Less-is-more Effects in Knowledge-based Heuristic Inference

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Abstract

Inference on the basis of recognition alone is assumed to occur prior to accessing further information (Pachur & Hertwig, 2006). A counterintuitive result of this is the "less-is-more" effect: a drop in the accuracy with which choices are made as to which of two or more items scores highest on a given criterion as more items are learned (Frosch, Beaman & McCloy, 2007; Goldstein & Gigerenzer, 2002). In this paper, we show that less-is-more effects are not unique to recognition-based inference but can also be observed with a knowledge-based strategy provided two assumptions, limited information and differential access, are met. The LINDA model which embodies these assumptions is presented. Analysis of the less-is-more effects predicted by LINDA and by recognition-driven inference shows that these occur for similar reasons and casts doubt upon the "special" nature of recognition-based inference. Suggestions are made for empirical tests to compare knowledge-based and recognition-based less-is-more effects.

Keywords: Heuristics; Recognition; Less-is-more; LINDA

The Less-is-more Effect

Suppose an individual is presented with the two cities *Milan* and *Modena* and asked to choose between the two along some criterion, for example to decide which has the larger population. In the classic work of Goldstein and Gigerenzer (2002), it is assumed that the participant will guess if they recognize neither of the items, they will use whatever additional knowledge is available to make a decision if they recognize both of the items and if they recognize only one of the items, they will choose this item as the larger without consulting any other cues or searching for further information about it (the *Recognition Heuristic* or *RH*). Recognition-driven inference of this type predicts a *less-is-more effect*, whereby individuals who recognize many of the items often perform worse than individuals who recognize fewer of the items (Goldstein & Gigerenzer, 2002). A number of studies have shown that this effect can be observed empirically (Frosch, Beaman & McCloy, 2007; Goldstein & Gigerenzer, 2002; Reimer & Katsikopoulos, 2004). It occurs because items that are more prominent (e.g., larger, more populous cities) are more likely to be encountered, hence more likely to be recognized. Recognizing one of the two items is thus a useful cue for choosing the recognized item; whereas if both items are recognized, additional knowledge is needed to make the decision and such additional knowledge may be very limited in discriminative power. In the terms provided by Goldstein and Gigerenzer (2002) a less-is-more effect, superior performance by an individual who recognizes fewer of the options, is expected when the recognition validity (the probability that a correct decision is made based upon recognition alone) exceeds the knowledge validity (the probability that a correct decision is made based upon the best available knowledge about the items).

The assumption underlying the RH is that items scoring higher on the criterion under consideration (larger cities, more successful ice-hockey teams, better tennis players etc). are ordinarily encountered more frequently. The counter-intuitive nature of the less-is-more effect makes its prediction by recognition-driven inference interesting, and has been used as a rhetorical device to promote the heuristic (Borges, Goldstein, Ortmann & Gigerenzer, 1999; Gigerenzer, 2007). Counter to this, failures to observe the effect have been cited in attempts to refute the RH (e.g., Boyd, 2001; Dougherty, Franco-Watkins & Thomas, 2008; Pohl, 2006). In describing the RH, Goldstein and Gigerenzer (2002) use the example of recognizing a city because it has appeared frequently in newspaper reports, a larger city is more likely to be so mentioned. Any individual who is presented with a city they recognize (but know nothing more about) and one they do not is therefore well-advised to choose the recognized city if judging which of the two is more populous. However, the recognizability of a particular city, for example, is a function of several factors, including its physical distance from the individual as well as its size. An appropriate analogy here might be the force of gravity. Local towns, like nearby planetary bodies, might have intrinsically less "pull" or prominence than distant cities (or distant galaxies) but their appearance in local news reports is enhanced by their closer physical proximity and both of these factors influence recognizability. A further moderating factor is the way in which the individual might shape the environment their own ends. In the newspaper example, the individual receiving the newspaper is implicitly assumed to be a fairly passive processor of the information contained within the newspaper and no consideration is given to the potential difference between an individual who actively seeks out a newspaper and one who does not or to potential differences between choice of reading matter (e.g., the *New York Review of Books* versus the *National Enquirer*) which may have very different content, and each of which might be sought out, or passively encountered, to different degrees by different individuals or groups of individuals.

A basic premise in what follows is that, for any given individual, there are several subgroups of items which the individual is able to recognize and about which they may also have partial knowledge. This is particularly likely if they are local to the individual in some way or if they form part of a set of items of special interest to that individual. For example, American cities include large, famous cities such as *New York* and *New Orleans*, and small cities associated with famous universities, such as *New Haven* and *Palo Alto*. The relative recognition of various subgroups (such as those with famous Universities) may not be simply correlated with size. Any individual with specialist knowledge or affiliation with any special-interest group, e.g., membership of a European academic community, might be more likely to recognize small but academic cities in the USA than all but the most famous large USA cities. For this fictional individual[1], there is a weaker relationship between recognition and magnitude for the subset of US cities with famous Universities than for the subset of US cities that do not possess famous Universities.

This assumption that differential access to various sub-groups of items may occur between individuals is not reliant upon anecdotal evidence or arguments of plausibility as above. By-item analysis of data taken from an experiment by McCloy, Beaman, Frosch and Goddard (in press) shows, when a group of 40 participants were asked to indicate which of a group of famous individuals they recognize, a significant interaction between the reason for the individual's celebrity and the participant's gender, $F(3, 43) = 13.44$, $p < .001$. For example, males recognized, on average, sports personalities 78% of the time (females = 55%) and rock stars 75% of the time (females = 66%). In contrast, females recognized fashion and show-business professionals 57% of the time (males = 33%). In what follows, we consider similar situations where, for an individual within the environment, there is no simple correlation between recognition and magnitude because subsets of the items are prominent for reasons unconnected to magnitude (e.g., the age, gender or special interests of the individual). The question we wish to address is whether less-is-more effects still occur in such situations and what forms of decision-rule, if any, will give rise to such effects.

LINDA

To formally examine the appearance of less-is-more effects, we suppose a pool of N items, split into several subsets A, B, C, ... Within each subset the participant is able to recognize a, b, c, ... items, respectively. In a typical test of recognition-driven inference, the experimenter selects items quasi-randomly from the pool. Since the constraints on the experimenter are unknown, a random selection from N is assumed and the basic case considered is where pairs of items are chosen, and the participant's task is to say which is larger. For purposes of exposition, attention is also restricted to situations in which there are just three subsets. The models can easily be extended to other cases (e.g., the participant is asked to choose between more than two items (Frosch et al., 2007; McCloy, Beaman & Smith, 2008) and/or the pool is split into more than three subsets).

On a given trial, suppose the participant recognizes i items from subset A, j items from subset B, and k items from subset C. Only two items are presented, so $0 \leq i + j + k \leq 2$. p_{ijk} is the probability of recognizing 0-2 items from A-C. p_{ijk} is dependent on how many items the participant can recognize in each of the subsets, but is independent of the decision rule adopted. The probability of success is α_{ijk}, given the recognition of i, j and k items from their respective subsets. α_{ijk} is dependent upon the decision rule adopted and distinguishes between models. The overall probability of success for any model is given by:

$$\Sigma_{ijk}\, p_{ijk}\, \alpha_{ijk} \qquad (1)$$

The distinguishing feature of the RH model is that the participant chooses the recognized item when only one item is recognized. So $\alpha_{000} = 0.5$ (no item recognized, pure guess); α_{100}, α_{010}, and α_{001} reflect the success of the recognition heuristic; α_{110}, α_{101}, α_{011}, α_{200}, α_{020}, α_{002} reflect use of knowledge. The alternative against which the RH is to be compared we refer to as LINDA (Limited INformation and Differential Access). As the name implies, this model requires two basic assumptions:

1. *The limited information assumption*. For each recognized item, the individual has reliable but limited information about its size (e.g. that the size is above the population median).
2. *The differential availability assumption*. Some subsets are more accessible than others so that subset A contains items that are more readily recognizable than subset B and so forth. The extent to which items in A are larger than items in B implements the recognition-criterion correlation which is the basis of the RH.

The limited information assumption is that some information is available at the time of decision-making against which to evaluate the usefulness of choosing the recognized item in any given case. This is strictly limited: above or below *median knowledge* corresponds in

[1] Who bears a strong resemblance to the second author.

information theoretic terms (Shannon & Weaver, 1948) to only 1 bit of information. The reliability of this information may also vary. The differential availability assumption states merely that, within any subset, a given individual may recognize more or less items. Thus, a member of the UK academic community may recognize more US cities with famous Universities than a UK-based baseball fan. The baseball fan, by contrast, may recognize more US cities with famous baseball teams.

Existence-Proofs for Knowledge-Based Less-is-more Effects.

For the LINDA model, consider the situation where the individual has accurate median knowledge of items from pool *N*, i.e., they accurately know whether each recognized item is above or below median. Subset *A* includes items in the top quartile of the size distribution, subset *B* includes items in the second highest quartile of the size distribution, and subset *C* contains all the remaining items. It is assumed for purposes of exposition that median knowledge is perfect, i.e., that the knowledge about a recognized item is accurate. This assumption can be relaxed but the general conclusions reported here hold for all reasonably high levels of accuracy (to just above chance). The size of the pool from which the test items are drawn is set at 100 but the same pattern of results is obtained for all large values of *N*. The key prediction is the relation between the proportion of correct decisions (calculated by equation (1)) and *n*, the number of items in the pool the participant can recognize. A less-is-more effect occurs, according to Goldstein & Gigerenzer's (2002) definition whenever performance of the inference task is demonstrably superior under conditions where fewer items from the pool of test items are recognized. McCloy et al. (2008) use a stricter definition, arguing that less-is-more effects should be restricted only to those areas of the graph where learning more items will continue to impair performance. We use the latter definition for our examples, although note that when this definition holds it necessarily implies that Goldstein & Gigerenzer's conditions are also met.

Example 1: Low validity for complete recognition. One way that less-is-more effects may be produced relates to how decisions are made when both items are recognized (in a 2-alternative forced choice task). LINDA is assumed to access limited and possibly inaccurate knowledge about the size of each recognized item, and use this knowledge to choose the item she believes to be larger. Suppose that choosing between two recognized items may, in some instances, be extremely difficult. An extreme version of this appears in Figure 1. When only one item is recognized, LINDA makes decisions on the basis of whether the item is judged above median (choose the recognized item) or below the median (choose the unrecognized item), as given in the appendix. Recognition-criterion correlations can be varied by varying the availability of the items in the subsets available to LINDA. For example, if all items in subset *A* (top quartile of the criteria) are recalled before all items in subset *C* (below median) then the recognition-criterion correlation is obviously higher than when all items in subset *C* are recalled before all items in subset *A*. In this simulation, we manipulated the recognizability of individual items within the subsets to obtain pre-set correlations between recognition and criterion. For the current example, we also assume that LINDA does not have the capacity to make a decision when both items are recognized, and so is obliged to guess, that is α_{110}, $\alpha_{101,}$ α_{011}. $\alpha_{200,}$ α_{020} and α_{002} were not calculated but all set at 0.5, as would be the case with simulations of the RH. The situation resembles one outlined in Goldstein and Gigerenzer (2002, pp. 84-85) in which German participants were experimentally exposed to the names of US cities without being presented with any further information which might be of use, and is also comparable with Schooler and Hertwig's (2005) ACT-R implementation of the recognition heuristic, which also assumed chance level performance when both items were recognized (Schooler & Hertwig, 2005, p. 614).

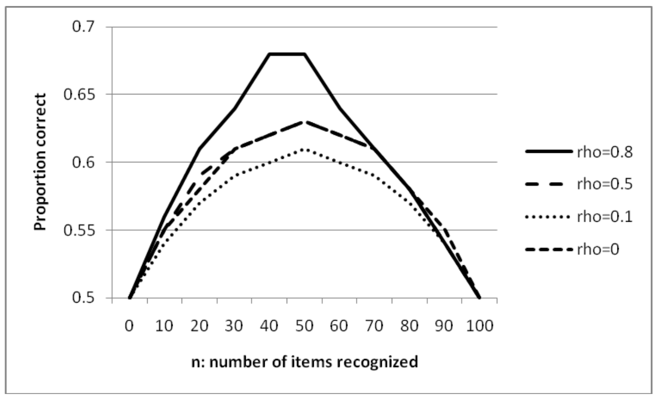

Figure 1: Proportion correct for the LINDA model when discrimination between two recognized items is at chance

Figure 1 shows clear less-is-more effects for all values of the recognition-criterion correlation tested. As more items are recognized (beyond a mid-point of 50% recognition rate) the proportion of correct inferences drops.

Unlike the RH model, which requires quite large criterion-recognition correlations to allow recognition validity to exceed knowledge validity, LINDA shows less-is-more effects for all values of the criterion recognition correlation, ρ, although the largest less-is-more effects occur for the largest values of ρ. For comparison, Figure 2 shows the predicted performance of the RH when knowledge validity is at chance and recognition validity takes the values of ρ reported in Figure 1. The validity of recognition is determined to some extent by ρ, which is determined for LINDA by the orderings of subset availability, and she experiences less-is-more effects occur even with low and zero values of ρ.

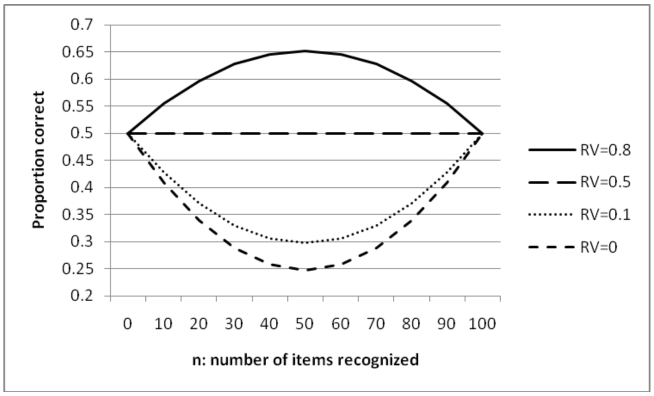

Figure 2: Predictions for the RH when recognition validity (RV) takes the values of the recognition-criterion correlation reported for LINDA. In this example, the RH is followed despite below-chance levels of validity (RV<0.5) which may not be realistic, but alternative strategies have yet to be suggested for these situations and, in particular, the point at which the RH is abandoned is not clearly outlined.

Example 1 relies upon the assumption that distinguishing between two recognized items is sufficiently difficult as to be effectively at chance. Both LINDA and RH are open to the criticism that, if knowledge validity for full recognition is chance, any non-random strategy able to operate when only one item is recognized will outperform knowledge and show less-is-more effects. This is a particular problem with the RH, where both knowledge validity and recognition validity are both set a priori for simulations such as this. Example 2 shows that low knowledge validity for full knowledge is not a necessary precondition for the appearance of less-is-more effects.

Example 2: Variation in subset availability. In order to formally compare LINDA with the RH model, we arranged that the models perform equally well when all items are recognized. Calculated probability of success when all items were recognized was 0.7525 for LINDA so knowledge validity was set at this level for the RH. The orderings of subsets in terms of recognition provide a potential rationale for variation in criterion-recognition correlation between individuals. Different orderings of subsets (and hence different recognition-criterion correlations) were simulated and the expected proportions correct using LINDA and the RH is given in Figure 2. We will use the notation *ABC* to denote subset availability, where *ABC* means that items from subset *A* are all more recognizable than the items from subset *B*, which in turn are all more recognizable than the items from subset *C*. A strict *ABC* recognition order obviously implies a high recognition-criterion correlation. Other recognition orderings (e.g., *ACB*) imply lower criterion-recognition correlations. *ABC* ordering is equivalent to a correlation between recognition and criterion of $\rho = .919$ and *ACB* ordering is equivalent to $\rho = .306$.

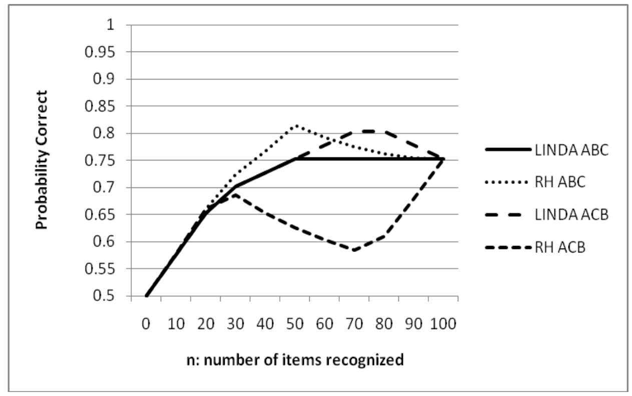

Figure 3. Performance of LINDA and the RH according to a recognition - criterion correlation determined by the recognizability of subsets. *ABC* ordering results in a high correlation and *ACB* a low (but still positive) correlation.

Figure 3 shows the performance of LINDA and the RH model for two different criterion-recognition orderings: *ABC* (items in the top quartile of the size distribution are most recognizable and items below median are least recognizable) and *ACB* (items in the top quartile are most recognizable, then items from below the median and finally items from the second quartile). *ABC* ordering corresponds to a strong criterion-recognition correlation ($\rho = .919$) and *ACB* ordering to a smaller, but still positive, correlation between criterion and recognition ($\rho = .306$).

The *ABC* ordering produces the expected effects from the literature. The RH model shows the less-is-more effect, while the knowledge-based LINDA model shows a monotonic relation between proportion correct and number of recognizable items. The situation is quite different for the *ACB* ordering: here, LINDA produces an inverted-U shaped function and a less-is-more effect. Less-is-more effects therefore do not imply use of the recognition heuristic – even given a positive criterion-recognition correlation – but may occur for other reasons. The inverted-U shaped functions that characterize the less-is-more effect indicate that a task becomes more difficult once the number of recognizable items passes a certain level. In the case of the RH model and the *ABC* ordering, this is because "easy" decisions (select the recognized item when only one item is recognized) are gradually outnumbered by "difficult" decisions (choose between items, both of which have been recognized) as the number of recognizable items increases. In the case of LINDA and the *ACB* ordering, moderate levels of recognition produce many easy decisions (discriminating a recognized item drawn from subset A from a recognized item drawn from subset C) but the decisions become more difficult when items of intermediate size, from subset B, begin to join the pool of recognizable items as the number of recognizable items increases.

Discussion

Whilst the two models give less-is-more effects in different circumstances, the effects are produced for essentially the same reasons. When few items are recognizable, the task is easier than when many items are recognizable. In the case of the RH model, for both Examples 1 and 2, when few items are recognizable the individual is more frequently confronted with the easy decision of selecting the one item recognized, rather than the problematic case of choosing between two recognized items, and this position is reversed when many items are recognizable. In Example 1 LINDA benefits from knowledge about the single item recognized which is not available to discriminate between two recognized items. For LINDA, performance in Example 2 for intermediate levels of recognition (up to 75 items) continues to improve as recognition rates rise because the discrimination required is still more likely to be between an item drawn from top quartile (subset *A*) and an item drawn from the bottom quartiles (subset *C*). Adding items from the second highest quartile (subset *B*), however makes the task more difficult this and leads to a drop in performance, and hence a less-is-more effect, at this point.

The fluency rule (discussed by Schooler & Hertwig, 2005) produces similar results and, once again, for similar reasons. Those items which are retrieved more quickly, dependent upon memory activation-level, are presumed to score more highly on the criterion (e.g., large cities are more quickly retrieved). For the fluency rule, intermediate rates of decay of activation allow for better discrimination between activated items than either fast or slow rates of decay. Over time, both slow and fast forgetting producing similar activation levels for dissimilar items (e.g., very large and very small cities). However, the fluency rule does not require or use any knowledge beyond the fact of fast retrieval. Thus, although it produces less-is-more effects of a kind, these are arguably recognition-driven based upon speed of access, rather than knowledge-driven, based upon some item-specific knowledge. The fluency rule is also reliant upon a fixed rate of decay from memory, an assumption which has recently been challenged (Berman, Jonides & Lewis, 2009; Lewandowsky & Oberauer, 2009; Lewandowsky, Oberauer & Brown, 2009; Nairne, 2002).

Testing LINDA.

LINDA demonstrates that less-is-more effects can occur for knowledge-based decisions and also that, when discrimination between two recognized items is sufficiently difficult, these effects can occur regardless of the recognition-criterion correlation. She therefore stands as an existence proof that less-is-more effects need not imply the use of recognition-driven inference but can be produced by strategies that invoke criterion knowledge. Any model that makes use of limited knowledge is likely to produce LINDA-like behavior.

Although LINDA reproduces the less-is-more effects observed with the RH, it is also worth noting that knowledge-based and recognition-based less-is-more effects are, or should be, empirically distinguishable. LINDA produces less-is-more effects similar to the RH when full knowledge has validity only slightly higher than chance but, unlike the RH, LINDA produces such effects regardless of the size of the recognition-criterion correlation (Figure 1). She also shows less inclination to produce such effects when knowledge validity is not artificially constrained and the recognition-criterion correlation is particularly high. Indeed, LINDA is more likely to show less-is-more effects when the recognition-criterion correlation is rather more moderate (Figure 3). Thus, although LINDA provides a plausible alternative account of existing less-is-more effects, there are experimental manipulations not yet investigated which should provide data that favor either one account or the other.

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Appendix

1. Derivation of the values of p_{ijk} in Equation (1):

Probability of recognizing no items:

$$p_{000} = [(N - a - b - c)/N] \times [(N - a - b - c - 1)/(N - 1)] = (N - a - b - c)(N - a - b - c - 1)/[N(N - 1)]$$

Probabilities associated with the recognition of only one item:

$$p_{100} = [2a/N] \times [(N - a - b - c)/(N - 1)] = 2a(N - a - b - c)/[N(N - 1)]$$

Probability of recognizing one item from the top quartile.

Similarly for second quartile and below median:

$$p_{010} = 2b(N\text{-}a\text{-}b\text{-}c)/[N(N - 1)]$$
$$p_{001} = 2c(N\text{-}a\text{-}b\text{-}c)/[N(N - 1)]$$

Probabilities associated with the recognition of both items:

$$p_{110} = 2ab/[N(N\text{-}1)]$$

(one item is in the top quartile and one item is in the second quartile)

$$p_{101} = 2ac/[N(N\text{-}1)]$$
$$p_{011} = 2bc/[N(N\text{-}1)]$$

(as above, substituting *v* and *w* where appropriate)

$$p_{200} = a(a - 1)/[N(N\text{-}1)]$$

(both items are in the top quartile)

$$p_{020} = b(b - 1)/[N(N\text{-}1)]$$
$$p_{002} = c(c - 1)/[N(N\text{-}1)]$$

(as above, substituting *v* and *w* where appropriate)

2. α_{ijk} parameters for the LINDA model demonstrated in Example 2.

$\alpha_{000} = 0.5$

No items are recognized, performance is chance.

$$\alpha_{100} = [0.5 \times (0.25N - a)/(N - a - b - c)] + [(0.75N - b - c)/(N - a - b - c)]$$

Probability correct if one item from the top quartile is recognized.

$$\alpha_{010} = 0.5 \times (0.25N - b)/(N - a - b - c) + (0.5N - c)/(N - a - b - c)$$

Probability correct if the recognized item is in the second quartile.

$$\alpha_{001} = (0.5N - a - b)/(N - a - b - c) + 0.5 \times (0.5N - c)/(N - a - b - c)$$

Probability correct if the recognized item is below median.

$\alpha_{110} = 0.5$

Two items are recognized: one item is in the first quartile and the second item is in the second quartile, so with median knowledge, performance is chance.

$\alpha_{101} = \alpha_{011} = 1$

One recognized item is above median and one is below so success is certain.

$\alpha_{200} = \alpha_{020} = \alpha_{002} = 0.5$

Both recognized items are from the same quartile, and so cannot be distinguished.

3. α_{ijk} parameters for the Recognition Heuristic model demonstrated in Example 2.

$\alpha_{000} = 0.5$

$$\alpha_{100} = 0.5 \times (0.25N - a)/(N - a - b - c) + (0.75N - b - c)/(N - a - b - c) = (0.875N - 0.5a - b - c)/ (N - a - b - c)$$

There is only one item recognized, it is in the top quartile.

$$\alpha_{010} = 0 + 0.5 \times (0.25N - b)/(N - a - b - c)] + (0.5N - c)/(N - a - b - c) = (0.625N - a - b - c)/ (N - a - b - c)$$

The recognized item is in the second quartile.

$$\alpha_{001} = 0 + 0.5 \times (0.5N - c)/(N - a - b - c) = (0.25N - 0.5c)/ (N - a - b - c)$$

The recognized item is below median.

$\alpha_{110} = \alpha_{101} = \alpha_{011} = \alpha_{200} = \alpha_{020} = \alpha_{002}$

All these cases involve recognition of both items, and it is assumed knowledge can be used with a certain probability of success. In the Example 2, this probability was chosen to ensure that the LINDA and RH models produced the same probability of success when all items were recognized.