The Effects of Peer Communication with Diagrams on Students' Math Word Problem Solving Processes and Outcomes

Yuri Uesaka (y_uesaka@p.u-tokyo.ac.jp)
Graduate School of Education, The University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan.

Emmanuel Manalo (emmanuel.manalo@aoni.waseda.jp)
Center for English Language Education in Science and Engineering
Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan

Abstract

This study investigated how providing students with opportunities to use diagrams in interactive communication with peers might affect their diagram use and problem solving processes. The participants were 42 junior high school students who were assigned to a condition with peer instruction opportunities (experimental) or without (control). The peer instruction opportunities were designed to facilitate students' diagram use in communication. The results revealed that, in post-instruction assessments, the experimental participants spontaneously used more diagrams and were more successful in problem solving. No differences were found in the timing with which participants started using diagrams. However, the experimental participants used more appropriate types of diagrams that also incorporated more relevant information. The findings therefore indicate that opportunities for peer communication with diagrams facilitate not only enhanced spontaneity in diagram use but also the construction of more appropriate, detailed diagrams, and these in turn likely contribute to better problem solving performance outcomes.

Keywords: Diagram use; communication development; peer instruction; math problem solving.

Introduction

Student Problems in Using Diagrams

Diagrams are effective tools for problem solving. Following Larkin and Simon's (1978) demonstration that reasoning with diagrams is computationally more efficient than reasoning with sentences, the effectiveness of diagrams in facilitating understanding and problem solving has been demonstrated in a wide variety of tasks. For example, Bucher (2006) showed that using diagrams promote the construction of rich mental models when students learn about the human circulation system. Hembree's (1992) meta-analysis of studies relating to math word problem solving revealed that constructing diagrams is the most effective heuristic among those proposed by Polya (1945).

The main concern in most previous diagrams research has been to show the effectiveness of using diagrams in many fields and the mechanisms that underlie that effectiveness (see, e.g., Ainsworth & Th Loizou, 2003; Cheng, 2002; Stern, Aprea, & Ebner, 2003). However, it is equally important to address in research the problems that students manifest in using diagrams in authentic learning situations. This is because some studies in the context of educational practice have pointed out that students are not as effective in using diagrams as teachers and researchers might expect. The problems they manifest include lack of spontaneous use, inappropriate choice of diagrams to use, and failure to make appropriate inferences when using diagrams (see, e.g., Cox, 1996; Uesaka & Manalo, 2006; Uesaka, Manalo, & Ichikawa, 2007). The most serious of these problems is the lack of spontaneity: students do not use diagrams spontaneously during problem solving even when their teachers have used plenty of diagrams in teaching them (Uesaka et al., 2007). If they fail to use diagrams off their own volition, they fail to benefit from the research-proven efficacies that diagram use brings to problem solving.

Although the problems in student diagram use have not been adequately addressed in the research literature, several studies have focused specifically on such problems. For example, Uesaka et al. (2007) suggested that a crucial reason behind the low use of diagrams is many students' perception that diagrams are tools for teachers' instruction rather than tools for their own problem solving use. In addition, Uesaka, Manalo, and Ichikawa (2010), through the conduct of experimental classes, developed a teaching method for enhancing the spontaneous use of diagrams: their findings suggest that enhancing both students' perceptions about the efficacy of diagram use and developing their skills in constructing diagrams are critical factors in resolving the lack of spontaneity problem.

However, the number of the previous studies that have examined influencing factors and teaching methods relating to student diagram use is limited. Notably, the effects of proposed teaching methods on other variables such as problem solving processes and outcomes have not been sufficiently examined. Investigating these aspects are important as findings could potentially be used directly in real educational settings to promote more successful student learning outcomes.

Diagrams as Communication Tools

Although previous diagrams research in psychology has tended to focus on their efficacy as tools for problem solving, diagrams are also effective tools for communication. This quality has been demonstrated in studies with patients who have problems in communication. For example, Lyon (1995) reported that diagrams facilitated communication with adults who, because of aphasia, found

communication through verbal means difficult (see also review by Sacchett, 2002). Also, in the context of education, Dufour-Janvier, Bednarz, and Belanger (1987) reported that math teachers use a lot of diagrams in explaining how to solve problems in class, and this practice contributes to the promotion of students' understanding of the processes involved. Thus, diagrams appear to work equally well as tools of explanation and as tools for problem solving.

Although these two aspects of diagrams – tools for problem solving and tools for communication – have been examined independently in different research areas, they are in fact related. Uesaka and Manalo (2007) reported that students' spontaneous use of diagrams in test situations was higher when they had previously been provided with opportunities to teach and communicate with other students in small group, peer instruction sessions. This finding suggests that, as a consequence of the peer teaching experience – which provided opportunities for the use of diagrams as communication tools – students internalized diagrams as tools for problem solving. Based on their analysis of protocol data taken during the instruction sessions, Uesaka and Manalo explained that, for the students who took part in peer instruction, appreciation of the efficacy of diagram use was enhanced as diagram construction and use directly assisted in their efforts to demonstrate how to solve the assigned problems to their peers – when, often, words on their own proved inadequate. Moreover, the practice they obtained in drawing diagrams during the peer instruction sessions likely contributed to enhancing their diagram construction skills. These explanations are congruent with the findings of Uesaka et al. (2010), in which interventions that promoted appreciation of the value of diagram use and provided practice for the development of diagram drawing skills were found to be effective in enhancing students' spontaneity in diagram use.

The strategy of providing students peer instruction and communication experiences to promote their spontaneous use of diagrams in problem solving is potentially very useful in educational settings. However, a few questions remain, the most important of which is whether the increased spontaneity in diagram use results in improved performance in problem solving (i.e., whether, as a result, students not only use more diagrams but actually solve more problems correctly). If such an outcome can be empirically demonstrated, it would enhance the potential educational value of this strategy. A second important question is how peer instruction might affect students' subsequent problem solving processes (i.e., how, as a result, they might approach problem solving differently). This question also remains unanswered, and addressing it would help toward better understanding the mechanisms that underlie this strategy.

Overview and Hypotheses of This Study

In line with the preceding discussion, the purpose of the present study was to examine the effects of providing peer instruction experiences on students' problem solving processes and performance outcomes. The study utilized experimental classes provided to 8th-grade students who were assigned to experimental or control conditions. Math problem solving instructions provided to the conditions were equivalent, except that those in the experimental condition were provided opportunities for interactive, small group peer instruction. To facilitate this, Aronson's (1978) "jigsaw method" was used so that students initially worked on solving an assigned problem on their own, and then moved on to explaining how they solved their problem to students who were initially assigned a different problem. As in the Uesaka and Manalo (2007) study, the assumption was that the peer instruction sessions would place experimental students in situations where they have to interactively communicate with the use of diagrams. In contrast, those in the control conditions were required only to explain how they solved their problem to a group, in a one-way, non-interactive communicative process.

Following the instruction sessions, whether the students in the experimental group actually spontaneously used more diagrams in subsequent problem solving was first assessed. After this, three hypotheses were tested. The first was that the students in the experimental condition would evidence better problem solving performance compared to those in the control condition.

The second hypothesis concerned the timing with which students would use appropriate diagrams in their problem solving efforts: here the prediction was that those in the experimental condition would start using appropriate types of diagrams sooner compared to those in the control condition. This hypothesis was derived from an assumption that, as a consequence of experiencing the effectiveness of diagram use during the peer instruction sessions, those students would subsequently use diagrams in problem solving in a more active and timely manner.

The third hypothesis was that students from the experimental condition would include more relevant information in the diagrams they construct in their subsequent problem solving efforts. This hypothesis was derived from an assumption that through their experiences in peer communication, experimental participants would develop a better appreciation of the usefulness of including more relevant details from the problem description into the diagrams they construct – to make those diagrams easier to understand and effective in clarifying the problem structure and associated method of solution.

Method

The experimental study that was carried out comprised of several investigations about the effects of interactive, peer communication on students' diagram use and problem solving performance. However, due to space constraints, only the investigations focusing on the impact of the interventions used on participants' problem solving performance, and the processes they used in problem solving (to understand the source of any observed differences), will be reported here.

Participants

The participants who voluntarily took part in this study were 8th-grade students (13–14 years of age). They were assigned to one of the two conditions (experimental and control) by using a randomized block design in which the students' school achievement scores were controlled. Only data from participants who attended all sessions were included in the analyses reported here: a total of 42 students (experimental, $n = 21$; control, $n = 21$). A basic skills assessment confirmed the equivalence of experimental and control participants' skills in table and graph construction at pre-instruction.

Materials

Math Word Problems Used in Instruction Sessions The study was carried out over six days, three of which – the second to the fourth days – were devoted to instruction. In each of the instruction days, two math word problems, with similar story contexts and requiring similar types of diagrams for their correct solution, were used. 'Arrangement problems', for which using a table and drawings to represent the situations described in the problems was deemed helpful, were used on the second day. 'Mobile phone problems', for which the construction of graphs was deemed effective, were used on the third day. 'Area problems', for which the use of tables was considered helpful, were used on the fourth day. As an example, one of the mobile phone problems is shown in Appendix 1.

Math Word Problem Solving Assessments Math word problem solving assessments were administered on the fifth and sixth days to examine the quantity and quality of the diagrams spontaneously produced by the participants. There were two types of math word problems (the water and pentagon problems) administered on the sixth day and used in the analysis carried out in this paper.

The water problem would have been facilitated by the use of graphs, and the pentagon problem would have been facilitated by the use of tables. Four university colleagues, including a qualified math teacher, independently considered the most effective kinds of diagrams to use in attempting to solve these problems, and all concurred on the kinds of diagrams noted above. Examples of the problems used are shown in Appendix 2. Examples of diagrams constructed by participants for two of the problems given are shown in Fig. 1.
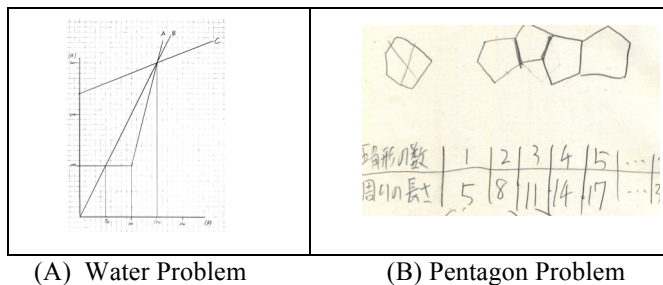

| (A) Water Problem | (B) Pentagon Problem |

Fig. 1: Example of diagrams participants produced during the math word problem solving assessment.

Procedure

Data collection and instruction sessions were conducted over 6 days at the University of Tokyo. Pre-instruction assessment was carried out on the first day, and post-instruction assessment on the final two days. As noted earlier, the instruction sessions were provided on the second to fourth days, with each session lasting approximately 50 minutes. Participants assigned to the same condition took the classes together. Instructions for both experimental and control conditions were provided by the same teacher.

During the instruction sessions, the teacher presented two math word problems each day and employed a consistent teaching procedure across both groups. Firstly, the teacher encouraged the students to carefully read and think about the problems given so that they would understand the nature of those problems. During this time, the teacher asked the students in the small group (of usually 4) to split into two smaller groups and assigned one of the two problems to each. Secondly, the teacher asked the students to solve on their own the problem they had been assigned. However, prior to letting the students attempt solving the problem, the teacher explicitly encouraged them to use diagrams – pointing out their usefulness for solving problems. The teacher also provided as much help as the students needed. For example, the teacher encouraged students who wanted to receive hints to gather in front of the board where the teacher then provided hints, as well as demonstrated steps that would lead to the correct solution of the problem and the use of appropriate diagrams.

During the instruction sessions, participants in the experimental condition were also later asked to explain to other students in their group how to solve the problem they were assigned. In doing so, they were encouraged to use diagrams. The experimental condition differed from the control condition on this particular point: the experimental participants were provided opportunities to communicate in more interactive circumstances with communication flow going both ways between explainer and the peer(s) he or she was explaining to. As noted by Uesaka and Manalo (2007), under such circumstances, diagram use becomes almost indispensable as students find it difficult to use only spoken words to explain the nature of the problem they are dealing with and the way to approach its solution.

In the control condition, some of the participants were asked to present their ideas about how to solve the problem they were assigned in front of the class. This kind of presentation of one's ideas about how a problem might be solved is quite common in typical Japanese classrooms. Thus, although they also explained, the control participants did so to a bigger group in less interactive circumstances, with communication flow going almost exclusively from explainer to listeners.

On the final two days, post-tests were administered and students' spontaneous use of diagrams and problem solving performance were evaluated. Only on the last day were the processes that participants used in attempting to solve the

math word problems video recorded. The data analyzed in this paper comes from this session during the sixth day.

Results

Spontaneous Use of Diagrams in Solving Problems

Spontaneity of Diagram Use Before conducting the statistical analyses, participants' responses to the two math word problems administered on the sixth day were firstly coded according to whether they constructed a diagram. A diagram was defined as any representation of the problem other than words (on their own), sentences, or numerical formulas. Tables were also counted as diagrams for the purposes of this study and defined as a depiction of at least a pair of values arrayed to represent two related variables. If a participant constructed at least one diagram when solving a problem, the participant's response to that problem was coded as "used diagrams (1)". Otherwise, the response was coded as "no diagram (0)".

A t-test was used to compare the average numbers of problems in which participants "used diagrams" in the two conditions. The result indicates that participants in the experimental condition spontaneously used diagrams in more problems than the participants in the control group ($t_{(40)} = 3.32$, $p < .01$; experimental group $M = 1.67$, $SD = 0.48$; control group $M = 1.10$, $SD = .62$).

Appropriateness of Diagrams Used Secondly, diagram types used by the participants were coded to confirm whether participants in the experimental condition constructed more of the types of diagrams deemed appropriate. As noted in the Method section, four university teachers determined the most appropriate types of diagrams to use in solving the problems given, these being a Cartesian graph for the water problem and a table for the pentagon problem. Thus, the appropriateness of the types of diagrams used by the participants was coded according to these views. If a participant constructed an appropriate type of diagram when solving a problem, the participant's response to that problem was coded as "used an appropriate diagram (1)". Otherwise, the response was coded as "did not use an appropriate diagram (0)".

Again, a t-test was used to compare the average numbers of problems in which participants "used an appropriate diagram" in the two conditions. The result indicates that participants in the experimental condition constructed more appropriate diagrams than the participants in the control condition ($t_{(40)} = 2.89$, $p < .01$; experimental group $M = .90$, $SD = 0.77$; control group $M = .33$, $SD = .48$).

Performance Outcomes in Problem Solving

To examine the first hypothesis, the students' performance outcomes in solving the two problems were analyzed. Before conducting the analysis, participants' responses to the problems were coded "correct (1)" or "incorrect (0)" according to a criterion set for each problem. For the water problem, a response was coded correct if participants' clearly indicated their understanding that Country B, then A,

and finally C were cheapest, respectively, according to increasing amounts of water consumed – even if they did not provide the exact numerical quantities of water (50 and 150 liters) that distinguished between the countries. For the pentagon problem, a response was coded correct if participants provided the correct circumferences for at least up to 10 sheets.

A t-test was used to compare the number of problems in which the participants in the two conditions obtained correct answers. The dependent variable was the number of correct answers and the experimental conditions comprised the independent variable. The result indicates that participants in the experimental condition obtained correct answers in more problems than participants in the control condition: $t_{(40)} = 2.49$, $p < .5$; experimental group $M = 1.19$, $SD = 0.67$; control group $M = .62$, $SD = .81$. This finding clearly suggests that when students are provided experiences that predispose them to use diagrams as communication tools (e.g., the peer teaching situation used in this study), both their subsequent spontaneity in diagram use and their actual problem solving performance are enhanced. The first hypothesis is therefore supported.

Processes of Solving Math Word Problems

Timing to Start Using Diagrams in Problem Solving To examine the second hypothesis, that the timing in using appropriate diagrams would differ between the two conditions, the post-instruction videotape recordings were analyzed from the viewpoint of when the participants started using appropriate diagrams and when they finished (or gave up) solving problems. (The video recording of three participants somehow failed, so their performances were excluded from the analysis.)

T-tests were used to compare how quickly participants in each condition started using appropriate diagrams. The dependent variable in this analysis was the product of the participants' starting times to use appropriate diagrams divided by the total amounts of time they spent engaged in solving each of the problems (i.e., how quickly they started using an appropriate diagram as a proportion of the total amount of time they spent in attempting to solve the problem), and the experimental conditions comprised the independent variable. The results indicate that the effect of the difference of the conditions was not significant in both problems (water problem: $t_{(21)} = .14$, n.s.; pentagon problem $t_{(28)} = 1.70$, n.s.). Participants who drew an appropriate diagram started to use it at an early stage in both conditions (water problem: experimental group $M = .17$, $SD = .15$; control group $M = .15$, $SD = .27$; pentagon problem: experimental group $M = .17$, $SD = .30$; control group $M = .05$, $SD = .06$). These results therefore lend no support to the second hypothesis.

Information Included in Participants' Diagrams To examine the third hypothesis, the information that participants put in their diagrams was analyzed. Before conducting the analysis, several details that were deemed important for each problem were determined. These details

were either directly obtainable from the text of the problem or they could easily be inferred from the information provided in that text. For the water problem, five 'relevant' points were set (e.g., whether 2400 yen which was provided in the text of the problem as the basic fee of country C was included), and for the pentagon problem three 'relevant' points were set (e.g., whether 8 cm as the circumference of two connected pentagons was included). The participants' diagrams were scored according to the number of 'relevant' information they included.

T-tests were used to compare the number of information included in participants' diagrams in the two conditions. Although the result in the pentagon problem was only marginally significant, the results in both problems indicate that the amounts of relevant information (means shown in Table 1, with SD in brackets) that experimental participants included their diagrams were greater compared to those in the control condition (water problem: $t_{(31)} = 2.58$, $p < .05$; pentagon problem: $t_{(35)} = 1.71$, $p < .10$).

Table 1: Mean amounts of detail included in diagrams

Problem \ Condition	Experimental	Control
Water Problem	2.94 (1.47)	1.67 (1.35)
Pentagon Problem	1.47 (1.12)	.83 (1.15)

Discussion

The findings of this study indicate that the use of peer instruction as a way of facilitating students' use of diagrams as communication tools is effective in enhancing not only the spontaneity with which they use diagrams in problem solving, but also their actual performance outcomes in problem solving. In other words, as a consequence of the peer instruction experience, students appear not only to use diagrams more spontaneously, but also to be able to more successfully solve math word problems. The findings also suggest that the better problem solving performance stems not from using diagrams in a more timely manner, but from using more appropriate types of diagrams that contain more relevant details. The relevance of these findings to diagrams research and to education will be considered here.

The finding that participants in the experimental condition evidenced greater spontaneity in diagram use at post-instruction confirms Uesaka and Manalo's (2007) report of similar results. Essentially, this finding assumes that the peer instruction sessions would make it more likely that participants would need to use diagrams in communicating with other students. In fact, this increased likelihood – although not verified in the present study – was confirmed through a protocol analysis carried out by Uesaka and Manalo (2007). The experience of using diagrams in communicative situations then provided participants not only with more practice in constructing diagrams, but also direct experience of the efficacies that diagram use brings to problem solving. As Uesaka and Manalo (2010) reported, both practice in diagram construction and appreciation of the benefits of diagram use are crucial components in promoting students' spontaneity in diagram use.

The present study, however, additionally demonstrated that the students in the experimental condition were subsequently more successful also in solving the problems. The connection that this finding establishes between facilitation of greater spontaneity in diagram use and improved problem solving performance outcomes had not been established in previous research. It provides strong support for the argument that promoting spontaneous diagram use in students is beneficial (see, e.g., Uesaka et al., 2007), and it points to peer instruction as an effective strategy for promoting such use. Trying to understand the effect of this strategy on students' problem solving processes would therefore be helpful, particularly for educators and researchers.

As previously noted, and contrary to one of the hypotheses posited in this study, the experimental participants did not employ diagrams in their problem solving any sooner than the control participants. In fact, when participants from both conditions employed any diagrams at all, they did so very early in the problem solving period. This suggests that the common instruction and encouragement provided by the teacher to participants in both conditions was adequate to instill in the participants the understanding that, when they do use a diagram for problem solving, they should do so from the beginning so that the diagram would be helpful not only in finding a solution to the problem but also in understanding the structure of the problem in the first instance.

Where the experimental and control participants differed was in their use of appropriate types of diagrams and the amount of relevant information they included in the diagrams. These two points of difference are probably sufficient in explaining the difference in performance outcomes between these two groups. Using inappropriate types of diagrams would less likely lead to the correct solution to a problem; likewise, lack of adequate relevant details would likely render a diagram less useful toward working out the correct solution to a problem.

So, how did the communicative experiences that experimental participants gained through peer instruction predispose them to use more appropriate types of diagrams and incorporate more relevant details in the diagrams they constructed? A possible answer is that, through explaining how to solve the assigned problems and answering other students' questions about how to solve those problems, the experimental participants developed a better understanding of the qualities of different types of diagrams that determine their suitability for different kinds of problems. (Note that the structures of problems used in the post-instruction assessments were the same as those used in the instruction sessions.) In the process of using diagrams to explain and communicate their ideas to the other students, the experimental participants would also likely have gained a better appreciation of the value of incorporating sufficient relevant details in diagrams – not only to make them easier to understand, but also to more usefully represent the relationships between different components of the

problem's structure. This process of change can also be understood from the Vygotskian perspective that interaction and communication with other people promotes the internalization and development of new skills and knowledge.

Perhaps the most valuable contribution of the present study is that it puts forward a method of instruction for promoting students' spontaneous diagram use that has genuine viability for application in most classroom settings. The peer instruction strategy utilized in this study appears to have considerable potential in the development of students' skills in using diagrams for communication and problem solving purposes.

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Appendix 1: Example of a Math Problem Used in the Instruction Sessions

In a mobile phone shop, two different types of plans are sold. When a customer asks your advice, which plan would you recommend as being cheaper according to amount of phone use?

Plan A: There is no basic fee, and no free calling time. The cost of calls is 30 yen per minute.

Plan B: The basic fee is 1500 yen including 100 minutes of free calling time, with 80 yen per minute charged thereafter.

Appendix 2: Examples of Math Problems Administered at Post-Instruction Assessment

Water Problem

The head of a company asked Taro to find out which of three countries – A, B, or C – would be best for establishing a factory which uses water. The different charging methods of each country are described below. Please imagine you are Taro, and come up with an explanation for the head of the company.

Country A: 1000 yen is charged as a basic fee, but you can use water without additional charge up to 100 liters. After 100 liters, 40 yen/liter is charged.

Country B: There is no basic fee. Water cost is 20 yen/liter.

Country C: In addition to 2400 yen as a basic fee, there is a charge of 4 yen/liter of water used.

Pentagon Problem

There are many sheets of paper in the shape of a regular pentagon, with each side being 1 cm. These sheets are arranged one by one with the rule that a new sheet shares only one side with already arranged sheets. Find the circumference when arranging 1, 5, 10 and 20 sheets.