

Linear Numerical Magnitude Representations Aid Memory for Single Numbers

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Abstract

Memory for numbers improves with age and experience. One source of this improvement may be children's learning linear representations of numeric magnitude, but previous evidence for this hypothesis may have confounded memory span with linear numerical magnitude representations. To obviate the influence of memory span on numerical memory, we examined children's ability to recall a single number after a delay, and the relation between recall and performance on other numeric tasks. Linearity of numerical performance was consistent across numerical tasks and was highly correlated with numerical memory. In contrast, recall of numeric information was not correlated with recall of colors. Results suggest that linear representations of numeric magnitudes aid memory for even single numbers.

Keywords: number representations; numerical estimation; memory

Introduction

Both in school and everyday life, children are presented with a potentially dazzling succession of numbers that they must remember. Some numbers must be remembered exactly, such as phone numbers and the answers to arithmetic problems ($6 \times 8 = 48$). Others only need to be remembered approximately, such as the number of children in one's class, the amount of money in one's piggy bank, or the temperature forecast for tomorrow's weather. When confronted with a series of numbers in either type of situation—e.g., a digit span task (Dempster, 1981) or a vignette (Brainerd & Gordon, 1994)—children's memory for numbers is much poorer than adults', and it improves greatly with age and experience. In this paper, we examine two theories attempting to explain this improvement in numerical memory—the *working memory theory* and the *representational change theory*—and come to a report on a novel memory task (memory for single numbers) that allowed us to test their predictions.

Working Memory Account

There are at least two potential explanations for age-related improvements in children's memory. The first proposal is that numerical information is better retained as children age because children's working memory also improves, thereby leading to better verbatim memory for numerical information when more than one number is presented sequentially (Dempster, 1981). This idea has been highly influential, and it has led to the digit span task being used widely as a measure of working memory span. An

important finding in this research is that the number of digits that can be accurately recalled at age 2 years is about 2, at age 5 about 4, at age 10 about 5, and among adults about 7 (+/- 2).

Representational Change Account

Another proposal for the source of improvements in numerical memory came from a recent study by Thompson and Siegler (2010). They proposed that poor recall of numerical information could be partly traced to children's developing representations of numerical magnitudes. Specifically, children's representations of the magnitudes of symbolic numbers appear to develop iteratively, with parallel developmental changes occurring over many years and across many contexts (Opfer & Siegler, in press). Early in the learning process, numerical symbols are meaningless stimuli for young preschoolers. For example, 2- and 3-year-olds who count flawlessly from 1-10 have no idea that $6 > 4$, nor do children of these ages know how many objects to give an adult who asks for 4 or more (Le Corre et al., 2006). As young children gain experience with the symbols in a given numerical range and associate them with non-verbal quantities in that range, they initially map them to a logarithmically-compressed mental number line (see Figure 1). Over a period that typically lasts 1-3 years for a given

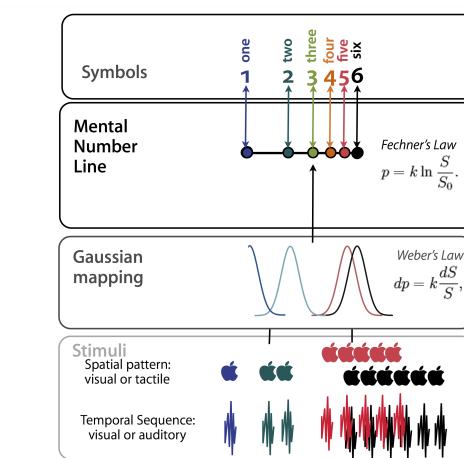


Figure 1. Depiction of a logarithmically-compressed mental number line. Within this representation, differences among numeric values are represented as a function of the difference in the logarithms of the numbers to be represented. Thus, differences between 1 and 2 seem larger than between 5 and 6.

numerical range (0-10, 0-100, or 0-1,000), children's mapping of symbolically expressed numbers to non-verbal representations changes from a logarithmically-compressed form to a linear form, where subjective and objective numerical values increase in a 1:1 fashion (Bertelletti et al., 2010; Opfer, Thompson, & Furlong, 2010; Siegler & Opfer, 2003; Siegler & Booth, 2004; Thompson & Opfer, 2010). Use of linear magnitude representations occurs earliest for the numerals that are most frequent in the environment (i.e., the smallest whole numbers) and is gradually extended to increasingly large numbers (Thompson & Opfer, 2010).

Support for the representational change account came from a task adapted from Brainerd and Gordon (1994). Thompson and Siegler (2010; Experiment 2) presented preschoolers ($M = 4.96$ years) with a series of numbers in a vignette and asked them to recall the numbers after a brief distracter (color/shape/object naming). For example, children heard, "Mrs. Conway asked students in her school district about their favorite foods. N_1 students liked spaghetti best, N_2 students liked pizza best, and N_3 students liked chicken nuggets best," were asked to name four colors/shapes/objects, and then asked, "How many students liked spaghetti best? How many students liked pizza best? How many students liked chicken nuggets best?" (see supporting materials for Thompson & Siegler, 2010, Experiment 3).

Thompson and Siegler (2010) made two observations supporting the idea that linear numerical-magnitude representations aid memory for numerical information. First, linearity of number line and number categorization were highly correlated with memory accuracy, whereas accuracy itself was not. Thus, a third variable (such as overall numeric proficiency) was unlikely to be a source of the correlation between numerical estimation and numerical memory. Further, memory accuracy was negatively correlated with the magnitude of the number given. This finding is important because if numeric symbols are mapped with a constant noisiness to a logarithmically-scaled mental number line (as in Figure 1), then signal overlap increases dramatically with numerical value, thereby leading to significant interference from adjacent values as the target number increases. Interference from highly similar exemplars is a well-known source of errors in recall (Schacter, Norman, & Koutstaal, 1998), yet it would not be predicted if children's memory for numbers depended solely on their memory span.

The Current Study

In this study, we investigated a potential source of concern in the evidence supporting the representational change account. That is, individual and developmental differences exist in memory span (Dempster, 1981), thereby leading to a potential confound existing between memory span (or memory strategies) and the development of numerical representations, thereby leading to a spurious correlation between linearity of numerical estimation and

span-based numerical memory. This concern seems justified by two considerations. First, previous studies have shown a correlation between working memory and linear numerical estimation performance (e.g., Geary, Hoard, Byrd-Craven, Nugent, & Numtree, 2007). Thus, working memory differences cannot be ruled out as a source of individual differences in numerical memory that would also correlate with linear numerical estimation. Second, the sum of numbers to be recalled and the number of distractors in Thompson & Siegler's (2010) study would have been at the edge of many children's memory span, leading many children to fail to recall numeric information if memory span were a contributor to numerical memory.

To address these concerns, the current study tested children's memory for a single number after a brief delay, thereby obviating any potential contribution of individual differences in working memory span to numerical memory. As in Thompson and Siegler (2010; Exp 2), we examined (1) the memory of preschoolers for numbers 0-20 because preschoolers vary in whether they represent these numbers as increasing linearly (Thompson & Siegler, 2010; Bertelletti et al., 2010), (2) the degree to which children's estimates of the positions of numbers on number lines increased linearly with actual numeric value, and (3) children's counting accuracy when asked to count numbers from 0-20. Additionally, we examined children's performance on a "give-a-number" task (Wynn, 1990) because performance on this task has been reported to show a similar logarithmic-to-linear shift (Opfer, Thompson, & Furlong, 2010) and no relation to counting accuracy (Le Corre et al., 2006), and to indicate children's realization that their counts denote cardinal values (Sarnecka & Carey, 2008).

Method

Participants

Thirty-two participants were recruited from 3 child-care centers in the Columbus metro area. Children were aged 3 years ($n = 10$, $M = 3.63$), 4 years ($n = 12$, $M = 4.51$), and 5 years ($n = 10$, $M = 5.41$).

Tasks

For all tasks, children were presented with 8 numbers in randomized order. We presented the same numbers used in Bertelletti, Lucangeli, Piazza, Dehaene, and Zorzi (2010): 2, 4, 6, 7, 13, 15, 16, and 18.

Counting task. On the counting task, children were presented with a stack of 8 poster board strips, one at a time, with a different number of white, equally-spaced poker chips attached to the strips. Each subject was told, "This game is a secret number game. You have to find out how many chips there are on this card." Children were neither encouraged nor discouraged to count, so that they would use their own strategies.

Number/color recall task. The numerical recall task immediately followed the counting task. After explaining the counting task to the subject, the experimenter continued, “Then, I’m going to tell you a password.” The experimenter pointed out a second experimenter, who was seated less than halfway across the room, and instructed the child that, in a quiet voice, he or she had to tell the second experimenter a “password” and then how many chips there were on the card. The “password” was one of eight colors of construction paper the experimenter kept in a stack to the right of the table, and meant as a distractor to prevent the child from simply rehearsing the number prior to confiding in the second experimenter.

Upon reaching the second experimenter, the child was asked to state the secret password (color), then how many chips there were (number). The second experimenter recorded the color the child reported, as well as the number of chips he or she remembered.

Numerical estimation task. Our numerical estimation task was adopted from Siegler and Opfer (2003). Children were presented with 8 sheets of paper. On each sheet was a 25 cm line, flanked by two vertical hatch marks. The value “0” was written below the vertical hatch mark representing the left end of the line, and the value “20” was written below the mark representing the right end of the line. Above the middle of the line was one of the 8 task numerals, centered within a circle. The experimenter told subjects, “Today, we’re going to play a game with number lines. What I’m going to ask you to do is show me where on the number line some numbers are. When you decide where the number goes, I want you to make a mark through the number line like this,” and demonstrated the mark. Children were not corrected on their responses. However, one child opted to mark outside the boundaries of the line, and was reminded, “The number goes somewhere *on* the line.”

Give-a-number task. The give-a-number task was adopted from Wynn (1990). Children were presented with a pile of 20 blue poker chips and told that the experimenter would ask them for a number of chips. The child’s task was to place what he or she believed to be the correct number of chips before the experimenter. Whether the child presented the experimenter with a correct or incorrect number of chips, the experimenter gave only neutral feedback. Children who claimed they didn’t know how to give the experimenter a numerical value were prompted with, “Can you try your best?” Finally, the experimenter would ask, “And how many is that?” Both the number of chips the child gave and the child’s verbal response were recorded.

Design and Procedure

Prior to being given the tasks above, children played one of two games to orient them to the experimenters and tasks. Board games of identical size were used in each game and were labeled “The Number Game.” Twenty-two colored squares of identical size were ordered consecutively on each

board. The first square was labeled “Start,” and the last square was labeled “Finish.” The squares between the first and last were consecutively numbered from 1 to 20. The sole difference between the two games was the arrangement of the numbers. In one game, the numbered squares were placed in a horizontal line across the board, arranged left-to-right. In the other game, the numbered squares were arranged in a circle, with numbers increasing in value in a clockwise direction. No effect of game type was observed in the data.

After the orientation games, experimenters revisited schools to administer the battery of tasks. The order of presentation of the tasks was counterbalanced using a Latin square design, with the exception that the numerical recall task necessarily followed the counting task. The time lapse between orientation and tasks for all but one child was one day. No children were tested more than 4 days following orientation. Children were tested individually during one 25-minute session occurring in a quiet room in their school.

Results

Our results are divided into two major sections. In the first section (“Description of Task Performance”), we describe changes in children’s performance in counting, numerical memory, numerical estimation, and give-a-number. In the next section (“Predictors of Numerical Memory”), we examine the relations among the tasks and the influence of numerical magnitude on memory.

Description of Task Performance

For each relevant task, we examined accuracy by calculating the Mean Absolute Percentage Error (MAPE) on the task, and we examined linearity by calculating the best-fitting linear function (R^2_{lin}) relating the number given to children against the number provided in response. To assess the effect of age, subjects were divided according to a median split of ages (4.45 years old).

Counting task.

To calculate MAPE for the counting task, we took the average of the Percentage Absolute Error (PAE), or $(|\text{Chips Shown}-\text{Number Counted}|)/20*100$, across all trials for a subject. To calculate linearity (R^2_{lin}), we regressed the number counted by a child on each trial against the number of chips that were actually shown.

There was an effect of age group on counting accuracy, $F(1,31) = 12.67, p < .01$ and linearity, $F(1,31) = 8.83, p < .01$. The average MAPE for all subjects was 13.52 (SD = 10.4), and average R^2_{lin} for all subjects was .69 (SD = .30). The MAPE was 19.1 (SD = 11.0) for younger children and 7.96 (SD = 5.9) for older. The average R^2_{lin} for younger children was .55 (SD = .31) and .84 (SD = .22) for older.

Number/color recall task.

As expected, the percentage of numbers and colors that were accurately recalled improved with age (numbers, $F[1,31]=7.6, p=.01$; colors, $F[1,31]=5.65, p=.02$).

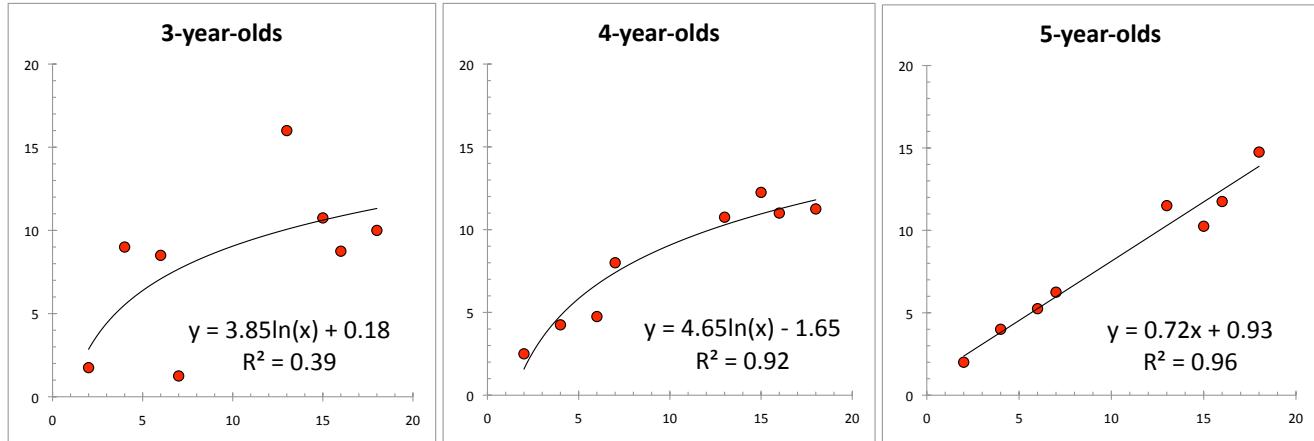


Figure 2. Numerical estimates by age group.

Improvements in number memory (younger, $M = 52\%$; older, $M = 74\%$) were nominally larger than improvements in color memory (younger, $M = 73\%$; older, $M = 88\%$), which was more accurate overall.

MAPE for the numerical recall task was determined similarly for the counting task, except that PAE was determined by $(|\text{Number Provided by Child} - \text{Number Recalled}|)/20 * 100$. Thus, if a child (correctly or incorrectly) counted 12 chips and then recalled there being 13 chips, PAE would be 5. PAE was averaged across all trials for a subject to determine the MAPE.

The average MAPE for all subjects was only 2.66 ($SD = 5.8$), with error in the younger group's recall (MAPE = 4.5) being marginally higher than the older group's (MAPE = .83), $F(1,31)=3.48$ $p=.07$. Thus, recall for a single number was much higher than had been observed in Thompson & Siegler (2010).

Numerical estimation We first examined development of numerical estimation by measuring age-related changes in accuracy of number line estimates. Accuracy of estimates was indexed by percent absolute error (PAE), defined as: $(|\text{to-be-estimated value} - \text{child's estimate}|)/\text{numerical range} * 100$. As expected, accuracy of number line estimates improved substantially with age, $F(1, 31)=9.64$, $p<.01$, with younger children's PAE being 30.3 and older children's being 18.1.

Previous work explained age-related changes in accuracy of number line estimates as coming from a logarithmic to linear shift in representations of numerical magnitude (see Opfer & Siegler, in press, for review). Consistent with this idea, we found that linearity of estimates improved with age (see Figure 2). Estimates of 3- and 4-year-olds increased logarithmically with actual value (3s: $R^2_{\log} = .39$, $R^2_{\text{lin}} = .35$; 4s $R^2_{\log} = .92$, $R^2_{\text{lin}} = .90$), whereas estimates of 5-year-olds increased linearly with actual value ($R^2_{\log} = .91$, $R^2_{\text{lin}} = .96$).

Give-a-number task. We next examined development of numerical recall by measuring age-related changes in

accuracy on the give-a-number task. Accuracy of estimates was again indexed by MAPE, $(|\text{number requested} - \text{number given by child}|)/20 * 100$. As expected, accuracy improved substantially with age, $F(1, 31)=30.58$, $p<.01$, with younger children's MAPE being 23.0 ($SD=11.8$), and older children's being 4.65 ($SD=6.1$).

Predictors of Memory Performance

Might improvements in memory accuracy—like improvements in accuracy of counting, numerical estimates, and give-a-number—be caused by a shift to linear representations of numerical value? Several observations suggest this might be the case.

	Memory:	Counting:	Number Line:	Give-A-Number:		
Task/Measure	% recall	Linearity	MAPE	Linearity	MAPE	Linearity
Counting:						
Linearity	.51**					
MAPE	-.43*	-.88**				
Number Line:						
Linearity	-.28	.42*	-.41*			
MAPE	-.39*	-.57**	.49**	-.84**		
Give-A-Number:						
Linearity	.51**	.79**	-.76**	.38*	-.52**	
MAPE	-.36*	-.59**	.61**	-.35†	.44*	-.76**

†<.10. *p<.05. **p<.01

Table 1. Relations among task performance.

First, as indicated in Table 1, accuracy of recall was negatively correlated with errors on all three number tasks: counting ($r = -.43$, $p < .05$), number line estimation ($r = -.37$, $p < .05$), and give-a-number ($r = -.36$, $p < .05$). In contrast, accuracy of recall for colors failed to correlate with accuracy of recall for numbers. Additionally, linearity on number tasks was a consistent predictor of numerical recall.

These correlations indicate that linear representations of numerical magnitude—whether assessed by counting, by numerical estimation, or by give-a-number—are central in children's ability to recall numeric information, as

suggested by the representational change hypothesis. Further, they show that these correlations are not simply driven by children's overall memory ability. If they were, ability to recall color would be at least as good a predictor of numerical recall as numerical accuracy or linearity.

Second, we observed that children who were most linear on one type of numeric task tended to be the most linear on another numerical task. Thus, linearity of counting correlated highly with linearity of numerical estimation ($r = .42, p < .05$) and linearity of give-a-number ($r = .79, p < .01$), and linearity of numerical estimation correlated with give-a-number ($r = .38, p < .05$). Thus, linearity of numerical performance appears to be a coherent source of individual differences, one that we have seen to predict individual differences in numerical memory.

Finally, we also examined whether there were differences in recall for large versus small numbers. This size effect is a straightforward prediction of recall depending on representations of numerical value. To test this, we examined memory for numbers that were below or above 10, and we divided children into two groups relative to the median split of ages for subjects in the study (4.61 years old). As predicted, an ANOVA showed a main effect of numeric magnitude on recall accuracy, $F(1,192) = 4.54, p < .05$, especially among the younger children (see Figure 2).

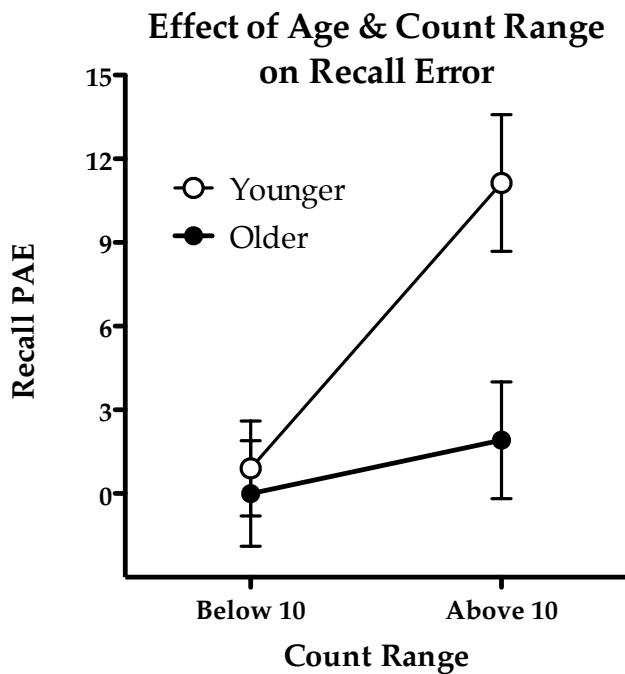


Figure 2. Relation between the magnitude of the number to be recalled and error in recall performance, for younger (white circles) and older (black circles) children.

General Discussion

Previous work has indicated that the development of linear representations of numerical magnitudes profoundly expands children's quantitative thinking. It improves children's ability to estimate the positions of numbers on number lines (Siegler & Opfer, 2003), to estimate the measurements of continuous and discrete quantities (Thompson & Siegler, 2010), to categorize numbers according to size (Opfer & Thompson, 2008), and to estimate and learn the answers to arithmetic problems (Booth & Siegler, 2008). Recent work has also indicated that the logarithmic-to-linear shift is associated with improved memory for numbers (Thompson & Siegler, 2010).

In this paper, we took a critical look at the representational change theory of development of numerical recall. We were particularly interested in whether it could account for changes in memory for single numbers. This issue is important because previous work could not rule out the influence of working memory span on numerical memory. In this way, we provided a particularly robust test of the theory.

Consistent with the representational change account, we found that linearity of numerical performance—whether linearity in counting, in estimating the position of numbers on number lines, or (to a lesser extent) in providing numbers of chips to verbally requested numbers—was positively correlated with accuracy of numerical recall. This positive association could not be explained simply by children getting more accurate on numerical tasks: accuracy was typically a poor predictor of numerical recall. Nor could this association be explained by numerically proficient children simply getting better at remembering items generally: accuracy at remembering colors was also a poor predictor of accuracy at remembering numbers. Rather, the high correlations of linearity among the numerical tasks suggests that there are stable individual differences in linearity of numerical representations, and these individual differences in linearity improve recall for even single numerical values.

Beyond demonstrating that linear spatial-numeric associations are associated with improved memory for numbers, the present results also help to explain the positive relation between linear numeric magnitude representations and arithmetic proficiency. That is, if learning linear spatial-numeric associations improves memory for single numbers as well as multiple numbers presented in vignettes, it is highly likely it also improves memory for numbers in other contexts, such as memorizing arithmetic facts. In this way, the present results suggest a plausible explanation for the observed association between numerical estimation and mathematics course grades (Opfer & Siegler, in press), and it suggests that numerical memory may moderate this link. Although this account is admittedly speculative, we believe it is an important issue for future research.

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