

# Unifying Deduction, Induction, and Analogy by the AMBR Model

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## Abstract

This paper presents a series of simulations performed with the AMBR model that demonstrate how deduction, induction, and analogy can emerge from the interaction of several simple mechanisms. First, a case of deductive reasoning is demonstrated when a problem is solved based on general knowledge. The system represents the target in different ways depending on the goal, and different solutions are generated. Second, the constructed solutions of the problems are remembered and later on used as a base for remote analogy. Finally, on the basis of the analogy made, a generalized solution of the class of problems is induced. One important characteristic of the model is that representation of the task, problem-solving, and learning are not viewed as separate modules. Instead, they are different aspects of one and the same joined work of the basic mechanisms of the architecture.

**Keywords:** cognitive modeling, analogy, deduction, generalization.

## Introduction

It has been suggested that analogy is the core of human cognition (Hofstadter, 2001; Holyoak, Gentner, & Kokinov, 2001). The reason is that “relational reasoning” can be found in a variety of cognitive processes.

Traditionally, however, models of analogy-making have been isolated from and contrasted to models of deductive reasoning (Gentner, 1983, 1989; Holyoak & Thagard, 1989; Hummel & Holyoak, 1997; Hofstadter, 1995). Somehow gradually analogy-making became a separate and important domain of study (Gentner, Holyoak, Kokinov, 2001).

When the AMBR model was first launched (Kokinov, 1988) it was suggested as a unified model of deduction, induction, and analogy. It has been claimed that deduction, induction, and analogy “are not separate cognitive mechanisms, but rather a slightly different manifestations of the same basic mechanisms” (Kokinov, 1988). A few experiments have been run to demonstrate that deduction, induction, and analogy have common properties – being primed by recent experience (Kokinov, 1990), and transferred knowledge being evaluated on the basis of the structural correspondence between the target and the memorized base (Kokinov, 1992). Even though AMBR was suggested as a unified model of these three “kinds” of reasoning, the line of its further development has gone in different directions like exploring the relations between analogy and memory (Kokinov, 1994a, Kokinov & Petrov,

2001, Grinberg & Kokinov, 2003; Petkov & Kokinov, 2009), analogy and perception (Petkov & Shahbazy, 2007, Petkov & Kokinov, 2009; Kokinov, Vankov & Bliznashki, 2009), analogy and judgement (Petkov & Kokinov, 2006). In this paper we return to the initial idea to model the three traditionally considered separate types of reasoning (since the time of Aristotle) by the same mechanisms.

Meanwhile other models of analogy started to explore the relations between analogy and induction (schema generalization). The SME has been used in the generalization of structurally similar situations (Kuehne, Forbus, Gentner, & Quinn, 2000; Lovett, Lockwood, Dehghani, & Forbus, 2007). The LISA model and its close relative DORA have been specifically addressing the generalization problem and its integration with the analogical mapping (Hummel & Holyoak, 2003; Doumas, Hummel, & Sandhofer, 2008).

There are not that many attempts to integrate analogy and deduction with the exception of PI (Holland, Holyoak, Nisbett, & Thagard, 1986) and of integrating the SME and qualitative reasoning (Forbus, 2001).

The current paper describes an attempt to demonstrate that deduction, induction, and analogy can be produced by a common pool of simple mechanisms (those postulated in the DUAL architecture) and that only the context and the specific interplay between these mechanisms will determine which of these processes will emerge out of the computational process. Thus this is an attempt to model examples of these three processes in a single simulation experiment – without tuning the parameters or changing the mechanisms for each of the cases.

## Brief description of the DUAL architecture

The DUAL architecture was launched (Kokinov, 1994b, 1994c) as a general cognitive architecture that will provide mechanisms for modeling various cognitive processes.

The knowledge in DUAL is represented by a huge number of interconnected micro-agents. Each agent’s symbolic aspect represents a small piece of knowledge while its level of activation represents the relevance of this piece of knowledge to the current context. The activation spreads through the network as in connectionists networks. There are two sources of activation – INPUT and GOAL nodes. Each agent may also have a residual activation that

slowly decreases in time. Thus, the pattern of activation dynamically represents the current context. The agents may perform symbolic operations – creation of new agents; changing weights of links, passing markers – with speed, proportional to their activation level.

Each active *instance-agent* emits a marker through the class hierarchy. The marker spreads up with a speed, proportional to the relevance of the respective concepts. When two markers cross somewhere, a *hypothesis for correspondence* between the two origins is created locally. In other words, the system performs a micro-analogy – it ‘notices’ that there is something in common between two items, and makes a micro-generalization of them.

The hypotheses are created independently by local computations; the hypothesis nodes interconnect themselves with supporting or inhibitory links; and a constraint satisfaction network gradually emerges.

When some agents are mapped to the arguments of a certain relation, the relations involving these nodes try to be transferred, enveloping the respective agents. Initially, the relations are transferred as *anticipation-agents* – a hypothesis that something is present in the environment or that a certain action can be performed. The anticipation may be verified by a simulated perceptual system. The agents on the GOAL list in turn activate further the chains of relations that lead to achieving the goals.

Finally, large structures emerge from the local dynamic interactions. Many pressures (for consistency, for goal completion) work in parallel to resolve the competition between the coalitions of hypotheses. Finally, some hypotheses win the competition (in different moments of time), whereas many losers fizzle out. The most promising hypotheses and anticipations remain in memory for further usage. Thus, the system learns during the problem solving and new generalizations and coalitions of generalizations enrich the system’s memory.

All mechanisms in DUAL work in parallel and influence each other. There is no separation between the processes of retrieval, mapping, formulation of hypotheses and anticipations; achieving the goal, and learning. Instead, they run in parallel and influence each other.

### Domain of the simulations

In order to demonstrate some of DUAL’s important abilities in a coherent set of simulations, we decided to apply the model to a series of negotiation problems which require a trade-off solution (see Gentner, Loewenstein, Thompson, & Forbus, 2009). An example of a trade-off problem is the classical story of the two sisters quarrelling over an orange (see Figure 1 for a simplified representation) which is compared to the conflict between Egypt and Israel.

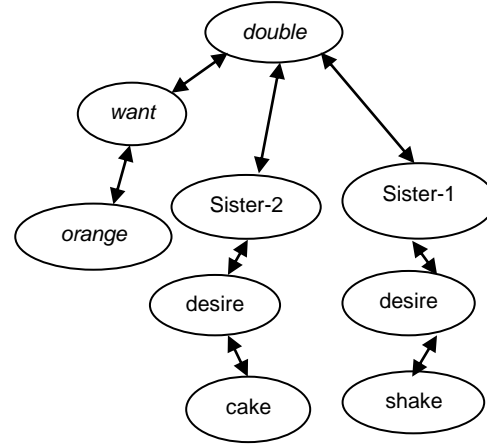


Figure 1. Simplified representation of the problem: two sisters quarrel for an orange and do not recognize that they want different parts of it and can divide it.

### Simulation 1: Transfer of a solution from general knowledge (Deduction)

The first simulation (Simulation 1A) models the mind of the first sister. Her goal is to make a shake, having an orange.

Thus, there are two agents that receive initial activation: a representation of an orange is on the INPUT; a representation of a shake is on the GOAL (Fig. 2).

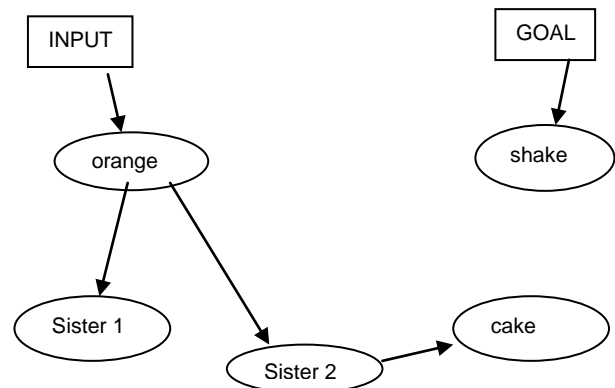


Figure 2. Part of the initial state of the first simulation.

The activation spreads through the class hierarchy – to the concepts of ‘orange’ and ‘shake’; upward to the more abstract concepts; and then back to some of their instances. Relatively easily, the general knowledge of the recipe for shake is activated (Fig. 3). There are other instances of orange and shake in the recipe and the mechanisms for marker-passing, the creation of hypotheses, and transfer do their job to produce the mapping between the given products and the recipe. Soon, the relations that are necessary for completion of the situation are transferred back from the recipe knowledge (Fig. 4). Namely, the sister should take the orange; this implies that she can squeeze out

the juice; this in turn is a necessary condition for making a shake.

Of course, some other instances of orange may be activated, mapped to the target, and some other relations may be transferred. The completion of the causal chain from the initial situation to the goal, however, increases dramatically the activation of the agents from this chain. Furthermore, the pressure of this active chain causes the respective mapping to win the competition. Thus, the system 'solves' the problem. As a consequence, the hypotheses and anticipations, relevant to this 'solution' are transformed into permanent agents and remain in the memory of AMBR for further use. The others fizzle out.

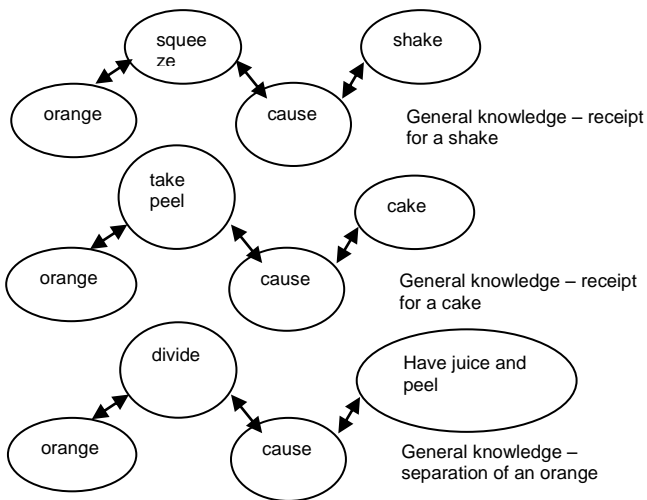


Figure 3. Part of representation of the general knowledge of the model.

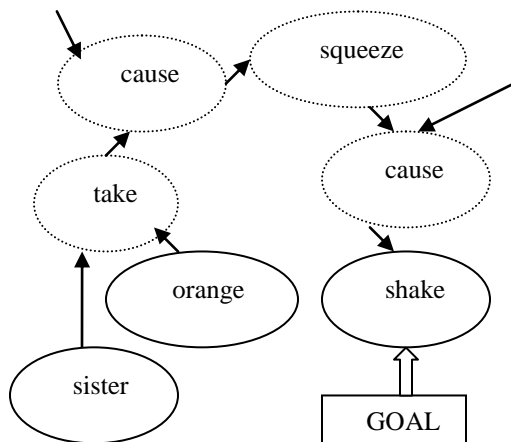


Figure 4. Part of the dynamically created representation of the situation. Some agents (dashed) are transferred from general knowledge.

The simulation has finished at the time of **68.06** AMBR cycles (compare with the longer duration of the other two simulations).

Simulation 1B is performed to simulate the mind of the second sister. It is analogous to the first one, except for the goal. Thus an orange is on the INPUT list, while a cake is on the GOAL list, and the system successfully transferred and learned a respective relational chain from the general knowledge: the sister should take the orange, should peel it, and should use the pieces of peel for making the cake.

The simulations successfully demonstrate the ability of the model to select from an un-separated general knowledge the relevant relations; to transfer them; and to combine them into a coherent solution. This approach differs from the traditional analogy-making models, in which the base situation is separated from the other knowledge.

## Simulation 2: Context sensitivity and the role of the goal (Deduction)

Simulation 2 simulates the mind of a third person – a judge. There is again an orange on the INPUT, but an agent, which represents the relation that both sisters should be satisfied, is attached on the GOAL (Fig.5). The same long-term memory that has been used for the previous simulation is used.

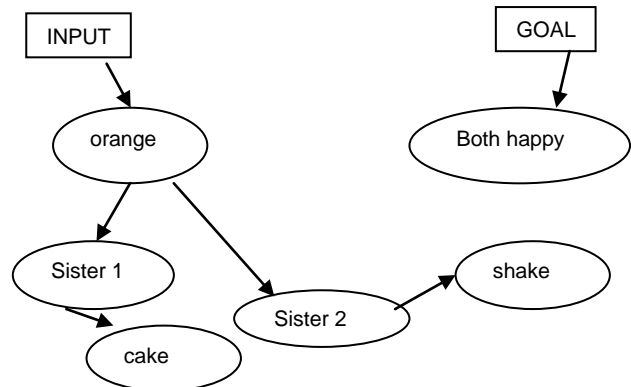


Figure 5. Part of the system's representation of the situation for simulation 2 (only a part of the chain is shown).

The goal, however, is different and this changes dramatically the further representation of the situation by the system. It is easy for the model to activate the recipes for making shake and cake, and to transfer the respective relations. In other words, it can combine the two solutions from the previous simulations. However, this is not enough for achieving the goal, because it is not possible that the two sisters take the orange at the same time. Thus, no chain of relations to the goal is created and the activation continues to spread. Since both the juice and the peel are active, another piece of knowledge 'springs up into the mind' of the model. The juice, the peel, the seeds, etc. are all parts of an orange. Now knowledge of how to separate an orange into its parts becomes active. Another chain of transferred relations reaches the goal, wins the competition, and finally,

a completely different representation of the situation has been made by the system (Fig. 6).

The simulation has finished at time **84.00**; which is longer than the first one.

The second simulation additionally demonstrated the ability of the system to use the basic DUAL mechanisms for solving problems that formally are not problems for analogy-making, but rather deductive tasks. Further, the simulation highlights the importance of the context-sensitivity of DUAL. Depending on the goal of the system, different relations may be transferred into the representation of the situation. The initial and the final representation of the situation may be viewed as two ends of a continuum of dynamic re-representations of the situation until the goal is reached.

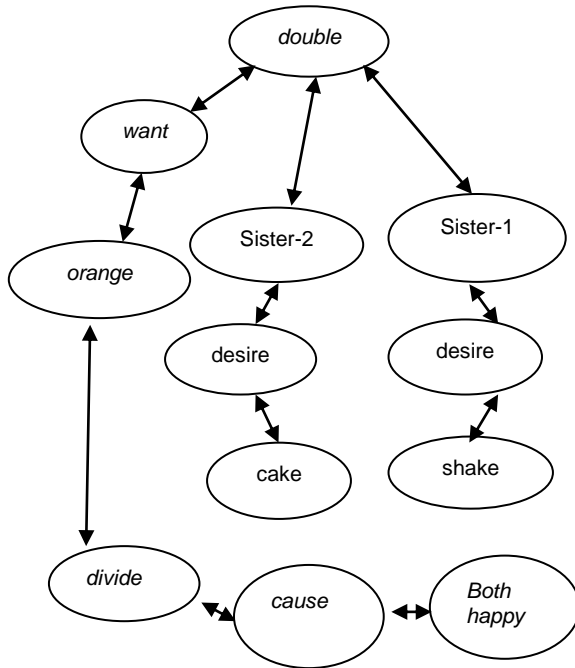


Figure 6. The long-term memory has been enriched with a new base after the simulation 2 (compare with figure 1).

The process of learning is not a separate sub-process. Instead, it is a natural consequence of the problem solving process. The set of winner hypotheses (the solution) is formed dynamically - its elements emerge at different moments of time. The increase of the relevance of these elements is modeled for different reasons (for the sake of the reasoning process). However, a side effect is that the system actually learns the solution for further usage.

The next simulation tests how to use this learned cases.

### Simulation 3: Remote analogy and generalization of the solutions

In the third simulation a representation of the classical Israel-Egypt problem is created and attached to the input of the system. The mind of a 'judge' is simulated. Thus, an instance of the relation 'both are satisfied' is attached to the

GOAL. DUAL-agents for Israel and Egypt are attached to the INPUT (Fig. 7). One instance of orange is also attached to the INPUT, simulating that the judge is by accident in front of a table with oranges on it. This is done to help the system retrieve the story about the two sisters. It is also hard for people to make such remote analogies (Gick & Holyoak, 1980) because it is difficult to activate the respective remote bases. May be a certain non-trivial context is necessary in order remote analogies to be initiated. The mechanisms of DUAL are context-sensitive and thus certain contexts may help them make the appropriate remote analogy.

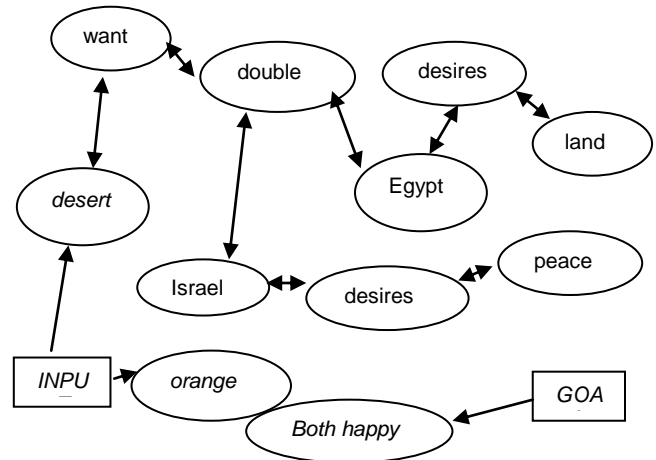


Figure 7. The initial state of the third simulation.

Egypt wants more land and taking the desert will satisfy it. Israel wants peace and taking the desert will ensure it. All this knowledge is encoded in the long-term memory as general knowledge, analogically to the encoding of the sister's recipes. Simulating the point of view of Egypt (putting 'land' on the GOAL), the system would transfer the respective relations from the general knowledge and would conclude that it should take the desert. The same is for the Israeli point of view.

However, there is a constraint that both Israel and Egypt should be satisfied on the GOAL list. The system cannot solve this problem by retrieving and applying general knowledge only. It cannot succeed in the same way as in simulation 2, because there is no such general knowledge in LTM that land and peace are two separate properties of the desert.

Thus, the model makes an analogy between the target and the base learned in the second simulation – how to divide the orange. Note that this analogy-making does not wait until the general knowledge is fully exhausted and deduction has failed (like in PI). Instead, everything runs in parallel.

Of course, initially the contextual orange is mapped to the sister's orange, and the goal agent – 'both satisfied' to the base's goal. However, soon the pressure for consistency ensures the right mapping: Israel and Egypt correspond to the sisters; and the desert to the orange.

The chain of relations to the goal is closed when the proposal to use separately the two properties of the desert (it can be used to live on; it can be used as a buffer zone for ensuring peace, if it is demilitarized) is generated. We have not yet simulated how the transferred separation of the desert properties may be used for solving the task. i.e. how can the land be used for living by Egypt and at the same time be demilitarized for ensuring peace for Israel. This is part of our further work but it was already demonstrated that DUAL is able to combine relations from general knowledge in order to complete a representation of a certain situation.

Generalization of the solution is simulated. Every winner hypothesis has a justification (which was the reason for its creation). Actually, this justification is the common superclass for the two mapped elements. Thus, every winner hypothesis is a super-class of the base and target elements and in turn a subclass of the common superclass found. Thus, it is a generalization of the two mapped elements. At the same time, the links among hypotheses, created from the structural correspondence mechanisms, keep all these winning hypotheses together – as a coalition that represents the whole generalized solution. Part of it is shown on Fig. 8.

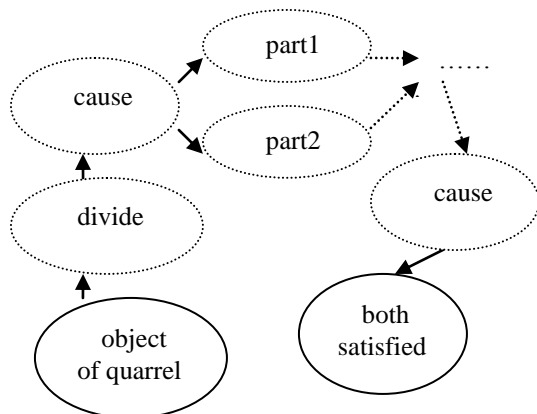


Figure 8. Part of the generalized solution obtained by the system during the third simulation (not all elements are shown).

The simulation finished at time **165.18**; much later than the previous two simulations.

The third simulation demonstrates two abilities of AMBR. First, it is able to use simultaneously general knowledge and remote analogous situations for problem-solving. Second, it is demonstrated that a generalization of the solution may emerge from the process of problem-solving without any specialized learning mechanisms.

## Conclusion

The idea to use analogy-making for solving complex problems is not new to the research in the field of cognitive science and artificial intelligence. Computational models of analogy-making have been applied successfully in solving

mathematical problems (Anderson & Thompson, 1989), negotiation problems (Gentner, Loewenstein, Thompson & Forbus, 2009), everyday physics problems (Klenk & Forbus, 2007), designs problems (Davis, Goel & Nersessian, 2009). The goal of the current paper was to show that the mechanisms underlying analogy-making are universal enough to be able to solve any kind of problems, including ones which are traditionally thought to be out of the scope of analogy-making. To this end, we attempted to show that the basic mechanisms of the DUAL architecture can be used to model a variety of reasoning tasks.

A series of simulations has been run in the domain of trade-off problems with the AMBR model without any changing and tuning in between. The model demonstrated its ability to use general knowledge in a deductive way in order to solve a specific task; to remember the solution, and then retrieve it and use it as a remote analogy to solve another problem, and finally construct a generalized solution to a class of trade-off problems. The simulations are run sequentially and continuously so that the results of the previous reasoning become available for subsequent problem solving by memorizing and learning. The model is yet to be further extended and specific predictions will be generated by further simulations, these predictions will then be tested against psychological data. At this point, the simulations are a proof of concept. They demonstrate that AMBR can model deductive, inductive, and analogical reasoning via the same simple mechanisms and that depending on the task and context each of these cognitive processes can emerge.

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