

The Ellsberg 'Problem' and Implicit Assumptions under Ambiguity

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Abstract

Empirical research has revealed that people try to avoid ambiguity in the Ellsberg problem and make choices inconsistent with the predictions of Expected Utility Theory. We hypothesized that people might be forming implicit assumptions to deal with the ambiguity resulting from the incomplete information in the problem, and that some assumptions might lead them to deviate from normative predictions. We embedded the Ellsberg problem in various scenarios that made one source of ambiguity (i.e., the implied distribution of the unknown number of the colored balls) explicit. Results of an experiment showed that more people chose consistently (and hence rationally) when the scenario encouraged them to think that the probability distribution of the number of balls was normal. The results give insight into the implicit assumptions that might lead to choices congruent with normative models.

Keywords: Ellsberg paradox, decision making under ambiguity, implicit assumptions, probability distributions.

Introduction

Since its inception the study of judgment and decision making has been concerned with the discrepancy between what we *do* and what we *ought to do* (Newell, Lagnado & Shanks, 2007). A long line of studies show that people make judgments and decisions that deviate from the principles of normative models, such as probability theory (for probability judgment) and Expected Utility Theory (EUT) (for decision making). Some tasks have become classics in the literature because they show such robust and systematic violations of normative principles.

The typical task used to demonstrate deviations from normative models is as follows: Provide people with a judgment or a decision problem which generally contains quantitative or statistical information, ask them to judge the likelihood of an event or make a choice between alternatives, and compare the obtained responses with what normative models dictate. If the response does not conform to the principles of a given normative model, then that response typically is labeled "fallacious", "erroneous", or "paradoxical". A general assumption behind this labeling is that it is possible to adhere to the principles of normative models given only the information provided in the problem description. Though it often is *possible* for people to engage in normative computations, the impoverished and/or abstract nature of many classic problems might lead people to make additional assumptions in order to develop a coherent picture of a particular problem (Nickerson, 1996). These additional assumptions could give rise to responses that are

incongruent with the principles of normative theories. Thus it is not a failure of normative computations per se, but a mismatch between the external description and the decision maker's internal representation of the problem (Stanovich & West, 2000; see also Krynski & Tenenbaum, 2007).

Nickerson (1996) emphasized how, in many of the classic probability judgment tasks (e.g., Bertrand's Box, Monty Hall), different implicit assumptions can lead to starkly different conclusions. A compelling example is that of an encounter with a man on the street who introduces you to his young son. You know the man to be a father of two; what is the probability that his other child is also a boy? Answers of $1/3$ and $1/2$ can both be justified depending on the implicit assumptions one draws (e.g., is the man equally likely to take walks with children of either gender or does he favor walks with a son?) (see also Bar-Hillel & Falk, 1982).

The essence of these discussions is that judgments defined as erroneous or paradoxical could be explained in terms of a mismatch between the information provided to the participant and the implicit assumptions that they form when they are presented with the problem. In this article we shift focus from probability judgments to a classic decision making problem – the Ellsberg Paradox. This is an infamous decision problem because peoples' choices in the problem systematically violate normative principles. In addition, the problem is impoverished in a similar way to those discussed in the judgment literature. Thus our basic hypothesis is that the 'paradoxical' behavior observed in the Ellsberg problem might result from the tacit assumptions people form when faced with incomplete information.

The Ellsberg Problem

In most of the decisions we make, we are faced with different sorts of outcomes with varying degrees of certainty. While we might be able to attach specific probabilities to different outcomes in some cases (i.e., when the outcome depends on a fair coin flip), we may encounter some events where assessing a probability value is not entirely possible (i.e., when predicting the outcome of the next U.S. presidential election). Ellsberg coined the term "ambiguity" for the latter case and claimed that most people prefer to bet on gambles with known probabilities rather than unknown (Ellsberg, 1961). His classic example is as follows:

Suppose that there is an urn known to contain 30 red balls and 60 black or yellow balls but the exact proportion of

black and yellow balls is not known. One ball will be drawn at random from the urn. You are offered to bet on two gambles with two alternatives.

Gamble 1

A: If the ball drawn is red, you will win \$100.

B: If the ball drawn is black, you will win \$100.

Gamble 2

C: If the ball drawn is either red or yellow, you will win \$100.

D: If the ball drawn is either black or yellow, you will win \$100.

Ellsberg suggested that A&D will be the most frequent choice pattern and B&C the least (Ellsberg, 1961). In other words, people will bet on alternatives with known probabilities (A&D) rather than ambiguous alternatives (B&C). However, the A&D pattern is an obvious violation of the sure-thing principle of EUT because it shows that people prefer to bet on a red ball rather than on a black ball in Gamble 1 whereas they also prefer to bet on a non-red ball rather than a non-black ball in Gamble 2. Another way to express this contradiction is that choosing A&D implies that the decision makers behave as if the number of red balls is higher than the number of black balls in Gamble 1, but the number of red balls is less than the number of black balls in Gamble 2.

The vast majority of empirical evidence has demonstrated that people indeed have a strong preference for A over B and for D over C (see Becker & Brownson, 1964; Slovic & Tversky, 1974; MacCrimmon & Larsson, 1979; for the two-color version see Raiffa, 1961; Yates & Zukowski, 1976; Kahn & Sarin, 1988; Curley & Yates, 1989; Eisenberger & Weber, 1995). In addition to the investigation of this original version, this tendency against ambiguity in decision making has been tested under different conditions (for an extensive review, see Camerer & Weber, 1992). For instance, it has been shown that unambiguous gambles are strictly preferred to ambiguous gambles, even when the expected value of the latter is higher (Keren & Gerritsen, 1999), and that people are willing to pay more for unambiguous gambles (Becker & Brownson, 1964).

Ambiguity Aversion

Ellsberg proposed that it is not irrational to display the A&D choice pattern, but rather that EUT fails to incorporate ambiguity as distinct from risk into choice behavior (Ellsberg, 1961). For decades, this hypothetical gambling situation has been thought to be a paradox because it contradicts one axiom of EUT while the “ambiguity aversion” it manifests is intuitively plausible. On the theoretical level, several attempts have been made to solve this paradox by modifying some aspects of EUT (e.g., Choquet integral in Choquet theory, see Schmeidler, 1989). On the empirical side, what accounts for ambiguity aversion has remained an enduring question in the literature (see Chow & Sarin, 2001; Fox & Tversky, 1995; Fox & Weber, 2002; Frisch & Baron, 1988; Goodie, 2003; Grieco &

Hogarth, 2004; Heath & Tversky, 1991; Hogarth & Kunreuther, 1989; Yates & Zukowski, 1976).

Ambiguity has been defined as: “the uncertainty about probabilities, created by missing information that is relevant and could be known” (Fellner, 1961; Frisch & Baron, 1988). The ambiguity seen in the Ellsberg problem has two components: The first is the proportion of the black and yellow balls, and the second is the procedure used in the arrangement of black and yellow balls in a way that makes us unable to know the probability distribution (and hence the proportion). For example, if the procedure used to determine the number of black and yellow balls was coin flipping (e.g. Heads all yellow; Tails all black), the urn then must contain either 60 black balls or 60 yellow balls. In contrast, if the number of black balls was determined via a random selection method (e.g., pulling numbered tokens out of a bag), then the number could be anything from 0 to 60.

This second component has not been emphasized by previous studies, however we think its role might be as important as the first one in creating paradoxical choices. This is because if the procedure used in the arrangement of the black and yellow balls is known, then although the exact proportions of each cannot be inferred, it is possible to deduce the probability distribution of the number of balls, and thereby, perhaps, reduce that component of ambiguity.

In the standard version of the Ellsberg problem because participants do not know the procedure used to determine the number of black and yellow balls, they are unable to make an inference about the probability distribution (e.g., Bertrand paradox, see Bertrand, 1889; Nickerson, 2004, p. 186-204). According to the principle of insufficient reason, when one does not know a probability distribution one has to assume a uniform distribution (which implies that the probability of winning is equally likely for each alternative in Gambles 1 and 2). Such an assumption leads to indifference (Baron, 2007).

However, people are not indifferent between the alternatives in each gamble. As noted above, Ellsberg (1961) argued that ‘ambiguity aversion’ directs preferences towards A&D. We suggest that this aversion, at least in part, comes from the implicit assumptions that people form in the presence of ambiguity, and/or the absence of the information that is required to make a pair of choices that is consistent with the principles of EUT. In particular people might form an implicit assumption about the arrangement of the black and yellow balls in the urn (i.e., how they were selected and placed in the urn).

To investigate this idea, we kept constant one component of ambiguity – the proportion of black and yellow balls – and manipulated the second component – the procedure used in their arrangement in the urn. We provided people with ‘missing information’ by embedding the classic Ellsberg problem within 3 different scenarios where the procedure used in arrangement of the black and yellow balls was explicitly stated and each yielded different (implied) probability distributions.

Selection of the scenarios and implied probability distributions

We focused on three possible probability distributions for the black (and yellow) balls, and hence, on three different scenarios where these probability distributions can be deduced.

In the first experimental group, the “50-50” group, the scenario was as follows: *The experimenter tossed a fair coin. If the coin toss came up heads, then all 60 balls are black. If the coin toss came up tails, then all 60 balls are yellow.* This scenario implies that the number of black balls could be either 0 or 60 with probability of .50.

In the second experimental group, the “equal probability” group, the scenario read: *The experimenter randomly picked a number out of a bag which contained numbers from 1 to 60. Then she put that number of black balls into the urn. For instance, if the number selected was 20, she put 20 black balls into the urn; and then made the total number of balls up to 90 by adding a further 40 yellow balls.* Thus implying that the number of black balls could be anything from 1 to 60 with an equal probability

In the third experimental group, the “normal distribution” group the scenario was: *The experimenter put 60 black and 60 yellow balls into a huge box and shuffled them for a short while. After that she randomly picked 60 balls out of the box and put those 60 balls into the urn described above.* This scenario suggests that the number of black balls could be anything from 0 to 60 but middle values (close to 30) are more probable than extreme values (close to 0 and 60).

Figure 1 shows the probability distributions implied in each scenario. (These figures were *not* provided to participants and neither were the “50-50”, “Equal Probability” etc. labels used in the problem descriptions.)

One practical reason for using these particular probability distributions was that they were convenient to be transformed into coherent scenarios. Second, and more importantly, they were the first possible distributions that quickly came to our minds. We thought this may also be true for other people. For instance, it could be quicker and easier to imagine that the number of black balls is anything between 1 and 60 with an equal probability, or likely to be something around 30, rather than unlikely to be something around 30 (i.e., a parabolic normal curve).

Although we could not find examples in the existing literature of this kind of manipulation with the Ellsberg problem, we made tentative predictions regarding the effect that providing people with scenarios yielding different (implied) probability distributions would have on choices.

We predicted that the “equal probability” scenario would lead to a choice pattern similar to the one observed in the original version of the Ellsberg problem because both imply that the number of black balls could be anything from 0 to 60 with an equal probability - even though this is not explicit in the original version. More importantly, we expected the choice pattern obtained in the “normal distribution” scenario to be different (and perhaps result in more EUT-consistent choices) from those observed in the

other scenarios since it is more informative in the sense that it implies a relatively small range for the possible number of black balls (i.e. the number of black (yellow) balls is more likely to be close to 30).

One could argue that the “50-50” scenario is as informative as the “normal distribution” scenario since it implies that number of black (yellow) balls is either 0 or 60, so that it could have a similar ‘consistency-increasing’ effect on the choice pattern as well.

The experiment examined these tentative predictions.

Method

Participants

One-hundred and forty first year psychology students (M age = 19.5, 89 female) at UNSW participated in the experiment as a part of their course requirement. They were randomly assigned to the four groups ($n = 35$).

Design and Procedure

All participants received a paper-and-pen version of the Ellsberg problem. All groups were given the three-color version (as described in the introduction) where they were first told: “Imagine an urn known to contain 30 red balls and 60 black or yellow balls (thus 90 coloured balls in total)”. With the exception of the control group (the “Original Ellsberg” group), this statement was followed by a scenario in which the procedure used in the arrangement of the black and yellow balls was explicitly explained (see above for descriptions).

Each scenario was followed by the statement that the exact proportion of black and yellow balls was still unknown, and that one ball would be randomly drawn from the urn. Participants were then asked to select one of the two alternatives that they would prefer to bet on in each gamble.

Results

Table 1 displays the number of each choice pairings in the two gambles for each group. Note that according to EUT, consistent choice pairings are “A&C” and “B&D” whereas inconsistent pairings are “A&D” and “B&C”. As can be seen in the table participants in the “Original Ellsberg” group demonstrated the standard pattern with A&D as the dominant choice pairing. This pairing was also the dominant one for participants in the “50-50” group and the “equal probability” group. Indeed there was no significant difference in the number of consistent choice pairings between either of these two groups and the “Original Ellsberg” group, $\chi^2(1, N=70) = 0, p > .05$.

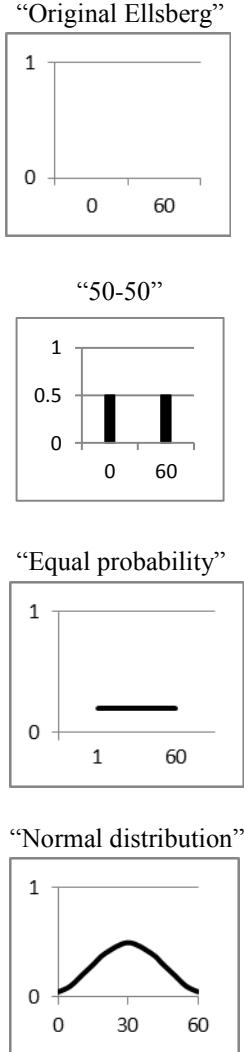


Figure 1: Implied probability distributions for the black balls in each scenario. The x -axis corresponds to the number of black balls and the y -axis to the probability values.

In stark contrast, the “normal distribution” group showed a significantly higher number of consistent choice pairings than the “Original Ellsberg” group, $\chi^2(1, N=70) = 5.72, p < .02$.

Discussion

The “normal distribution” group demonstrated more consistent choice preferences compared to both the “Original Ellsberg” group and the other two experimental groups. The increase in consistency appears to result primarily from an increase in the selection of A&C and a corresponding decrease in the selection of A&D. One way to explain this increase could be a reduction in ambiguity aversion which led people to choose C (ambiguous alternative) in Gamble 2. So why does the “normal distribution” scenario lead to a reduction in ambiguity aversion and to the more ‘rational’ choice pattern?

Table 1: The number of participants according to their choice pairings and to consistent/inconsistent choices across four groups. ($n = 35$ in each group).

Choice pairings	Groups			
	“Original Ellsberg”	“50-50”	“Equal”	“Normal”
A&C*	7	7	8	14
A&D	21	18	17	11
B&C	1	4	5	1
B&D*	6	6	5	9
\sum Consist.	13	13	13	23
\sum Inconsist.	22	22	22	12

* Choice pairings consistent with EUT

In the “normal distribution” scenario, participants were told that the 60 black and yellow balls were placed in the urn after the experimenter put 60 black and 60 yellow balls in a box, shuffled them for a while, and randomly picked 60 balls out of the box. Although it is not explicitly stated, this procedure implies that the probability distribution of the number of black (yellow) balls is normal. Therefore, the probability of the number of black (yellow) balls being around 29, 30, 31 is higher than it being 1, 2, 3 or 58, 59, 60.

This information is crucial because it (might) suggest to the participant that the distribution of balls in the urn is highly likely to be something like 30 red, 30 black and 30 yellow balls. Armed with this additional information people need no longer be ambiguity averse or indifferent and can make choices consistent with EUT.

Why is the increased consistency manifested primarily in more A&C and not more B&D choices? We conjecture that A remains more attractive than B because it represents a choice of 30 red (for sure) over “30-ish” black. Likewise C is more attractive than D because C comprises 30 red (for sure) or “30-ish” yellow, whereas D has two uncertain options (“30-ish” black or “30-ish” yellow).

Thus the key mechanism appears to be a reduction in at least one component of ambiguity (the method of arrangement) that is provided by the more informative “normal distribution scenario”. The normal distribution is more informative, for instance, than the “equal probability” scenario (cf. Larson, 1980) because in the latter scenario, since the number of black balls can be anything from 1 to 60, it is almost impossible to make even a rough estimation about the number of balls. Thus the scenario is still ‘impoverished’ and leads to a similar pattern of choices as observed in the original version. Indeed, equal probability distributions are considered to be one of the least informative distributions in probability theory (Jaynes, 1968).

To test this notion of ‘informativeness’ a follow-up experiment could elicit estimates of the number of black (yellow) balls from participants after they make a decision

in each gamble. If the implied distributions are being assumed then those participants given the “normal distribution” should give estimates with a narrow range for the number of black (yellow) balls (i.e., around 30).

This questioning might also shed light on why the 50-50 scenario, which is arguably as informative as the ‘normal distribution’, still resulted in a similar pattern of choice preferences (A&D) as the original version. One possibility is that the all-or-none nature of the 50-50 distribution makes option C (relatively) less attractive because the decision maker might reason that there is either 60 black (yellow) balls or 0. If there are *no* yellow balls then C (red or yellow wins) looks like a poor choice because there are only 30 red balls. In contrast D (black or yellow wins) looks better because there has to be 60 balls of one of those colors.

A further interesting feature of these data is that the participants given the “normal distribution” scenario still chose option A with almost the same frequency as those in the Original Ellsberg group (compare the totals for the top two rows in Table 1). This might suggest that participants in the “normal distribution” group were still ambiguity averse when presented with Gamble 1 since choosing A is the indication of ambiguity aversion.

It is possible that participants chose the alternative A in Gamble 1 because they were still ambiguity averse, but when they came to make a decision in Gamble 2, they realized that choosing D would lead to an inconsistent preference after choosing A. Thus they chose the alternative C, not because they were less ambiguity averse, but because D seemed inconsistent after choosing A.

Our ability to test this idea is limited because all participants completed the experiment with paper and pen and were free to answer the gambles in any order. A follow up in which presentation order was reversed, and order of completion was controlled, might provide insight into this alternative explanation.

If people are less ambiguity averse in Gamble 2 of the “normal distribution” group, then there should not be any change in terms of the number of A&C pairings chosen even when the participants are presented with Gamble 2 first. In other words, they should have no problem with choosing alternative C first although it is ambiguous. On the other hand, if the participants are still ambiguity averse but are trying to be consistent across gambles, then when given the reversed order they should choose alternative D first (because it is unambiguous), and then alternative B (because choosing B is consistent with choosing D).

These results provide an important first step in our understanding of the types of implicit assumption that might underlie choice patterns in decisions under ambiguity. We think these results are a useful bridge between the literature on probability judgment (e.g., Nickerson, 1996) and risky (ambiguous) choice and reinforce that in both domains ‘erroneous’ behaviour can be attributed to the impoverished nature of the tasks under investigation (see also Krynski & Tenenbaum, 2007 for a causal-model based approach to disambiguation of probability problems).

These results suggest that (some) people are able to adhere to the principles of normative theories not as a result of providing them with the complete specification of a problem, but with a particular type of information (a scenario which encourages them to think that the black and yellow balls are normally distributed). This implies that a good match between our implicit assumptions and the description of a decision problem makes it possible to narrow the apparent gap between what we normally *do* and what we *ought to do*.

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