Learning Linear Spatial-Numeric Associations Improves Memory for Numbers

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Abstract

Memory for numbers improves with age and experience. We tested the hypothesis that one source of this improvement is a logarithmic-to-linear shift in children's representations of numeric magnitude. In Experiment 1, we found that linearity of representations improved with age and that the more linear children's magnitude representations were, the more closely their memory of the numbers approximated the numbers presented. In Experiment 2, we trained children on a linear spatial-numeric association, and we found that children who learned to represent numbers as increasing linearly with numeric magnitude also improved their memory for numbers. These results suggest that linear spatial-numeric associations are both correlated with and causally related to development of numeric memory.

Keywords: spatial-numeric associations; number representations; numerical estimation; memory

Introduction

Remembering numeric information is an important part of daily life. Sometimes it is necessary to remember numeric information exactly (e.g., social security numbers, phone numbers, flight numbers, street addresses), whereas other times remembering the general gist of numeric information will suffice (e.g., savings account balances, temperatures, number of students in a lecture hall). Across both types of memory, children's memory for numbers improves greatly with age and experience (Dempster, 1981; Brainerd & Gordon, 1994). Here we tested the hypothesis that children's numerical memory improves with age due to changes in how children represent numerical magnitudes.

Development of Numerical Representations. Children's representations of the magnitudes of symbolic numbers appears to develop iteratively, with parallel developmental changes occurring over many years and across many contexts (Opfer & Siegler, in press). Early in the learning process, numerical symbols are meaningless stimuli for young preschoolers. For example, 2- and 3-year-olds who count flawlessly from 1-10 have no idea that 6 > 4, nor do children of these ages know how many objects to give an adult who asks for 4 or more (Le Corre et al., 2006). As young children gain experience with the symbols in a given numerical range and associate them with non-verbal quantities in that range, they initially map them to a logarithmically-compressed mental number line. Over a period that typically lasts 1-3 years for a given numerical range (0-10, 0-100, or 0-1,000), children's mapping of symbolically expressed numbers to non-verbal representations changes from a logarithmically-compressed form to a linear form, where subjective and objective numerical values increase in a 1:1 fashion (Bertelletti et al., 2010; Opfer, Thompson, & Furlong, 2010; Siegler & Opfer, 2003; Siegler & Booth, 2004; Thompson & Opfer, 2010). Use of linear magnitude representations occurs earliest for the numerals that are most frequent in the environment, that is the smallest whole numbers, and is gradually extended to increasingly larger numbers (Thompson & Opfer, 2010).

Changes in numerical representations occur not only with increasing age, but also with specific experiences designed to train linear spatial-numeric associations. For instance, Opfer and Siegler (2007) provided second graders with corrective feedback on the location of numbers near 150, the maximally discrepant point between a logarithmic and linear function forced to pass through 0 and 1,000 (see Figure 1). After receiving feedback, children adopted a linear representation that spanned the entire 0-1,000 numeric range.

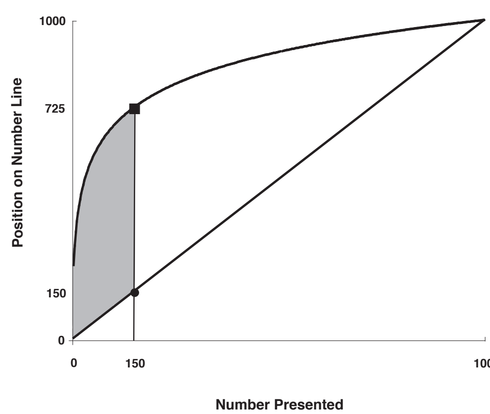

Figure 1: Logarithmic and linear functions. Distance between representations is greatest at 150 (725 vs. 150); this means that the logarithmic function increases more than the linear representation between each successive pair of numbers up to 150, but increases less than the linear function above 150. Thus, numbers below 150 are more discriminable in the logarithmic representation, and numbers above 150 are more discriminable in the linear representation.

Whether occurring with age or with specific training experiences, the logarithmic-to-linear shift in children's representations of symbolic quantities expands children's quantitative thinking profoundly. It improves children's ability to estimate the positions of numbers on number lines (Siegler & Opfer, 2003), to estimate the measurements of continuous and discrete quantities (Thompson & Siegler, 2010), to categorize numbers according to size (Opfer & Thompson, 2008), and to estimate and learn the answers to arithmetic problems (Booth & Siegler, 2008).

Relation between Numerical Representations and Numerical Memory. Recently, Thompson and Siegler (2010) found that individual differences in the numerical representation (logarithmic or linear) that children used to estimate numbers on 0-1,000 number lines was associated with their ability to recall large numbers (>150). Their reasoning was that children who possessed a linear representation of numbers were better able to differentiate the large numbers (see Figure 1), and thus to remember them after a delay.

If true, this account has important theoretical and practical implications. Theoretically, it might explain the previously observed association between age and ability to remember numbers (e.g., Brainerd & Gordon, 1994). Practically, it suggests that children's memory for numbers could also be improved by engendering the logarithmic-to-linear shift observed in the training studies (e.g., Opfer & Siegler, 2007; Opfer & Thompson, 2008). Testing this practical implication is also theoretically interesting because it would provide evidence for a *causal* link between numerical representations and memory, as opposed to just a correlation that might be equally well-explained by a third variable (such as increasing mathematical proficiency).

The Current Study. The current series of experiments were designed to test for a causal link between children's numerical representations and their numerical memory. In Experiment 1, Kindergartners, second graders, and adults estimated numbers in the 0-1,000 range and recalled numbers presented in meaningful vignettes. The purpose of Experiment 1 was to investigate the unique contributions of both age and quality of numerical representations to accuracy in numerical recall, as well as to identify children who would benefit from training in Experiment 2.

In Experiment 2, Kindergartners and second graders received training on the number-line estimation task, following the procedure used in Opfer and Siegler (2007). Our goal in Experiment 2 was to investigate whether adoption of linear spatial-numeric associations on the number line estimation task would improve recall of numerical information. We were particularly interested in memory for large numbers (>150) because they were much larger than those for which children received training (150), yet were predicted to elicit the greatest improvements by the logarithmic-to-linear shift account.

Experiment 1: Age Differences in Numerical Estimation and Numerical Recall

Participants

Participants were 14 Kindergartners (*Mean age* = 6.25 years, *SD* = 0.39 years; 50% girls), 63 second graders (*Mean age* = 8.31 years, *SD* = 0.33 years; 45% girls), and 28 adults (*Mean age* = 20.07 years, *SD* = 2.3 years; 50% women).

Tasks

Numerical estimation Participants were asked to estimate the position of 22 sequentially presented numbers on a line, where the left end was labeled "0," the right end "1,000," and no other marks. The numbers to be estimated (from Opfer & Siegler, 2007: 2, 5, 18, 34, 56, 78, 100, 122, 147, 150, 163, 179, 246, 366, 486, 606, 722, 725, 738, 754, 818, and 938) were centered above the midpoint of each line. After participants made each of their estimates, another problem appeared on the computer screen.

Numerical recall Participants listened to six short vignettes and were asked to recall the numbers in the vignette after a brief distracter. For example, children heard, "Mrs. Conway asked students in her school district about their favorite foods. N_1 students liked spaghetti best, N_2 students liked pizza best, and N_3 students liked chicken nuggets best," were asked to name four colors/shapes/objects, and then asked, "How many students liked spaghetti best? How many students liked pizza best? How many students liked chicken nuggets best?" (see supporting materials for Thompson & Siegler, 2010, Experiment 3). Each story involved three "small numbers" (5, 18, 53, 79, 164, 237), three "medium numbers" (419, 487, 524, 548, 625, 632), or three "big numbers" (725, 759, 817, 846, 938, 962). Numbers were presented in random order within vignettes, and each number was presented equally often with each vignette.

Procedure

Children were tested individually during one 25-minute experimental session occurring in a quiet room in their school; adults were tested individually during one 20-minute experimental session in a laboratory on a college campus. Participants always completed the number line estimation task first, and no feedback was given on participants' performance.

Results and Discussion

Numerical estimation We first examined development of numerical estimation by measuring age-related changes in accuracy of number line estimates. Accuracy of estimates was indexed by percent absolute error (PAE), defined as: ([|to-be-estimated value – participant's estimate|]/numerical range) * 100. For instance, PAE = 45% if a child clicked at the location for 600 when asked to estimate the number 150 on a 0-1,000 number line, ([|150-600|]/1,000) * 100. That is, the *higher* the PAE, the *less* accurate the estimates. As expected, accuracy of number line estimates improved substantially with age, $F(2, 102) = 80.87$, $p < .0001$, $\eta^2 = .61$ with Kindergartners' PAE being 31% (*SD* = 9%), second graders' 17% (*SD* = 8%), and adults' 3% (*SD* = 0.9%).

Previous work explained age-related changes in accuracy of number line estimates as stemming from a shift from logarithmic to linear mappings between symbolic and spatial values (see Opfer & Siegler, in press, for review). To test this idea, we compared the fit of the logarithmic and linear regression functions for the relation between the mean estimates of each age group and actual numeric value. Consistent with the logarithmic-to-linear shift hypothesis, we found that Kindergartners' mean estimates were best described by a logarithmic function (log $R^2 = .82$, lin $R^2 = .43$), second graders' about equally by each function (log $R^2 = .91$, lin $R^2 = .88$), and adults by the linear function (log $R^2 = .66$, lin $R^2 = 1.0$). To ensure that these fits did not arise from averaging over distinct cognitive profiles, we used the same procedure to find the best-fitting function for each individual's estimates. Only 21% of Kindergartners produced a series of estimates better fit by the linear than logarithmic function, whereas 46% of second graders and 100% of adults produced a series of estimates better characterized as linear than logarithmic. Thus, each analysis provided a consistent picture of developing numerical estimation, with Kindergartners most likely using a logarithmic representation, second graders using logarithmic and linear ones about equally often, and adults relying largely on linear representations.

Numerical recall We next examined development of numerical recall by measuring age-related changes in accuracy of memory. Accuracy of memory was again indexed by PAE, [(|to-be-remembered value – number participant remembered|)/1,000] * 100. Please note that the *higher* the PAE the *less* accurate were the numbers recalled. As expected, accuracy of memory improved substantially with age, $r = -0.63$, $F(1, 102) = 67.51$, $p < .0001$, with Kindergartners' PAE being 35% (*SD* = 8%), second graders' 19% (*SD* = 9%), and adults' 7% (*SD* = 3%).

Relation between numerical estimation and numerical recall Might improvements in memory accuracy—like improvements in accuracy of numerical estimates—be caused by a logarithmic-to-linear shift in representations of numerical value? Several observations suggest this might be the case.

First, memory accuracy was highly correlated with performance in numerical estimation (Figure 2). Overall, number line estimation accuracy explained 61% of the variance in recall accuracy; age explained only 2% more variance in accuracy of numeric recall than did entering number line estimation accuracy into the regression model alone (Model 1 = 61% variance, $F(1, 102) = 159.44$, $p < .0001$; Model 2 = 63% variance, $F(2, 101) = 85.24$, $p < .0001$). When age was entered before number line estimation accuracy, number line estimation accuracy explained 16% more variance in accuracy of numeric recall than did entering age into the regression model alone (Model 1 = 63% variance, $F(1, 102) = 67.56$, $p < .0001$; Model 2 = 79% variance, $F(2, 101) = 85.24$, $p < .0001$). Thus, accuracy of number representations can explain most individual differences in memory accuracy.

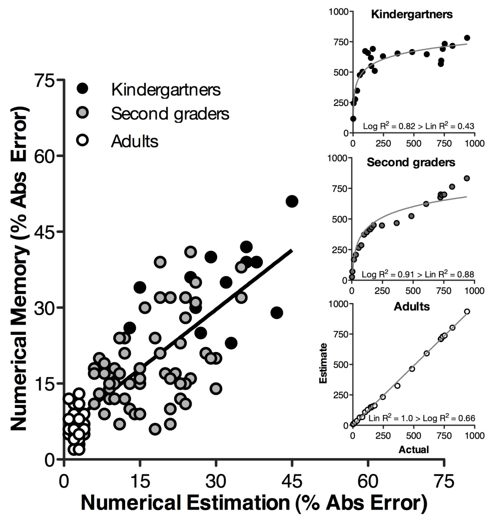

Figure 2: Percent absolute error on the numerical estimation task is strongly correlated with percent absolute error on the numerical memory task for Kindergartners (black circles), second graders (gray circles), and adults (white circles). The inset figures illustrate a logarithmic-to-linear switch in numerical estimation across the age range.

A second set of observations came from the predicted effects of numerical magnitude on memory accuracy. That is, if numeric symbols are mapped with a constant noisiness to a logarithmically-scaled mental number line, then signal overlap increases dramatically with numerical value, thereby leading to significant interference from adjacent values as the target number increases. In contrast, if numeric symbols are mapped with constant noisiness on a linearly-scaled mental number line, then signal overlap is greatest for neighboring values but does not otherwise increase with numeric value. On a 0-1,000 mental number line, for example, the difference between the two representations would be greatest around 150 (see Figure 1), leading to a distinct pattern of predicted errors: for numbers greater than 150, use of a logarithmic representation would interfere much more with memory than use of a linear representation, whereas for numbers less than 150, accuracy would favor the logarithmic representation or neither representation (depending on overall noisiness).

To test this prediction, we conducted a 2 (numerical range: below 150, above 150) x 2 (best fitting function on the number line estimation task: logarithmic, linear) ANOVA on PAE scores for the recall task. There was a main effect of numerical range, $F(1, 103) = 155.63$, $p < .0001$, $\eta^2 = .52$, and best fitting function, $F(1, 103) = 38.20$, $p < .0001$, $\eta^2 = .27$. There was also a significant numerical range x best fitting function interaction, $F(1, 103) = 41.97$, $p < .0001$, $\eta^2 = .14$. For numbers below 150, memory accuracy was high regardless of the numerical representation employed on the number line estimation task, $F(1, 103) < 1$, $p > .05$. However, for numbers greater than 150, memory accuracy was much lower among participants who produced a logarithmic series of estimates on the number line estimation task than among participants who produced a linear series of estimates (PAE = 31% vs. 13% respectively, $F(1, 103) = 71.51$, $p < .0001$, $\eta^2 = .41$). Thus, memory accuracy—particularly memory for large numbers—was associated with use of linear representations.

If improvements in memory accuracy—particularly memory for large numbers— can be explained by a logarithmic-to-linear shift in representations of numerical value, then the largest age differences in memory accuracy would also come in memories for large numbers. To test this prediction, we conducted a 2 (numerical range: below 150, above 150) x 3 (age group: Kindergartners, second graders, adults) ANOVA on PAE scores for the recall task. As expected, there was a main effect of numerical range, $F(1, 102) = 90.87$, $p < .0001$, $\eta^2 = .43$, age group, $F(2, 102) = 73.07$, $p < .0001$, $\eta^2 = .59$, and a significant numerical range x age group interaction, $F(2, 102) = 9.87$, $p < .0001$, $\eta^2 = .09$. For numbers below 150, the effect of age on memory accuracy was relatively small (PAE Kindergartners, 15% > second graders, 5% = adults, 2%), $F(2, 102) = 9.47$, $p < .0001$, $\eta^2 = .16$. For numbers above 150, the effect of age was much larger (PAE Kindergartners, 41% > second graders, 23% > adults, 8%), $F(2, 102) = 53.61$, $p < .0001$, $\eta^2 = .51$.

In summary, a logarithmic-to-linear shift in representations of numerical value accurately predicted (1) improving accuracy of numerical estimation, (2) an age-related change in pattern of numerical estimates, (3) a strong correlation between numerical estimation performance and memory accuracy, and (4) the finding that developmental changes in numerical memory occurred much more for numbers greater than 150 than less than 150.

In combination with previous findings (Thompson & Siegler, 2010), the results of Experiment 1 provide converging correlational evidence that linear spatial-numeric associations improve numerical memory. Additionally, the results show that age alone cannot account for the association between quality of representation and numerical memory, an issue that could not be explored in Thompson and Siegler's data. This is important because it raises the possibility that manipulating the quality of numeric representations would improve numeric memory. In the next study, we sought evidence of a causal link between linear spatial-numeric associations and numerical memory.

Experiment 2: Effects of Training on Numerical Estimation and Numerical Recall

Participants

Children from Experiment 1 who produced a logarithmic series of estimates on the number-line estimation task were included in Experiment 2 as were additional Kindergartners and second graders who were recruited to participate in the training procedure. Participants were 23 Kindergartners (*Mean age* = 6.23 years, *SD* = 0.39 years; 61% girls; 48% were later assigned to the treatment group) and 64 second graders (*Mean age* = 8.31 years, *SD* = 0.34 years; 59% girls; 47% were later assigned to the treatment group).

Tasks

The numerical estimation and recall tasks were equivalent to the tasks described in Experiment 1.

Procedure

Children were randomly assigned to a treatment group, who received corrective feedback on their placement of 7 numbers (around 150) on the number line, or a control group, who completed the same problems but without feedback on their estimates (see Opfer & Siegler, 2007, for a detailed description of the training procedure). During training, children made a hatch mark for the to-be-estimated number, and then the experimenter told the child whether the estimate was near (within 10%) or far (beyond 10%) from the correct location. After the experimenter indicated the correct placement and labeled the number the child mistakenly indicated, the child described why the corrected mark showed the right location for the number. After this training, both groups completed a 22-problem number-line posttest, followed by the numerical recall task described in Experiment 1.

Results and Discussion

Effect of feedback on numerical estimation To assess the effectiveness of the training regime, we conducted a 2 (test phase: pretest, posttest) x 2 (condition: control, treatment) x 2 (grade: Kindergarten, second grade) ANOVA on number line PAE scores. As expected, accuracy increased significantly from pretest to posttest, $F(1, 83) = 41.69$, $p < .0001$, $\eta^2 = .30$, with accuracy also being greater in the treatment than control condition, $F(1, 83) = 7.08$, $p < .01$, $\eta^2 = .08$, and greater for older than younger children, $F(1, 83) = 71.88$, $p < .0001$, $\eta^2 = .46$. Against the idea that pretest to posttest gains occurred through regression to the mean, we also observed a significant test phase x condition interaction, $F(1, 83) = 6.30$, $p < .05$, $\eta^2 = .05$. Post-hoc analysis indicated that these gains from pretest to posttest were larger in the treatment group (M = 8%, SD = 7%) than in the control group (M = 3%, SD = 6%), $F(1, 85) = 16.31$, $p < .0001$, $\eta^2 = .16$. Finally, a test phase x condition x grade interaction, $F(1, 83) = 6.32$, $p < .05$, $\eta^2 = .05$, indicated that feedback reliably induced pretest-to-posttest gains among second graders (treatment: pretest, *M* = 21%, posttest, *M* = 11%; control: pretest, *M* = 22%, posttest, *M* = 20%) but not Kindergartners (treatment: pretest, *M* = 31%, posttest, *M* = 27%; control: pretest, *M* = 34%, posttest, *M* = 30%).

Why might feedback have induced changes in accuracy, and why might it have induced changes only in second graders? Opfer and Siegler (2007) had suggested that feedback on the placement of numbers like 150 on a 0-1,000 number line caused children to make analogies to the placement of more familiar numbers (such as the location of 15 on a 0-100 number line). Previous research (e.g., Siegler & Opfer, 2003; Siegler & Booth, 2004) has indicated that second graders typically place numbers on a 0-100 number line linearly, so such an analogy would be quite useful to them, allowing them to map the structure of the 0-100 to the 0-1,000 number line. In contrast, Kindergartners typically place numbers on a 0-100 number line logarithmically, which would preclude such structure mapping and thus reduce any benefit of receiving feedback on their estimates. If this idea were correct in the present case, then we would see evidence of a logarithmic-to-linear shift in second graders, but not Kindergartners.

To assess whether a logarithmic-to-linear shift occurred in one, both, or neither age group, we next regressed the numbers to be estimated against the estimates provided by children. As expected (given the design of the study), estimates on pretest were fit better by the logarithmic than by the linear functions regardless of age or condition (Kindergartners, control: log R^2 = .74, lin R^2 = .30, treatment: log R^2 = .73, lin R^2 = .36; second graders, control: log R^2 = .97, lin R^2 = .70, treatment: log R^2 = .96, lin R^2 = .72). By post-test, however, feedback sent Kindergartners and second graders in quite different directions. Without feedback, Kindergartners in the control group continued generating logarithmic series of estimates on posttest (Ks, control: log R^2 = .90, lin R^2 = .56); indeed, the fit of the logarithmic regression function *increased* with mere practice. In contrast, feedback dramatically decreased the fit of the logarithmic *and linear* functions (treatment: log R^2 = .37, lin R^2 = .21), which would not be expected if feedback caused Kindergartners to map their 0-1,000 estimates to their 0-100 estimates. In contrast, among second graders, feedback did induce a logarithmic-to-linear shift: on post-test, second graders who did not receive feedback continued to generate logarithmic series of estimates (log R^2 = .96, lin R^2 = .77), whereas second graders who did receive feedback generated estimates better fit by the linear than by the logarithmic functions (log R^2 = .77, lin R^2 = .97).

Transfer of learning to numerical recall We next examined whether the logarithmic-to-linear shift that we induced in numerical estimation would also improve accuracy of numerical recall. Similar to Experiment 1, accuracy of numerical estimation and numerical recall were highly correlated (Figure 3), but we were interested in whether a causal connection existed. To examine this issue, we separated Kindergartners and second graders into two groups—learners (N = 31), those children who learned to produce a linear series of estimates on the number-line posttest, and non-learners (N = 56), those children who continued to produce a logarithmic series of estimates on the number-line posttest. Our hypothesis was that the accuracy of learners' recall would be higher than that of non-learners, and this difference would be especially strong for large numbers. To test this hypothesis, we conducted a 2 (numerical range: below 150, above 150) x 2 (learner status: non-learner, learner) ANOVA on PAE memory scores. As expected, memory was more accurate for small than large numbers, $F(1, 85) = 188.01$, $p < .0001$, $\eta^2 = .67$, and more accurate among learners than non-learners, $F(1, 85) = 12.81$, $p = .001$, $\eta^2 = .13$. Additionally, we observed a significant numerical range x learner status interaction, $F(1,85) = 7.94$, $p < .01$, $\eta^2 = .03$. For numbers below 150, memory accuracy was high regardless of whether children learned to produce a linear series of estimates on the number line estimation task (non-learners PAE = 6%, learners PAE = 5%), $F(1, 85) < 1$, $p > .05$. For numbers greater than 150, however, memory accuracy was much lower among non-learners than learners (PAE = 35% vs. 24% respectively, $F(1, 85) = 14.02$, $p < .0001$, $\eta^2 = .14$). Thus, as in Experiment 1, memory accuracy—particularly memory for large numbers—was associated with acquisition of linear representations.

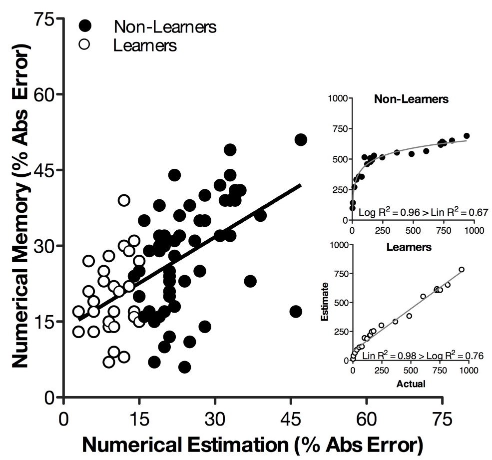

Figure 3: Percent absolute error on the numerical estimation task is strongly correlated with percent absolute error on the numerical memory task for non-learners (black circles) and learners (white circles). The inset figures illustrate a logarithmic-to-linear switch in numerical estimation across non-learners and learners.

Could something other than learning linear representations be responsible for learners having more accurate recall than non-learners? We tested two alternative explanations. The first idea was that age alone improved recall. This idea seemed plausible because learners (*M* = 8.23, *SD* = .59) tended to be older than non-learners (*M* = 7.5, *SD* = 1.07), $t(85) = 3.49$, $p < .001$, $d = .84$, possibly leading them to have better memory. To test this idea, we examined second graders alone because roughly half of the 64 second graders (n = 29) qualified as learners, and their ages were very close (learners, M = 8.35, SD = .35; non-learners, M = 8.28, SD = .33, $t(62)$ = .84, p > .05, *ns*). Here too we found that memory was greater for learners than non-learners (learners, PAE = 19%, SD = 7%; non-learners, PAE = 24%, SD = 9%, $t(62)$ = 2.01, p < .05, d = .62). Another possibility was that feedback alone improved memory, regardless of whether it actually led to learning linear representations. Against this hypothesis, however, we found no main effect of feedback on memory accuracy (treatment, PAE = 18%, SD = 9%; control, PAE = 19%, SD = 12%; F < 1). Thus, actually learning linear representations from the feedback appeared both necessary and sufficient for the average child to improve memory accuracy.

General Discussion

Previous work has indicated that a logarithmic-to-linear shift in children's representations of symbolic quantities profoundly expands children's quantitative thinking. It improves children's ability to estimate the positions of numbers on number lines (Siegler & Opfer, 2003), to estimate the measurements of continuous and discrete quantities (Thompson & Siegler, 2010), to categorize numbers according to size (Opfer & Thompson, 2008), and to estimate and learn the answers to arithmetic problems (Booth & Siegler, 2008). Recent work has also indicated that the logarithmic-to-linear shift is associated with improved memory for numbers (Thompson & Siegler, 2010), but it was unclear whether there was a causal link between the two.

We found evidence that a logarithmic-to-linear shift in estimating the position of numbers on number lines was both correlated with and causally related to improved memory for numbers. In Experiment 1, linearity of numerical estimates increased with age, and the more linear children's magnitude representations were, the more closely their memory of the numbers approximated the numbers presented. These results provided a replication of earlier results, and they also revealed that the association between accuracy of numerical estimates and numerical memory could not be accounted for by age differences alone.

To test the idea that linear magnitude representations were causally related to number memory, in Experiment 2 we trained children on a linear spatial-numeric association on the number line task. Here, we found that children who learned to represent numbers as increasing linearly with numeric magnitude also improved their memory for numbers. This improvement was particularly large for numbers greater than 150, though children were not given feedback on their estimates in this range. Theoretically, this finding is interesting because it is a prediction that comes uniquely from the logarithmic-to-linear shift account.

Beyond demonstrating that linear spatial-numeric associations improve memory for numbers, we believe the present results also help to explain the positive relation between linear numeric magnitude representations and arithmetic proficiency. That is, if learning linear spatial-numeric associations improves memory for numbers in vignettes, it is highly likely it also improves memory for numbers in other contexts, such as memorizing arithmetic facts. Thus, the present results suggest a plausible explanation for the observed association between numerical estimation and mathematics course grades (Opfer & Siegler, in press), an important issue for future research.

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