in Figure 4A–B aggregate across the entire 10-min experiment. However, participants initially knew nothing about the relevant timing statistics, so performance is unlikely to have been stable over time. Rather, we would expect group differences to emerge progressively with experience.

To assess change over time, AUC values were calculated in a sliding window over the 10-minute testing period. For each plotted point, an AUC value was computed based on trials weighted according to a Gaussian function of their distance from that point ( $\mu = 0$ ,  $\sigma = 60$  sec). The window was centered at points ranging from 0.5 min to 9.5 min. The resulting timecourses, shown in Figure 4C, depict the gradual development of differences in the two groups' behavior. The results also suggest differences began to appear as early as the first minute or so of testing.

**Choice reversals** A phenomenon of particular interest to decision-making researchers is the *reversal* of intertemporal choices. That is, instances where decision makers do not merely forego a delayed reward outright, but reverse their own initial decision to pursue the same reward.

### Survival analysis, trials exceeding 1 sec

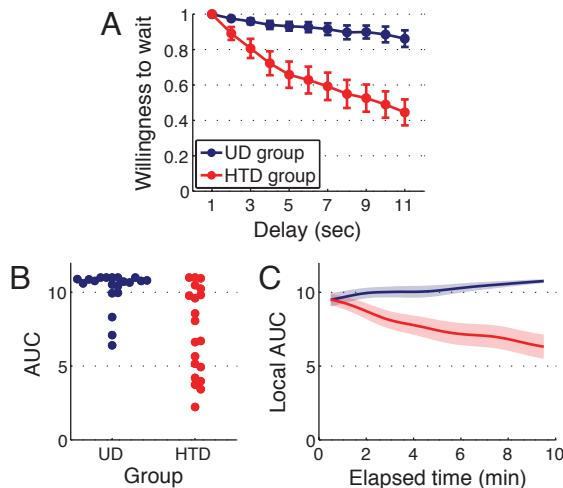


Figure 5: Survival analysis restricted to trials in which participants waited at least 1 sec. The first point in the survival curve is therefore fixed at 1. This analysis reflects participants' willingness to *continue* waiting for a delayed outcome after having begun to wait. Panels A–C correspond to the panels in Fig. 5.

In some situations, choice reversals are viewed as reflecting non-rational dynamic inconsistency, potentially arising from a failure of self control (Ainslie, 1975; Mischel & Ebbesen, 1970). In contrast, the present task (in the HTD condition) required such behavior for optimal performance: the best-performing strategy is initially to wait for the delayed outcome but to quit if it fails to arrive after about 2 sec. Reversals in this context do not signify dynamic inconsistency (i.e., they do not signal a violation of stationarity) because time passage is a source of new, predictive information about the timing of the awaited reward.

We ran a secondary analysis focusing specifically on dynamic reversals. This analysis excluded trials where participants opted for the small, immediate reward immediately (an unproductive strategy in either condition). Focusing only on trials where the participant waited at least 1 sec, this analysis evaluated willingness to continue waiting beyond that point.

Figure 5A shows survival curves restricted to these trials. Because trials quit in the first 1 sec are excluded, the first point in each survival curve is fixed at 1 while the other points remain free to vary. The curves reveal that even after participants had begun to wait, they gave up waiting earlier in the HTD condition than in the UD condition. Figure 5B shows AUC values summarizing these curves for individual participants. The minimum possible AUC is 1. All but 3 individuals in the UD condition showed high willingness to continue waiting; that is, dynamic reversals were virtually absent. Reversals occurred far more frequently in the HTD condition. The median AUC in the UD group (10.74) significantly exceeded that in the HTD group (7.38; rank-sum  $p < 0.001$ ). Consistent with findings shown previously, this effect developed progressively over time (see Figure 5C).

This analysis shows that group differences do not depend on HTD-group participants becoming categorically unwilling to pursue the delayed outcome, but instead reveal the gradual development of a less patient waiting policy.

### Discussion

We have shown evidence that decision makers adaptively modulate their willingness to wait for delayed outcomes in response to the timing statistics of their environment. The adjustments occurred after a short period of relevant experience. Two aspects of the results deserve special emphasis.

The first is that participants in the UD condition were remarkably successful at waiting patiently for delayed rewards. Tasks that require patience or delay tolerance are often assumed to present a challenge to human decision makers. In contrast to this assumption, the present results suggest such demands posed little difficulty when the value of persistence was supported by direct experience with timing statistics.

The second key point is that participants in the HTD group tended to develop significantly less patient waiting policies, which constituted an adaptive response to the timing statistics in effect. The observed behavior was isomorphic to behavior usually interpreted as reflecting self-control failure: partici-

participants chose to begin waiting for a delayed outcome, but reversed their choice before the outcome was obtained. We created a situation where such reversals, far from being anomalous, were optimal by objective criteria. Without introducing any direct element of temptation, we have shown impatience-like behavior arising as a result of valid statistical learning.

Our results agree well with the idea that intertemporal decision-making mechanisms tend to be effective at maximizing reward rate over time (Kacelnik, 2003). There is a natural connection between the present work and topics in the optimal foraging literature such as patch-departure decisions (Brunner, Kacelnik, & Gibbon, 1996).

Several factors might contribute to participants' overall superior performance in the UD condition, which called for high persistence, compared to the HTD condition. The reward-maximizing strategy in the HTD condition is arguably more complex: decision makers can err by quitting either too soon or too late (whereas in the UD condition the only error is to quit too soon). Individuals in the HTD condition might therefore require more experience to reach equivalent performance. A related idea is that the relevant properties of the uniform distribution might be easier to learn; followup work should introduce additional measures of temporal beliefs to track how beliefs evolve with experience in each condition. A further possibility is that individuals may hold an initial bias toward patient strategies in the context of this task. Several individuals remained highly patient even in the HTD condition (see Figure 4B). Such a bias could stem from high valuation of individual delayed rewards, from a motive to seek information, from prior beliefs about the time-interval distribution, or perhaps from a motive to appear outwardly consistent.

The present work has addressed the topic of dynamically inconsistent decision behavior by examining circumstances under which such behavior would be appropriate. An account of dynamic inconsistency based on statistical learning and inference (even in cases where errors may occur) is a potential alternative to accounts positing competition among multiple internal systems or objective functions. To the extent that the failure to tolerate delay involves an aversive affective experience such as frustration (Mischel et al., 1972), the present results highlight that affective processing may promote adaptive responses in some contexts (cf. Bechara, Damasio, Tranel, & Damasio, 1997) and should not be seen solely as an impediment to advantageous decision making.

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