Children's use of Structure Mapping in numerical estimation

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Abstract

How do young children connect number words to the magnitudes they represent? Here, we test whether 5- to 7-year-old children, like adults, use Structure Mappings (SM) to link number words and approximate magnitudes. We show that 6- and 7-year-olds' number line estimates are recalibrated in response to the distribution of numbers being estimated, providing evidence for SM in these older children. We also find that 5-year-olds show improved estimation performance when given visual access to their previous estimates, suggesting that, while these youngest children do not use SM in some estimation tasks, they nonetheless understand the structural relationship between the count list and approximate magnitudes.

Keywords: Language acquisition, number, approximate magnitudes, word learning, number words.

Introduction

Beginning in infancy, humans can represent the approximate numerical magnitude of sets using the Approximate Number System, or ANS (for review, see Dehaene, 1997). Upon learning the verbal count sequence, children gain access to another way to store and manipulate numerical information—the count list. The count list is a symbolic number system that allows for the precise representation of numerical quantities. These two systems become linked to each other early in development: Children in preschool and kindergarten provide bigger estimates for larger numbers, indicating an ability to map number words onto nonverbal numerical representations (Le Corre & Carey, 2007; Lipton & Spelke, 2005; Barth, Starr, & Sullivan, 2009). However, these early mappings are not stable, and change over development. Accuracy on estimation tasks improves with age, with counting ability, and with explicit training (Siegler & Opfer, 2003; Lipton & Spelke, 2005; Booth & Siegler, 2006; Le Corre & Carey, 2007; Ebersbach et al., 2008; Barth, et al., 2009; Mundy & Gilmore, 2009; Siegler & Ramani, 2009; Thompson & Opfer, 2010). However, while much is known about the developmental trajectory of estimation ability, surprisingly little is known about the learning mechanisms that children use to construct and refine mappings between number words and approximate magnitudes.

Recent research has argued that adults rely on at least two distinct mechanisms for attaching number words to magnitudes (see Sullivan & Barner, 2010, for review). Associatively Learned Mappings (ALM) involve the creation of item-by-item links between individual words and magnitudes, resulting in many mutually independent mappings. Structure Mappings (SM), in contrast, support estimation for larger numbers, and are formed by creating a single link between the verbal and nonverbal number systems on the basis of their shared structure (Gentner & Namy, 2006; Carey, 2009; Gentner, 2010). In particular, SM requires noticing the ordinal structure of each system – e.g., that the word "fifty" comes later in the count sequence than "forty", and should therefore be used to label larger sets. As a result, each number word mapped through SM will be mapped in relation to all other mappings in the count list. These two mechanisms make distinct predictions regarding the effects of new learning experiences on existing mappings. In the case of ALM, changes to the mapping for one number word should have little effect on the mappings of other words, since they are mapped independently. For SM, in contrast, changes to any individual mapping in the system should have consequences for all other mappings.

Evidence for ALM comes from the developmental literature, where it has been shown that young children learn the referents of number words sequentially (Wynn, 1990), and that even after learning the referents of many number words, some children still fail to demonstrate a structural knowledge of the relationship between the count list and numerical magnitudes (Lipton & Spelke, 2005; Le Corre & Carey, 2008; Barth et al., 2009). Additional evidence for ALMs come from research on adults that has shown that estimates for numerical magnitudes smaller than about 20 are not influenced by misleading feedback (Sullivan & Barner, 2010).

Evidence for SM also comes from multiple lines of research. First, several studies have shown that providing adults with misleading feedback about an individual mapping or about the range of magnitudes being tested shifts estimation behavior for most of the number line, and especially for large numbers (Izard & Dehaene, 2008; Sullivan & Barner, 2010). This provides evidence that SM guides the mappings of relatively large number words to ANS representations of their referents. In the absence of misleading feedback, adults' patterns of estimation are also influenced by the distribution of numbers being estimated (Sullivan, Juhasz, Slattery, & Barth, in press), suggesting that adults dynamically adjust their mappings in response to the estimates they have already made. There is also some evidence that children use structure mappings by at least the age of 7. Thompson and Opfer (2010) found that 2nd graders can analogically extend knowledge about numbers with a familiar range (e.g., 1-100) to perform estimates for numbers within an unfamiliar ranges (e.g., 1-10,000). Taken together, these studies provide evidence that adults, and possibly children, deploy structure mappings when relating the verbal number system to nonverbally presented numerosities.

Despite this evidence that adults and older children use SMs, little is known about how such mappings might be acquired. One possibility is that early estimation abilities are not supported by SMs—children's success (or failure) at estimation tasks may be driven primarily by the strength of their ALMs. By this view, SMs may be learned gradually over time and supported by a small set of ALMs. Another possibility, however, is that even very young children use SM, allowing them to make internally consistent (ordinal), though perhaps inaccurate, estimates. Finally, it possible that young children are able to use structure to guide estimation, but do not do so in standard experimental tasks, due to the memory and processing requirements of the measures typically used. In typical estimation tasks, children provide estimates one-at-a-time, without access to previous responses (Siegler & Opfer, 2003; Lipton & Spelke, 2005; Booth & Siegler, 2006; Le Corre & Carey, 2007; Ebersbach et al., 2008; Barth, et al., 2009; Mundy & Gilmore, 2009; Siegler & Ramani, 2009; Thompson & Opfer, 2010). For example, in some tasks children view arrays of dots and are asked to estimate how many there are (Lipton & Spelke, 2005; Le Corre & Carey, 2008; Barth et al., 2009; Mundy & Gilmore, 2009). In other studies, they are shown a line with endpoints marked (e.g., 0 and 100) and are asked to estimate where a number within this range (e.g., 23) belongs on the line (Booth & Siegler, 2006; Ebersbach et al., 2008; Siegler & Ramani, 2009; Thompson & Opfer, 2010). In both types of estimation paradigm, children must keep track of previous responses and how these relate to the current trial in order to use SM. This raises the possibility that children only reliably use SM in situations where previous estimates remain perceptually available (and, as a result, that children's number word mapping abilities may have previously been underestimated due to the use of tasks requiring large memory components). Consistent with this, Thompson and Opfer report that memory for number words predicts individual differences in estimation performance (Thompson & Opfer, 2011).

To address these possibilities, we conducted a study of number line estimation with the aim of answering three questions about the developmental trajectory of number word learning and the learning mechanisms that guide number-word mappings. First, we asked whether children's estimates, like adults', are affected by the distribution of numbers they are asked to estimate. If children use ALM to guide their estimates, then their responses should be independent of one another, and thus should not be affected by differences between distributions of numbers being estimated. If they use SM, however, then this type of distributional information should affect their responses, since previous estimates are used to calibrate future estimates. While previous studies have demonstrated that estimation ability improves greatly between the ages of five and seven (e.g., Siegler & Opfer, 2003), the learning mechanisms guiding this development are still unclear. By determining whether children's estimation behavior is affected by the distribution of numbers being estimated, we can assess the relative roles of ALM and SM in supporting estimation throughout development. Second, we explored the possibility that young children might have mappings between number words and magnitudes, but that the memory and processing demands of traditional estimation tasks might prevent or dissuade them from accurately deploying these mappings. To assess this possibility, we manipulated whether children could see their previous number line estimates, by sometimes allowing them to make multiple estimates on a single line. We reasoned that if children have knowledge of how number words relate to magnitudes, but fail to use this knowledge due to a failure to recollect previous responses, then making previous estimates visually available may improve estimation performance. Finally, in order to assess children's structural knowledge of number-word mappings, we deployed a new method for analyzing estimates. While many previous studies rely primarily on measures of accuracy and linearity of estimates, they have not dissociated accuracy (e.g., how much a given estimate deviates from the correct response) from ordinality (e.g., whether, if a larger number is being estimated on trial n than on trial $n-1$, children also provide a larger estimate for trial n than for trial n-1). If children rely on SM guide estimation, then we might expect their estimates to exhibit ordinality before they become accurate. By dissociating measures of accuracy from measures of ordinality, we can explicitly measure how children's structural knowledge of the count list develops over time, and thus better understand the learning mechanisms that guide children's acquisition of adult-like understanding of number words.

Materials and Methods

Participants Eighty-five children participated. Seventy-seven children completed at least 24 trials and were included in the final analyses. This included 26 5-year-olds, 25 6-year-olds, and 26 7-year-olds.

Materials Stimuli consisted of a horizontal black line 23 cm long (the number line). The number line was centered on a 4.25" x 11" piece of paper. Printed on the left of the number line was the numeral "0" and on the right was the numeral "100". The numbers to be estimated were presented auditorily, and ranged from 3-97.

Procedure Each participant was shown the number line and was told, "This is a number line. See? It goes from 0 all the way to 100" while the experimenter gestured from left to right across the length of the line. The experimenter continued, "Each number has its own special place on the number line. Today, you're going to show me where certain numbers go on the number line. Look! 0 goes here [gesture to leftmost endpoint] and 100 goes here [gesture to the rightmost endpoint]. And all of the other numbers have their own special places on the number line. I'm going to give you a pencil, and your job will be to draw an up-and-down line to show me where each number goes. Are you ready?" Participants were then given 24 estimation trials[1]. On each trial, the number to be estimated was presented, and the child was given a new, differently colored pencil to mark each answer with (to differentiate estimates when they were marked on the same sheet).

Participants were randomly assigned to one of two conditions: the Standard condition and the Visual Comparison condition. In the Standard condition, participants made estimates for numbers one at a time, marking each estimate on a new number line (see Booth & Siegler, 2006; Siegler & Opfer, 2003; Barth & Paladino, 2011). In the Visual Comparison condition, participants made estimates one at a time, but provided multiple estimates on the same number line. As a result, children in the Visual Comparison condition could refer to previous estimates in order to calibrate subsequent estimates. Estimates for the first 12 trials were recorded on one number line, and the last 12 trials were recorded on a separate line.

Participants in each condition were asked to make estimates for one of two possible distributions of numbers. In the Small Number Distribution, 24 numbers were selected between 1-100 such that 4 were smaller than 10 and the rest were selected at random. The Large Number Distribution contained the 24 numbers generated by subtracting the Small Number set from 100 (Barth & Paladino, 2011; Sullivan, et al., in press).

Participants' estimation behavior was also qualitatively coded online for evidence of reference-point use, counting, and other strategies. Those data are not reported here.

Analyses

Dependent Measures Each child's responses were measured on the number line and converted to their numerical estimate equivalent. Indecipherable responses were excluded (N=9/1848 trials). Responses that were located immediately to the right of the number line's endpoint were included in the final analyses (N=28/1848 trials) as these were frequently accompanied by a child's explanation (e.g., "this one has to be off the list"). These responses resulted in some estimates that were larger than 100 (see also Cohen & Blanc-Goldhammer, in press, for a discussion of how the bounds of a number line can constrain estimates in undesirable ways, and why the assessment of numerical knowledge can be facilitated by using unbounded number line tasks). Analyses excluding these 28 trials were also conducted, with identical results to those reported.

Our analyses focused on two measures of estimation performance. First, we measured whether the child's estimates respected the ordinality of the count list. A trial was labeled as ordinal if the child provided an estimate in the correct direction relative to a previous estimate, regardless of its accuracy (e.g., by providing a larger estimate on trial *n* than on trial *n-1* if a larger number was requested on trial *n* than on trial *n-1*). Second, we calculated the linear slope of the relationship between estimate and the number being estimated (e.g., Siegler & Opfer, 2003; Booth & Siegler, 2006; Ebersbach et al., 2008; Lipton & Spelke, 2005; Barth, et al., 2009).

Methods All analyses reported below were conducted using the LME4 package of *R* (Bates & Sarkar, 2007; *R* Development Core Team, 2010). In all models, Subject was considered a random factor, while Comparison Condition and Distribution were considered fixed factors. Ordinality scores resulted in binomial data, and were therefore subjected to logit analyses. We report parameter estimates (ß), p-values estimated from Markov Chain Monte Carlo (MCMC) simulations, and standard error estimates.

Results

We predicted participants' estimation behavior from a model containing age and the magnitude of the number being estimated. Consistent with previous research, there was an effect of Age (ß = 11.5, SE = 1.6, $p < .0001$), an effect of Magnitude (ß = .66, SE = .11, $p < .0001$), and an interaction of Magnitude and Age (ß = -.20, SE = .02, $p < .0001$). These data replicate the finding that children's estimation behavior differs according to magnitude, and that this effect is mediated by age. Next, we analyzed the effect of Magnitude on estimates for each age group separately. Here, ß represents a simple slope measure, with perfect performance as ß = 1. Predictably, 5-year-olds performed the worst (5-year-olds: ß = .36, SE = .03, $p < .0001$). Six-year-olds' estimates had a slope closer to 1, indicating more adult-like performance (ß = .57, SE = .02, $p < .0001$), and 7-year-olds performed extremely well (ß = .74, SE = .02, $p < .0001$). We also compared log and linear fits for each participant's estimates. Like in previous reports (Siegler & Opfer, 2003; Booth & Siegler, 2006), we found that the estimates of younger children were more likely to be best fit by a log-curve than those of older children, which were more linear. This demonstrates that younger children's estimates are somewhat inaccurate, and do not display an adult-like linear relationship between number and estimates (Siegler & Opfer, 2003; but see Ebersbach et al., 2008; Barth & Paladino, 2011; and Cohen & Blanc-Goldhammer, in press, for alternative explanations of the source and importance of logarithmic estimation patterns).

Consistent with the trend of improved performance in estimation accuracy as a function of age, the likelihood that participants provided ordinal responses also differed significantly as a function of age (ß = 1.02, SE = .16, $p < .0001$). This demonstrates that it at least one source of the developmental shift in estimation ability is a maturing understanding of the structural relationship between number words and magnitudes. However, even though young children's estimates were less accurate and less likely to be

[1] Approximately 60 of the participants were given the opportunity to complete a second set of 24 trials in the opposite condition. Due to significantly higher rates of error and numerous experimenter notes of inattention during the second 24 trials, data for these trials were not analyzed further.

ordinal than older children's, all participants demonstrated high levels of ordinality: Nearly 70% of all estimates made by 5-year-olds were ordinal. Despite lacking accurate mappings between number words and magnitudes, even the youngest children produced ordinal responses, suggesting that these children have access to SMs.

We next assessed whether participants in each age group were sensitive to our Distribution manipulation. Previous research has shown that adults' estimation behavior is affected by the range of numbers they are asked to estimate, suggesting that they use Structure Mapping when estimating (Sullivan et al., in press; Sullivan & Barner, 2010). Here, we predicted participants' estimates from a model containing the number being estimated and the Distribution condition (Small Number Distribution vs. Large Number Distribution) in order to assess whether young children's' estimates also shift in response to distributions presented. We found that 5-year-olds did not show any effect of Distribution (5-year-olds: ß = -6.5, SE = 4.6, $p > .15$). However, 6-year-olds showed an effect of Distribution and an interaction of Distribution and Number (ß = -16.25, SE = 4.98, $p < .01$; interaction: ß = .23, SE = .06, $p < .0001$), and 7-year-olds showed an interaction of Distribution and Number (ß = 3.99, SE = 4.80, $p > .25$; interaction: ß = -.11, SE = .04, $p < .025$). Six and 7-year-olds, but not 5-year-olds, dynamically recruited information about the range and distribution of numbers being estimated and incorporated it into their subsequent estimates, providing evidence that these older children adjust their number-to-space mappings in response to information about the range of numbers to be estimated. This recalibration of estimation behavior suggests that older children recruit knowledge of the structural relationships between number words and numerical magnitudes in order to alter their estimation behavior in response to the specific demands of the estimation task.

One possible explanation of our youngest participants' relatively poor performance on the number line task and insensitivity to the Distribution manipulation is that these children lack sufficient knowledge of the structure and logic of the count list (Lipton & Spelke, 2005). If this is the case, then our Visual Access Condition manipulation, which selectively gave participants access to their previous estimates, should have had no effect on estimation performance. Said differently, if young children have weak knowledge of the relationship between number words and approximate magnitudes, then their performance should not differ even when past estimates are visible. However, if 5-year-olds have a firm grasp of the structural relation of number words to numerical magnitudes, but simply have difficulty recalling the location of previous estimates, then visual access to previous estimates should facilitate estimation performance.

Estimation performance differed as a function of condition (Standard vs. Visual Access) for both 5- and 6-year-olds. Five-year-olds showed an interaction of Condition and Magnitude (ß = -.18, SE = .06, $p < .01$).

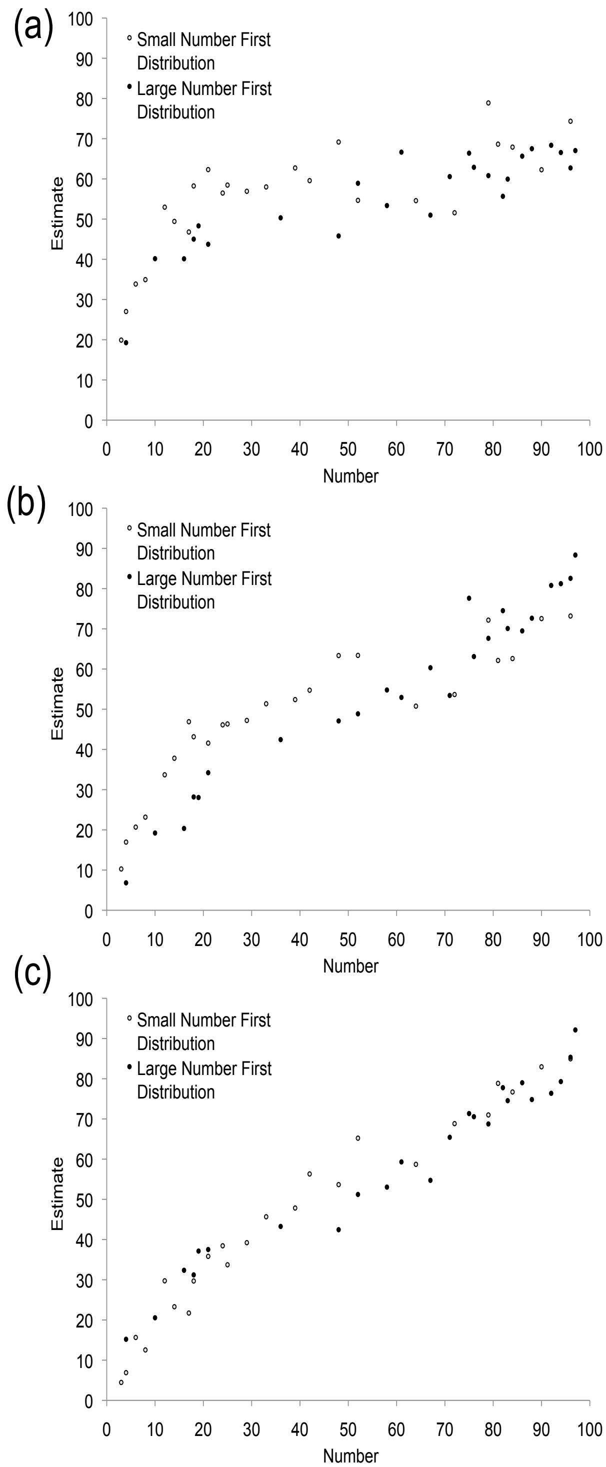

Figure 1: Estimation performance for (a) five-year olds; (b) six-year olds, and (c) seven-year-olds. Data points are means. Black markers indicate Large Number Distribution; gray markers indicate Small Number Distribution.

These youngest participants were more likely to provide smaller estimates for smaller numbers and larger estimates for larger numbers in the Visual Comparison condition relative to the Standard condition. This resulted in more accurate performance in the Visual Comparison Condition than in the Standard Condition (slope for Standard Condition: ß = .21; Visual Comparison Condition: ß = .38). In contrast, 6-year-olds showed a main effect of Condition (ß = -8.50, SE = 4.30, $p < .05$), but no interaction. This suggests that 6-year-olds were somewhat sensitive to our condition manipulation—however, without an interaction, we cannot definitively say that access to previous estimates improved their estimation performance. Finally, 7-year-olds showed neither an effect of condition nor an interaction. This suggests that these older children, whose estimates tend to be quite accurate and linear even in a standard number-line estimation task (e.g., Booth & Siegler, 2006), do not show improved performance from having visual access to previous estimates, likely because their performance was already quite accurate and internally consistent.

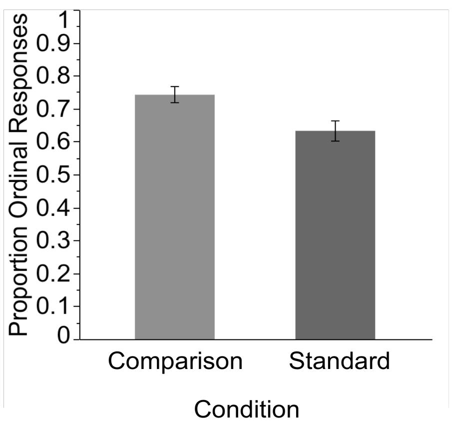

Figure 2: Proportion of ordinal responses in 5-year-olds' in the Visual Comparison and Standard conditions.

Because the slope of estimates can be strongly affected by outliers, we also explored the effect of condition on Ordinality, which reflects children's knowledge of the ordering, but not the distance, between numbers. Condition did not predict any differences in Ordinality for 6- or 7-year-olds (6-year-olds: ß = -.10, SE = .44, $p > .8$; 7-year-olds: ß = -.01, SE = .62, $p > .9$). In contrast, 5-year-olds provided a significantly larger proportion of ordinal responses in the Comparison condition than in the Standard condition (ß = -.60, SE = .30, $p < .05$). Together, the effects of Condition on accuracy and ordinality suggest that even our youngest participants use their knowledge of the count list to guide estimation accuracy and ordinality.

Discussion

The present study demonstrates that SM develops during childhood, and extends previous research on the development of estimation ability by decomposing and recontextualizing the sources of error in estimation performance. By measuring ordinality (and not simply accuracy), by manipulating the role of working memory in estimation, and by asking children to estimate a biased sampling of numbers, we have shown that even very young children rely on structural knowledge of the mappings between number words and approximate magnitudes when estimating. Taken alongside previous recent research showing that structural alignment can improve estimation behavior in older children (Thompson & Opfer, 2010) and that adults rely on SMs when making estimates (Sullivan & Barner, 2010; Sullivan et al., in press; Izard & Dehaene, 2008), these data support the view that the development of estimation abilities depends critically on knowledge of the structural similarities between the verbal and nonverbal number systems. To our knowledge, this is the first study to characterize how SMs are refined over the course of development, and to show the SMs may emerge even before children use them reliably in typical estimation tasks.

We found that children at all age levels tested demonstrated structural knowledge of the relationship between number words and the magnitudes they represent. Six- and 7-year-olds recalibrate their estimation behavior in response to the distribution of numbers being estimated. Additionally, although 5-year-olds performed poorly when making one estimate per line, they performance improved significantly when they were given access to previous estimates. These results clearly show that 5-year-olds understand the ordinal structure of the count list, and can use it to guide their estimates, even though traditional estimation tasks have previously failed to demonstrate this. Together, these findings suggest that the ability to recruit structural information about the number system to flexibly recalibrate and refine estimates develops greatly between the ages of 5 and 7, but that even the youngest estimators, when given visual access to previous estimates, can use this information to improve the accuracy and ordinality of estimation behavior.

While this study provides the first developmental data tracking the development of SM in number word mappings, it raises several additional questions. First, this study leaves open why SMs change over time. Although the ability to remember (and use) previous responses to calibrate future responses likely varies as a function of working memory, it may also depend critically on participants' relative familiarity with the count list. Consistent with this, estimation ability improves with counting ability (Lipton & Spelke, 2005; Davidson, Eng, & Barner, under review). However, counting ability is not wholly predictive of estimation performance, and even relatively weak counters can provide larger estimates for larger magnitudes (Le Corre & Carey, 2008; Barth et al,. 2009). One possible explanation of these conflicting data is that even children who are just beginning to learn the count list possess SMs for number word mappings, but the likelihood that they will accurately deploy these mappings in estimation tasks is not only mediated by number knowledge, but also by memory and other processing limitations. The present study provides suggestive evidence that cognitive limitations (like memory constraints) may greatly influence estimation behavior, and in doing so, influence the conclusions we draw about the development of number knowledge. By this line of reasoning, improved memory for number increases the likelihood that estimates will remain accurate and internally consistent, unless the estimation task removes such demands on memory. However, other accounts of the relationship between estimation and memory have provided a nearly opposite account—some have proposed that improving estimation ability actually improves children's memory for numbers (Thompson & Siegler, 2010). Future research on the development of number knowledge will benefit from exploring how working memory ability and knowledge of the count list contribute to differences in estimation behavior early in development.

A second question raised by this study is how children initially form SMs, and what types of information they use in this process. Previous studies have argued that ALMs may provide the basis for the construction of accurate SMs (Sullivan & Barner, 2010; Carey, 2009). However, this hypothesis has not been directly tested in children, and little is known about which number words are associatively mapped before children begin to show evidence of using SM in development. In order to explore this, studies currently in progress are probing children's use of associative and structure mapping early in acquisition, using calibration techniques that have been used to ask this question in adults (e.g., Izard & Dehaene, 2008; Sullivan & Barner, 2010). Because estimation ability has been shown to be predictive of other measures of academic success (e.g., Siegler & Ramani, 2009), understanding the mechanisms that support accurate estimation may be crucial to developing effective interventions and to understanding the cognitive underpinnings of math success.

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