How is knowledge transmitted in a small world network through communicative interaction?

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Abstract

Social science studies have shown that people are connected in a "small world." In this network, people are connected with short path lengths and are highly clustered. To clarify how people in a small world acquire knowledge through communicative interactions, we constructed a multi-agent model and subsequently conducted a computer simulation. Results of the computer simulation showed that in a small world network, agents acquire correct and diverse knowledge. We discuss the adaptive nature of a small world network for knowledge acquisition.

Keywords: knowledge acquisition; small world network; communicative interaction; multi-agent model

Introduction

Category learning is one of the most researched aspects of knowledge acquisition in cognitive science. In their studies, researchers often create experimental settings where participants learn (artificial) categories by receiving feedback. As a result, most research on category learning has clarified people's knowledge acquisition through individual learning (e.g., Cohen & Lefebvre, 2005; Kruschke, 2008).

However, in the real world people acquire knowledge not only through individual learning, but also through communication with others. Pentland (2007) argued that a research perspective involving social networks containing individuals is necessary to clarify human behaviors. Goldstone and Janssen (2005) emphasized the importance of research on collective behavior. For example, they point out that "interacting ants create colony architectures that no single ant intends," and "populations of neurons create structured thought, permanent memories and adaptive responses that no neuron can comprehend by itself" (p.424). By implication, interactions among individuals produce unique processes of knowledge acquisition that are not clarified by research on micro processes of an individual's knowledge acquisition.

In this paper, we discuss knowledge transmission which occurs through interactions of individuals. This topic is very important in the consideration of knowledge acquisition, because people often communicate with each other in ways that result in learning (e.g., Mason, Conrey, & Smith, 2007). Research about knowledge transmission reveals not only the roles of collective behavior for knowledge transmission, but also individual cognitive aspects (e.g., Brighton, 2002; Kalish, Griffiths, & Lewandowsky, 2007; Griffiths & Kalish, 2007; Smith, Kirby, & Brighton, 2003).

In the present study, we focus on one of the most intriguing network structures of people's connectivity, called the *small world*, and examined the role of this connectivity in knowledge transmission.

Connections in a small world

Social science researchers have shown that connectivity among individuals is not random, but has some regularity (although not complete). Milgram (1967) and Travers and Milgram (1969) empirically showed that people were connected with short path lengths (around six degrees of separation), which is known as the "*small world phenomenon.*" This is even more evident in recent research using email and online relationships (Dodds, Muhamad, & Watts, 2003). Another feature of individual connectivity is highly clustered relationships, where one's acquaintances also have a high probability of knowing each other (we call this probability clustering coefficient). Watts and Strogatz (1998) called a network having these two features a *small world network*. They proposed a very simple mathematical model representing the small world network: Imagine the network starting from a ring lattice with 1000 vertices, each connected to its 10 nearest neighbors by edges. Then, each edge is randomly rewired with probability p by disconnecting one of its vertices and connecting it to a randomly chosen vertex. Watts and Strogatz (1998) showed that this model replicates the small world network with

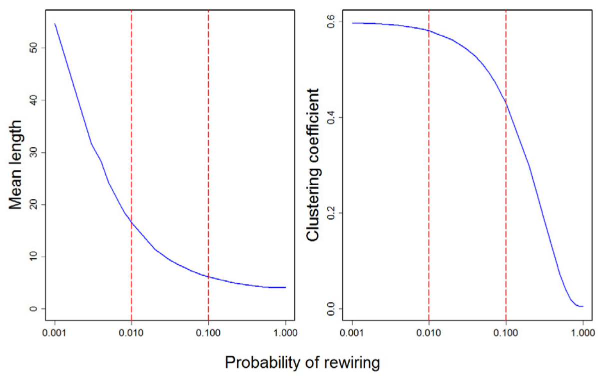

Figure 1. Relationship between rewiring probability and path length and clustering coefficient. These plots are based on mean values over a sample of 100 different graphs.

intermediate values of p ($0.01 \le p \le 0.1$; see Figure 1). Hereafter, we refer to the model of Watts and Strogatz (1998) as the *WS model*.

Knowledge transmission in a small world network

How is knowledge transmitted among individuals in a small world network? Cowan and Jonard (2004) used computer simulation to examine how effectively knowledge is transmitted by interactions among agents in the WS model. They found that agents in the small world network (p=0.09) showed the highest level of knowledge accumulation.

Cowan and Jonard (2004) were mainly interested in economic issues such as innovation in knowledge and knowledge transmission from a quantitative perspective. However, from a cognitive science perspective, we are particularly interested in the quality of knowledge transmission in a small world network. For example, cognitive studies of human memory have shown that individual memory mechanisms can produce incorrect knowledge (e.g., false memory, Roediger & McDermott, 1995). The findings of Cowan and Jonard (2004) would presumably include the transmission of both correct and incorrect knowledge through interactions among agents in a small world network. Thus, agents in a small world network can "effectively" acquire both correct and incorrect knowledge.

In the present study, we constructed a multi-agent model and subsequently conducted a computer simulation in order to examine how correct or incorrect knowledge is transmitted through interactions among agents in the WS model.

Multi-agent network model for knowledge transmission

Model

Network We set up a network in which 1000 agents exist. The connectivity of the agents was represented using the WS model. In this network, each agent had 10 edges for nearest neighbors when the probability of rewiring was zero. Thus the two parameters, the number of vertices and the number of edges for each vertex, were 1000 and 10, respectively. In order to represent various networks, from regular to random networks, we constructed eight networks, using eight values of rewiring probability: 0, 0.01, 0.04, 0.07, 0.1, 0.4, 0.7, and 1.

Knowledge Each agent learns 100 objects. We assumed that agents have three discrete knowledge states, $S_{correct}$, $S_{incorrect}$, and $S_{missing}$ for each object. $S_{correct}$ and $S_{incorrect}$ represent opposite ideas about an object. We defined $S_{correct}$ and $S_{incorrect}$ as the correct and incorrect knowledge states, respectively, according to actual fact. $S_{missing}$ represented the state where agents have no ideas about an object. For example, imagine the question, "Are there any direct flights from Tokyo to Buenos Aires?" The following three states are assumed for this question.

$S_{correct}$: "There are no direct flights."
$S_{incorrect}$: "There are direct flights."
$S_{missing}$: "I have no idea."

Cognitive features of agents Each agent had the following two cognitive features. First, the default knowledge of each agent was set as follows. The number of $S_{correct}$ of 100 objects was determined by random sampling from a normal distribution, $N(10, 5^2)$ for each agent. The number of $S_{incorrect}$ was determined by random sampling from a normal distribution, $N(9, 5^2)$ for each agent. Hence, at first, an agent had 10 $S_{correct}$, 9 $S_{incorrect}$, and 81 $S_{missing}$ for 100 objects on average. Second, each agent was assumed to have a limit on the number of objects to acquire. This limit was determined by random sampling from a normal distribution, $N(80, 10^2)$ for each agent. Therefore, each agent could have a total of 80 $S_{correct}$ and $S_{incorrect}$ out of 100 objects on average. If the number of knowledge states of $S_{correct}$ and $S_{incorrect}$ exceeded the limit, then one of objects whose knowledge state was $S_{correct}$ or $S_{incorrect}$ was randomly chosen and changed into $S_{missing}$.

Communication We assumed that each of the knowledge states for the 100 objects is revised through communication between two agents. In particular, we assumed that the agents calculate their degrees of confidence for the knowledge states, and stochastically revise their knowledge states using their degrees of confidence.

The three knowledge states are represented by three values; $S_{correct}$, $S_{incorrect}$, and $S_{missing}$ equal 1, 0, and 0.5, respectively. Let i and j denote agents who communicate with each other, and k_{im} and k_{jm} denote knowledge states that i and j have for object m. When agent i communicates with agent j about object m, i and j respectively calculate their continuous degrees of confidence about object m, $Conf_{im}$, $Conf_{jm}$, based on the following equations:

$$Conf_{im} = \left(1 - \alpha_i^{0.1 t_{im}+1}\right) k_{im} + \alpha_i^{0.1 t_{im}+1} k_{jm} \qquad (1)$$

$$Conf_{jm} = \left(1 - \alpha_j^{0.1 t_{jm}+1}\right) k_{jm} + \alpha_j^{0.1 t_{jm}+1} k_{im} \qquad (2)$$

The parameters, α_i and α_j, represent weight for knowledge state in calculating $Conf_{im}$ and $Conf_{jm}$. For example, when $\alpha_i > 0.5$, agent i is influenced by agent j's knowledge state more than agent i's knowledge state. When $\alpha_i < 0.5$, agent i is influenced by agent i's knowledge state more than agent j's knowledge state. When $\alpha_i = 0.5$, agent i is equally influenced by agents i and j 's knowledge states. Hence this parameter represents psychological trait of agents on strength of influence by other agents. α_i and α_j are determined by random sampling from a normal distribution, $N(0.5, 0.1^2)$; and t_{im} and t_{jm} reflect the number of communications on object m. In particular, t_{im} and t_{jm} start at 0 and are updated by 1 after each communication.

Based on the calculated $Conf_{im}$ and $Conf_{jm}$, agents i and j stochastically update knowledge states for m. Table 1 shows probabilities of i's knowledge update of object m into $S_{correct}$, $S_{incorrect}$, or $S_{missing}$ based on $Conf_{im}$ [1].

Figure 2 shows the relationship among t_{im}, α_i, and $Conf_{im}$ when agent i (k_{im}=1, knowledge state is $S_{correct}$) communicates with agent j (k_{jm}=0, knowledge state is $S_{incorrect}$). In this situation, these two agents have conflicting knowledge states. $Conf_{im}$ is affected by j's knowledge state when t_{im} takes a small value. When t_{im} equals 0 (i.e., agent i has not had communication with other agents about object m), the knowledge state is likely to shift into $S_{missing}$. For example, when $\alpha_i = 0.5$ (i.e., agent i weights k_{im} and k_{jm} equally in calculating $Conf_{im}$), i's knowledge state shifts into $S_{missing}$. However, as t_{im} takes a larger value, $Conf_{im}$ is not affected by j's knowledge state. This control represents cognitive processes where the knowledge state is flexible at first, but becomes gradually fixed and unlikely to change through communications.

Table 1: Probabilities of i's knowledge update of object m into $S_{correct}$, $S_{incorrect}$, or $S_{missing}$ based on $Conf_{im}$.

	$S_{correct}$	$S_{missing}$	$S_{incorrect}$
$Conf_{im}$ >0.5	$Conf_{im}$	1- $Conf_{im}$	0
$Conf_{im}$ =0.5	0	1	0
$Conf_{im}$ <0.5	0	$Conf_{im}$	1- $Conf_{im}$

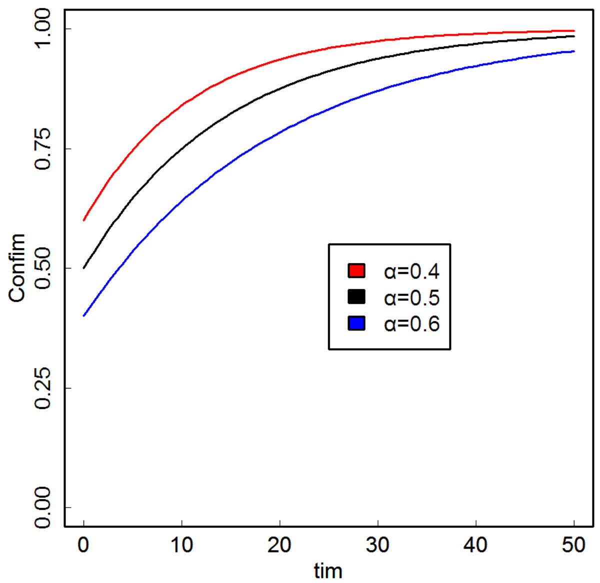

Figure 2. Relationship among t_{im}, α_i, and $Conf_{im}$. This figure shows the relationship when k_{im} and k_{jm} are 1 and 0, respectively.

[1] We assume that the discrete variables (i.e., k_{im} *or* k_{jm}) correspond to behaviors in communication (i.e., the content that agents convey in communication), and that the continuous variables (i.e., $Conf_{im}$ or $Conf_{jm}$) correspond to latent, psychological degrees of confidence on object m.

Overview of simulation

We define one communication with four steps. Table 2 shows the outline of the four steps.

One period is defined as 2000 communications. For each simulation, 3000 periods of 2000 communications were conducted at least. From period r ($r \geq 3000$), we calculated the following convergence criterion, CC_r.

$$CC_r = \sum_{r=r-999}^{r} \left(1 - \frac{CN_{r-1}}{CN_r}\right) \tag{2}$$

$$CN_r = \frac{CKS_r}{CKS_r + IKS_r} \tag{3}$$

where CKS_r and IKS_r denote numbers of $S_{correct}$ and $S_{incorrect}$ knowledge states for 1000 agents at period r. Thus CN_r represents relative proportion of $S_{correct}$ to $S_{incorrect}$ in period r. In principle, CN_r does not equal to CN_{r-1} because there is a possibility in every communication that knowledge states of communicating agents change. When CN_r is increasing through periods, CC_r will deviate from zero and take positive value because CN_{r-1}/CN_r tend take less than one. In contrast, when CN_r is decreasing through periods, CC_r will deviate from zero and take negative value because CN_{r-1}/CN_r tend take more than one. These two patterns suggest that numbers of $S_{correct}$ and $S_{incorrect}$ do not converge. When CN_r fluctuates randomly, CC_r will approach zero, suggesting that numbers of $S_{correct}$ and $S_{incorrect}$ have converged. According to these features of CC_r, when CC_r meets the following condition, we assumed that CN_r has converged at period r.

$$-0.001 < CC_r < 0.001 \tag{4}$$

We conducted 30 simulations for each of the eight networks.

Table 2. Four steps of one communication

First step: A pair of connected agents, i and j, is determined from a network.
Second step: Object m is determined from 100 objects.
Third step: Agents i and j calculate their degrees of confidence $Conf_{im}$ and $Conf_{jm}$, respectively.
Fourth step: Based on the calculated $Conf_{im}$ and $Conf_{jm}$, agents i and j update their knowledge state k_{im} and k_{jm}, respectively.

Results and discussion

We shall discuss following two points about acquired knowledge; correctness, and diversity.

Correctness

As to the correctness of acquired knowledge, we examined CN_r that can be assumed to represent converged correctness of knowledge state.

Figure 3 shows the relationship between probability of rewiring and CN_r. CN_r tends to increase as a network becomes random. However, increase of CN_r is diminishing after p>0.1. Increase of CN_r between p=0.01 and p=0.1 was about 0.1 (0.543-0.643). In contrast, that between p=0.1 and

p=1 was about 0.04 (0.643-0.688). Figure 1 shows that path lengths substantially become shorter as probability of rewiring increases until p=0.1. Thus, this relationship between path lengths and CN_r implies that path length influences CN_r in a network.

Cowan and Jonard (2004) showed that agents in a small world network (i.e., p=0.09) could achieve the maximum level of knowledge accumulation. Results of our simulations showed that agents in the random world network (i.e., p=1) could achieve correct knowledge at the maximum level. However, the value of CN_r in p=0.1 is closed to CN_r in p=1, suggesting that agents in a small world network could acquire correct knowledge.

Our simulation architecture mainly differs from Cowan and Jonard (2004) in the following two ways. First, the criteria of learning (i.e., knowledge accumulation or correctness of knowledge) differed. Second, the algorithms of communication between agents differed. Therefore, our simulation and Cowan and Jonard (2004) basically examined different aspects of knowledge acquisition. In common between our simulation and that of Cowan and Jonard (2004) is the network in which the agents are set. Therefore, our results and those of Cowan and Jonard (2004) indicate that the network structure among agents plays important roles in knowledge acquisition, irrespective of the criteria of learning (i.e., quantity or quality of knowledge) and the algorithm of communications.

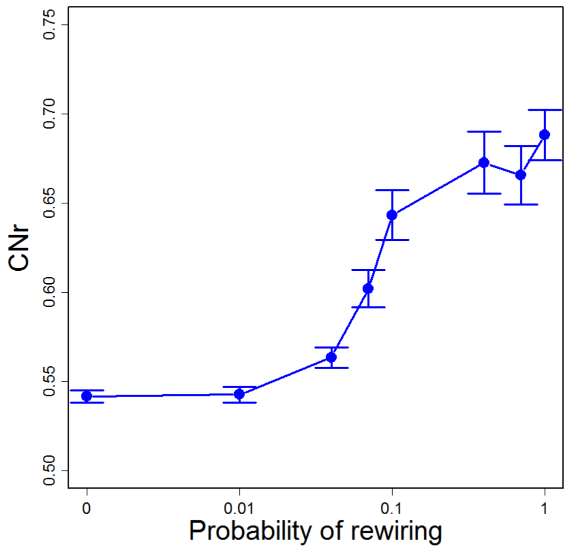

Figure 3. Relationship between rewiring probability and CN_r. Error bars denote 95% confidence interval.

Diversity of acquired knowledge

Next, we analyze acquired knowledge from different perspective. So far, our analyses have been based on each agent's acquired knowledge, and found that each agent acquired $S_{correct}$s and $S_{incorrect}$s for 100 objects after period r in which CC_r met the criterion of equation (4). These results

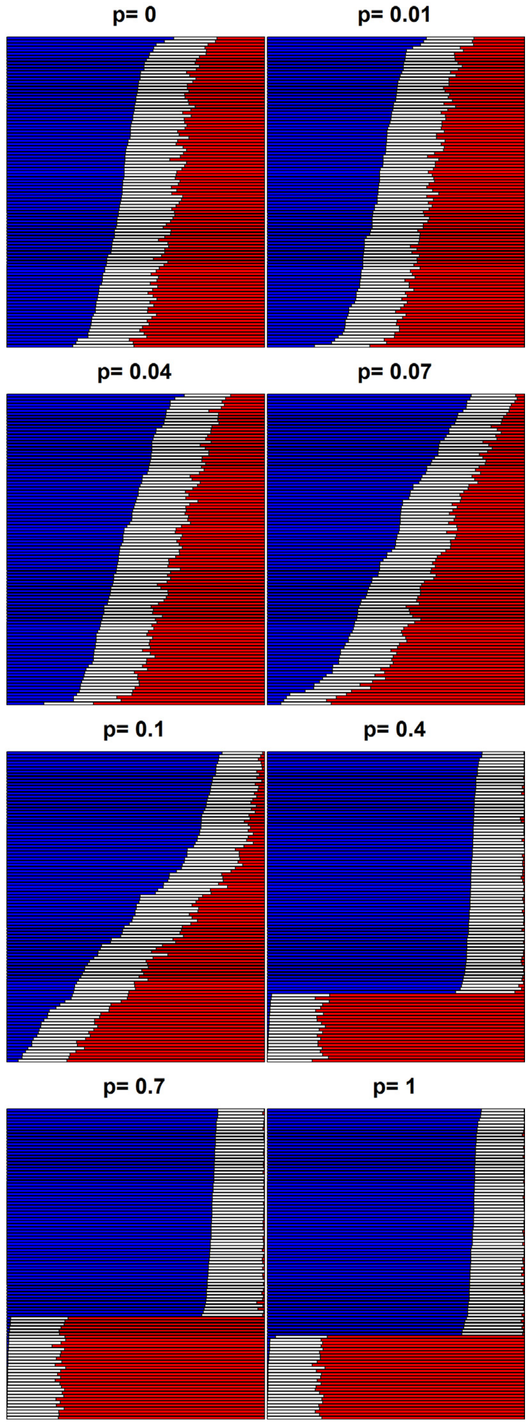

Figure 4. Proportions of $S_{correct}$, $S_{incorrect}$, and $S_{missing}$ for each of 100 objects. Each figure has 100 rows, denoting each of 100 objects. Blue, red, and white areas in each row illustrate proportions of $S_{correct}$, $S_{incorrect}$, and $S_{missing}$. These are one of results of 30 simulations for each network. Each tendency of the 8 networks shown in Figure 5 was almost identical to results of the other 29 simulations.

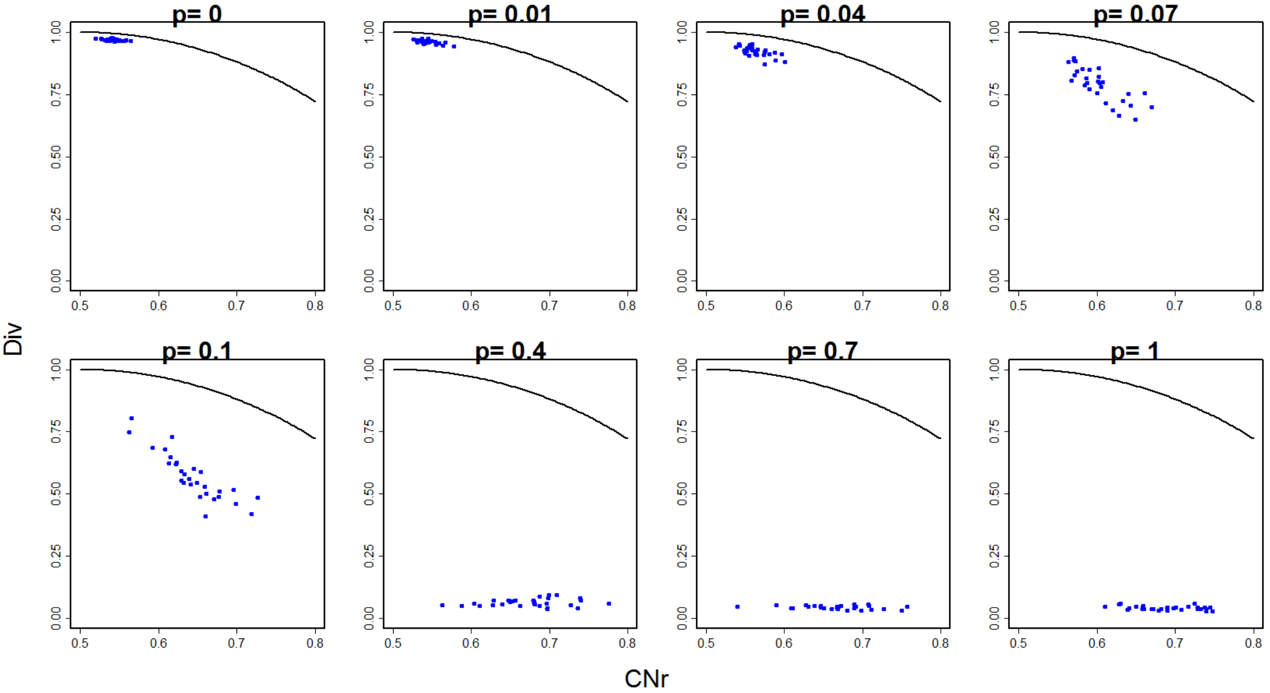

Figure 5. Relationship between CN_r and *Div*. The curve represents the maximum *Div* predicted from CN_r

raise another question about distributions of $S_{correct}$ and $S_{incorrect}$ for each of 100 objects. There are two possibilities about distributions of $S_{correct}$ and $S_{incorrect}$ for each object. The first possibility is that all agents have the same knowledge state. For example, all agents have $S_{correct}$ for some objects and $S_{inorrect}$ for others. In this case, it follows that objects do not have diversity in that all agent have the same knowledge state. The second possibility is that agents have different knowledge states for a single object. In this case, objects have diversity in that opposing knowledge states exist. Here we examine distributions of $S_{correct}$ and $S_{incorrect}$ for each of 100 objects. We calculated proportions of $S_{correct}$, $S_{incorrect}$, and $S_{missing}$ for each of 100 objects.

Figure 4 illustrate these proportions for 8 networks. When p value is 0, 0.01, or 0.04, 0.07, and 0.1, each object contained $S_{correct}$, $S_{incorrect}$. This tendency was not observed when p>0.1. Although most objects contained only $S_{correct}$, and a few objects contained $S_{incorrect}$, there were few objects that contained both $S_{correct}$, and $S_{incorrect}$.

Note that diversity is basically correlated with correctness. If an object has diversity, it means that $S_{correct}$ and $S_{incorrect}$ are equally likely to be acquired by agents for the object. That is, there is trade-off relationship between correctness and diversity. We examined the relationship between correctness and diversity.

In order to examine this issue, we quantified the diversity of acquired knowledge (*Div*) using following equation:

$$Div = \frac{1}{100}\sum_{m=1}^{100}\left(-c_m \log_2 c_m - i_m \log_2 i_m\right) \qquad (3)$$

where c_m denotes the relative proportion of $S_{correct}$ to $S_{incorrect}$ for object m, and i_m denotes the relative proportion of $S_{incorrect}$ to $S_{correct}$ (i.e., $c_m+i_m=1$). Thus the equation corresponds to the calculation of mean entropy among the 100 objects, using c_m and i_m. Figure 5 shows the relationship between CN_r and *Div*. As discussed above, CN_r and *Div* were correlated. Note that when p=0.1, *Div* took higher values than when p=0.4, 0.7, and 1. However, values of *CN* were analogous when p=0.1, 0.2, 0.7, and 1 (see Figure 4).

Therefore, the difference of acquired knowledge between networks of $p \geq 0.1$ relates to the diversity of the acquired knowledge. Although the correctness of the acquired knowledge was analogous among networks of $p \geq 0.1$, the diversity of the acquired knowledge was quite different. Acquired knowledge showed some diversity when p=0.1 and this result was not observed when p>0.1. According to the *WS* model, this difference seems to derive from the clustering coefficients (see Figure 1). The clustering coefficients are generally high when $p \leq 0.1$. However, the clustering coefficients dramatically drop when p>0.1. This boundary between high and low clustering coefficients around p=0.1 seems to produce the difference of diversity in acquired knowledge between networks.

In the real world, people can usually have contradicting knowledge states for one object. Individual characteristics, such as personality or intelligence should produce such contradicting knowledge states. We do not deny this argument. However, the findings in the present study suggest that network structure, in particular cluster of individuals, also plays very important roles in producing contradicting knowledge states.

General discussion

In this paper, we examined how correct or incorrect knowledge is transmitted in the *WS* model by using computer simulation. In particular, we focused on a small world network, which is characterized by short path lengths and high clustering coefficients (Watts & Strogatz, 1998). We found that agents effectively acquired correct knowledge through communicative interactions in the small world network, which is regarded as the representation of people's connectivity in the real world (Dodds Muhamad, & Watts, 2003; Milgram, 1967; Travers & Milgram, 1969).

Cowan and Jonard (2004) showed that agents in a small world network effectively accumulated knowledge through communicative interactions. Hence these findings suggest that a small world network is an adaptive structure in that knowledge can be transmitted effectively and correctly. We also found that acquired knowledge in the small world network has some diversity, which would be consistent with the real world fact that people can have different knowledge states for an identical object.

The present study shows that interactions among individuals play important roles in knowledge acquisition. We do not claim that network structures suffice to explain human cognition. However, some previous studies have also shown that interactions between individuals can explain the knowledge structures that people possess (e.g., Brighton, 2002; Griffiths & Kalish, 2007; Smith, Kirby, & Brighton, 2003). Hence, research about human cognition from the macro and social network level can make contributions for clarifying not only interactions among individuals, but also individual cognitive processes.

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