

A Reinforcement-and-Generalization Model of Sequential Effects in Identification Learning

Matt Jones (mcj@colorado.edu)

University of Colorado, Department of Psychology, 345 UCB
Boulder, CO 80309 USA

Abstract

Responses in identification-learning tasks depend on events from recent trials. A model for these sequential effects is proposed, based on previous work in category learning and founded on theories of reinforcement learning and generalization. The model is compared to two other theories in their predictions of the influence of previous stimuli and previous feedback. Two experimental paradigms are introduced that allow separate assessment of these two effects. Results support the reinforcement-and-generalization model.

Keywords: Identification; reinforcement learning; generalization; sequential effects; mathematical models.

Introduction

Sequential effects arise in essentially any repeated psychological task. Although the majority of experimental designs and statistical analyses aim to average out these effects, they often contain useful information. Sequential effects can facilitate inferences about perceptual representations (Jones, Maddox, & Love, 2005), concept representations (Jones, 2009), and learning and decision processes that integrate knowledge from past experience (Sakamoto, Jones, & Love, 2008).

This paper focuses on sequential effects in identification learning, a well studied task in which subjects learn to assign a unique response to each of a given set of stimuli. Sequential effects are well established in this domain (e.g., Garner, 1953), and several extant models attempt to explain these phenomena (e.g., Holland & Lockhead, 1968; Luce & Green, 1974; Stewart, Brown, & Chater, 2005). A new model is proposed here, based on theories of reinforcement learning and generalization, that attempts to unify sequential effects in identification with previous work on sequential effects in category learning (Jones, Love, & Maddox, 2006; Jones et al., 2005). Two experiments test the predictions of this model and compare it to previous proposals.

Models of sequential effects in identification can be distinguished by the separate influences they ascribe to past stimuli and past feedback. However, disentangling these is difficult in a standard identification task, because the stimulus and feedback observed on any past trial are perfectly confounded. The present experiments offer two new solutions to this problem, by using probabilistic feedback and irregular stimulus-response mappings that varied across subjects. Analyses of sequential effects in both experiments support the reinforcement-and-generalization model. Implications for other benchmark phenomena of identification learning are also discussed.

Sequential Effects in Identification Learning

On each trial in a standard identification-learning task, the subject is presented with a single stimulus, selects a response, and is given feedback with the correct answer. Often the stimuli are arranged along a single perceptual dimension, such as loudness or line length, the responses are numbers on a keypad (e.g., 1-9), and the assignment preserves the natural ordering of the stimuli (e.g., the shortest line is mapped to response 1, etc.). This basic paradigm has been used for decades to explore perceptual representations as well as the decision processes that allow people to discriminate items in their environment.

Identification learning has long been known to exhibit sequential effects, whereby the identities of the stimuli on recent trials systematically bias the current response. The first such finding was in a loudness-judgment task by Garner (1953), who observed an *assimilation effect*, whereby the current response was biased toward the correct answer on the previous trial.

The assimilation effect has played an important role in the development of models of identification learning. Two broad explanations for this phenomenon have been proposed. The first ascribes the effect to the previous stimulus, by assuming the representation of the current stimulus is confused with or contaminated by other stimuli in memory, so that its perceived value is biased toward those past stimuli (DeCarlo & Cross, 1990; Garner, 1953).

The second explanation contends that assimilation is due not to the previous stimulus, but to the previous feedback. These theories propose a *relative judgment* (RJ) strategy, in which subjects base the current response on the feedback they received on the previous trial, combined with an adjustment for the difference between successive stimuli (Holland & Lockhead, 1968; Luce & Green, 1974; Stewart et al., 2005). For example, if the previous feedback was 6 and the current stimulus appears two steps greater, then the subject will respond 8 on the current trial. This strategy alone does not produce assimilation, but it does if one also assumes subjects systematically underestimate the differences between stimuli (e.g., stimuli 3 steps apart can lead to a response adjustment of only 2 steps). In this case the response becomes biased in the direction of the previous feedback (Stewart et al., 2005).

Reinforcement-and-Generalization Model

Assimilation in identification learning can also be interpreted as a manifestation of a broader phenomenon that

Jones et al. (2006) termed the *decisional recency effect*. This is a general tendency to select responses or perform actions that have been recently reinforced, and it arises in essentially any repeated task involving rewards or feedback (see Jones et al., 2006, for a review). Decisional recency is also a natural prediction from the framework of reinforcement learning, in which the values of actions are updated based on the rewards they induce (e.g., Sutton & Barto, 1998). When such updates are made in proportion to the difference between predicted and observed outcomes (known as delta-rule learning), estimated reward values depend primarily on more recent feedback (Estes, 1957).

Investigations of sequential effects in category learning have found evidence for both the decisional recency effect and a separate *perceptual recency effect* arising from the previous stimulus (Jones et al., 2005, 2006). These two effects can be dissociated using a probabilistic task, in which different occurrences of the same stimulus can lie in different categories. This allows separate assessment of the influences of past stimuli and past feedback.

Perceptual recency is observed in analyses that hold the previous feedback constant and examine the influence of the previous stimulus. For example, subjects categorizing rectangles by height will respond to the current stimulus as though it were taller than it actually is, if the previous stimulus was short (Jones et al., 2006). This suggests the representation of the current stimulus is biased away from the values of recent stimuli, consistent with the large body of physiological evidence that sensory processing is founded on adaptation and contrast (e.g., Sekuler & Blake, 1994). However, it is in direct opposition to the stimulus-confusion account for assimilation in identification learning.

Decisional recency is observed in analyses that hold the previous stimulus constant and examine the effect of the previous feedback. These show a bias towards selecting the category that was reinforced on the previous trial (Jones et al., 2005, 2006). Furthermore, this effect is moderated by the similarity between present and previous stimuli. When successive stimuli are identical or highly similar, the current response depends very strongly on the previous feedback, whereas the effect becomes null or slightly negative as the stimuli become increasingly dissimilar.

The dependence of decisional recency on similarity further supports the reinforcement-learning interpretation. In reinforcement learning, expectations about a given stimulus cannot normally be based on prior knowledge about that exact case but must rely on past experience with other, similar stimuli. This is the problem of generalization. A large body of research on generalization shows that people and other animals generalize between stimuli to the extent they perceive them as similar (e.g., Shepard, 1987). Nearly all reinforcement-learning models embody this principle. Therefore, if decisional recency reflects reinforcement learning from the previous trial, we should expect its magnitude to depend on how strongly the subject generalizes between the previous and present stimuli, which in turn will depend on their similarity.

Drawing on the above evidence, Jones et al. (2006) developed a mathematical model of sequential effects in category learning that embodies both decisional and perceptual recency effects. This paper proposes a natural adaptation of that model to identification learning, referred to as the reinforcement-and-generalization (RG) model. The model's two primary assumptions are that the perception of the current stimulus is biased away from the previous stimulus, and the current response is biased toward the previous feedback to a degree determined by the similarity between present and previous stimuli.

The perceptual portion of the identification model is unchanged from that of the categorization model. The perceived value of the current stimulus, Ψ_n , is assumed to depend on both the present and previous stimuli, S_n and S_{n-1} . Provided that stimuli are scaled linearly with their perceptual representations, a reasonable assumption is that the effect of the previous stimulus is also linear (e.g., DeCarlo & Cross, 1990). The coefficient c is negative if the perceptual effect is contrastive and positive if it is assimilative.

$$\Psi_n = S_n + c \cdot S_{n-1} \quad (1)$$

The decision portion of the model assumes the response depends on two sources of evidence. The first is the current percept, mapped onto the response scale by the learned stimulus-response map f . The second is generalization of reinforcement from the previous feedback, F_{n-1} , whose impact depends on the similarity between successive stimuli, $\text{sim}(S_n, S_{n-1})$, together with a scaling parameter, β . The only change in the identification model is that it predicts the expected value of the response, $E(R_n)$, rather than the probabilities of selecting among (nominal) category labels.

$$E(R_n) = f(\Psi_n) + \beta \cdot \text{sim}(S_n, S_{n-1}) \cdot (F_{n-1} - f(\Psi_n)) \quad (2)$$

Experiment 1: Variable Feedback

The three models discussed above differ in how they ascribe sequential effects to the previous stimulus and feedback. Experiment 1 tested the models' predictions using a probabilistic identification task, which allowed separate assessment of these two influences on the current response. Stimuli were horizontal lines of varying length, and responses were the numbers 1 through 9. Subjects were told there was one line length for each response, but in fact there were ten lengths, referred to here as A through J. Each feedback value followed two different stimuli equally often – A and B for 1, B and C for 2, and so on.

Experiment 1 also included a between-subjects manipulation of sequential dependencies among trials, designed to affect subjects' use of the RJ strategy. The 18 possible stimulus-feedback pairings were partitioned into two types: *Lower trials*, in which the stimulus was the shorter of the two possible values for the feedback given (A-1, B-2, ... H-9), and *Upper trials*, in which the stimulus was the longer possible value (B-1, C-2, ... J-9). In the *Independent* condition, all trials were sampled independently. In the *Autocorrelated* condition, 80% of trials were of the same

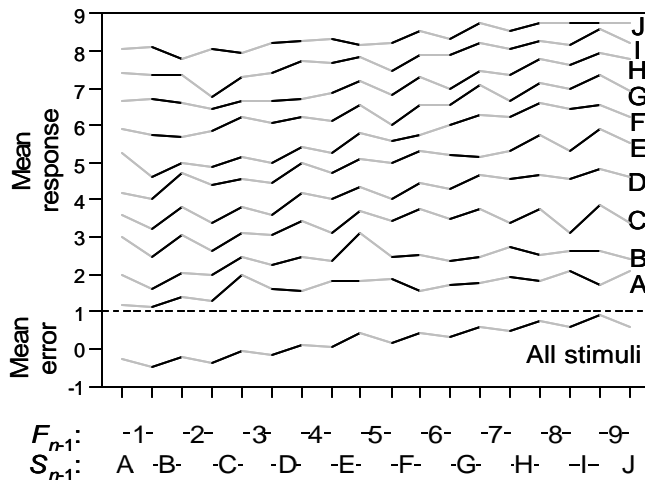


Figure 1. Mean response in Independent condition of Experiment 1, as a function of present stimulus (separate curves), previous stimulus (S_{n-1}), and previous feedback (F_{n-1}). Lower section shows mean error collapsed over present stimulus.

type as their predecessors. Because the RJ strategy is only reliable (i.e., produces the correct answer) when successive trials are of the same type, it was predicted that autocorrelation would serve to induce or increase subjects' use of RJ. The results below support this assumption.

Method

33 and 35 undergraduates were randomly assigned to the Independent and Autocorrelated conditions, respectively. Stimuli were horizontal lines with lengths from 2.54 to 5.08 cm in steps of 0.28 cm. The stimulus on each trial was presented in a random position on an LCD monitor, and once the subject pressed a response key (1-9), the chosen and correct responses were both displayed. Stimulus and feedback remained for 1000 ms. Trials were separated by 500 ms of blank screen. Each subject completed 400 trials in blocks of 50.

Results and Discussion

Sequential effects were assessed by computing the mean response as a function of the present stimulus, previous stimulus, and previous feedback. The results for the Independent condition are shown in Figure 1 (Autocorrelated results are discussed below). Different curves correspond to values of the current stimulus, and the abscissa shows all 18 possibilities for the stimulus-feedback pair on the previous trial. Each grey segment represents a comparison of two values of S_{n-1} , with F_{n-1} and S_n held fixed. These segments tend to slope downward, indicating a negative effect of the previous stimulus. Each black segment represents a comparison of two values of F_{n-1} , with S_{n-1} and S_n held fixed. These segments tend to slope upward, indicating a positive effect of the previous feedback. The pattern is clearest in the lowest curve, which shows average error collapsed over S_n . All nine S_{n-1} comparisons are negative

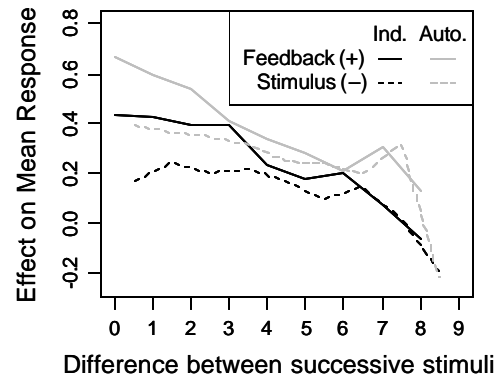


Figure 2. Dependence of sequential effects on similarity between present and previous stimuli in Experiment 1. Previous-stimulus effect is negative but is inverted for comparison to previous-feedback effect.

and all eight F_{n-1} comparisons are positive. Both patterns are reliable by binomial tests ($p_s < .01$). The net slope of this curve is positive, replicating the classic assimilation effect, which is now be seen to result from a stronger positive effect of F_{n-1} compared to the negative effect of S_{n-1} .

The negative influence of the previous stimulus rules out the stimulus-confusion model, so it is not considered further. However, both RJ and RG models predict this result, as well as the positive influence of the previous feedback. To distinguish these models, we consider how the sequential effects depend on the autocorrelation manipulation and on the similarity between successive stimuli.

The effect of stimulus similarity can be seen in Figure 1 as the increased jaggedness of the curves near the main diagonal, indicating that both sequential effects are stronger when S_n is more similar to S_{n-1} . This can be formalized as follows. For the previous-feedback effect, the influence of F_{n-1} (corresponding to the slope of each black segment in Figure 1) can be averaged over all $[S_{n-1}, S_n]$ pairs that differ by a given number of steps. This yields the average effect of the previous feedback on the current response, conditioned on the difference between present and previous stimuli. A parallel approach can be used for the previous-stimulus effect, except that each comparison involves two values of S_{n-1} (e.g., $S_{n-1} = A$ vs. B , conditioned on $F_{n-1} = 1$ and $S_n = C$), and thus two differences between S_{n-1} and S_n . For ease of exposition, I use the average of these two differences (1.5 in the above example), so that stimulus differences for the previous-stimulus effect range from .5 to 8.5.

Figure 2 shows the previous-feedback and previous-stimulus effects as a function of the difference between successive stimuli. The previous-stimulus effect is negative, but it is inverted to facilitate comparison. Four significant aspects of this graph are discussed in turn: the curves all slope downward, the feedback effect has a steeper slope than the stimulus effect, the Autocorrelated curves lie above the Independent curves, and the curves for the two conditions are parallel. To test the reliability of these

Table 1: Sequential effects as functions of similarity, from individual-subject fits to Experiment 1

Condition	Previous-feedback		Previous-stimulus	
	Intercept	Slope	Intercept	Slope
Independent	.488	-.057	-.231	.019
Autocorrelated	.637	-.069	-.389	.027

patterns, separate curves were estimated for each subject with the simplifying constraint that all curves be linear. Thus a slope and intercept for the previous-stimulus and previous-feedback effects (as functions of stimulus dissimilarity) were estimated for each subject. Mean values by condition are displayed in Table 1.

Starting with the Independent condition, the mean slope for the previous-feedback effect is significantly negative ($t_{32} = 6.18$, $p < 10^{-6}$), indicating this effect decreases as successive stimuli become dissimilar. This is a central prediction of the RG model. The mean slope for the previous-stimulus effect is marginally positive ($t_{32} = 1.83$, $p = .076$), suggesting this effect may also weaken with similarity. Under the RG model, this suggests that perceptual contrast levels off as stimulus differences become large, rather than continuing to increase at a constant rate. Equation 1 does not predict this, but it is a natural elaboration of the model that is consistent with more detailed studies of perceptual contrast (e.g., Petzold, 1981).

The simple version of the RJ model does not predict either sequential effect to depend on similarity. However, DeCarlo and Cross (1990) proposed that people rely more on this strategy (as opposed to responding to the absolute value of the current stimulus) when successive stimuli are similar, and this assumption predicts the pattern seen here.

Direct comparison shows that the previous-feedback effect has a steeper (absolute) slope than the previous-stimulus effect ($t_{32} = 7.16$, $p < 10^{-7}$). This is consistent with the RG model (although the opposite pattern is also), because it assumes perceptual contrast and generalization are unrelated processes. The RJ strategy alone does not predict the slope difference, but recall that Stewart et al. (2005) proposed people underestimate stimulus differences in mapping them to response differences. This is the assumption needed for the model to predict the overall assimilation effect, and it also causes the effect of the previous stimulus (and hence its dependence on similarity) to be attenuated relative to the effect of the previous feedback.

Turning to a comparison between conditions, the intercepts for both sequential effects were greater in the Autocorrelated condition ($t_s > 3$, $p_s < .01$). There was no significant interaction between effect type (feedback vs. stimulus) and condition ($F_{1,66} = .12$, $p > .5$). For slopes, there was no difference between conditions for either sequential effect ($t_s < 1$, $p_s > .1$), and there was no condition-by-effect interaction ($F_{1,66} = .41$, $p > .5$). Therefore, the effect of autocorrelation was a uniform increase in both sequential effects, to equal degrees, independent of stimulus similarity.

The difference between conditions supports the assumption behind the autocorrelation manipulation, that it

would increase reliance on RJ. However, it also reveals two aspects of the RJ strategy that are incompatible with it as an explanation for sequential effects. First, the fact that the slopes of the similarity effects were unchanged implies that reliance on RJ is independent of similarity. This directly contradicts the assumption needed for RJ to explain the similarity effect to begin with. Second, the fact that the two sequential effects are strengthened equally implies that subjects correctly scale stimulus differences to response differences. This directly contradicts the assumption needed for RJ to explain the overall assimilation effect. To be clear, the data do not rule out RJ altogether. Subjects do seem to engage this strategy when it is reliable (as it would be in a standard, deterministic identification task). The point, however, is that RJ cannot explain the observed sequential effects. Therefore this strategy must act on top of more fundamental mechanisms operating separately on stimulus and response representations, as in the RG model.

Experiment 2: Variable Mappings

Experiment 2 varied the stimulus-response map between subjects, so that stimuli and feedback would be decoupled when considering the data for all subjects together. Subjects' task was to learn to assign the numerals 1 through 9 to the letters A through I. The assignments were irregular, which renders strategies such as RJ useless (e.g., knowing that 3 maps to G is irrelevant to the answer for 4) and allows us to assume that processing consists of stimulus identification, retrieval of stimulus-response associations, and response selection. Our interest is in the errors that occur at the stimulus and response stages, due to processes such as perceptual or motor confusion, generalization, encoding and retrieval errors, and sequential effects. These processes can all be summarized by a confusion matrix among stimuli, specifying the distribution over which stimuli are accessed given the stimulus actually presented, and a confusion matrix among responses, specifying the distribution over which response is selected given the one that is retrieved.

The experiment hinges on a counterbalancing of stimulus-response maps, due to Shepard (1957), that allows separate estimation of the stimulus and response confusion matrices. To estimate the response-confusion matrix, one considers the distribution of actual responses conditioned on which response was correct. When these probabilities are averaged over subjects, patterns of stimulus confusion average out to a constant, because for any pair values for the actual and correct responses, the associated stimuli are counterbalanced over subjects. The stimulus-confusion matrix is estimated in a similar way, by computing the distribution over the correct stimulus for the response that was chosen (i.e., $f^{-1}(R)$, where f is the mapping), conditioned on the stimulus that was actually presented.

The primary aim of Experiment 2 was to test the RG model's predictions for how stimulus and response distributions are affected by the previous trial. However, because the use of irregular mappings necessitated symbolic

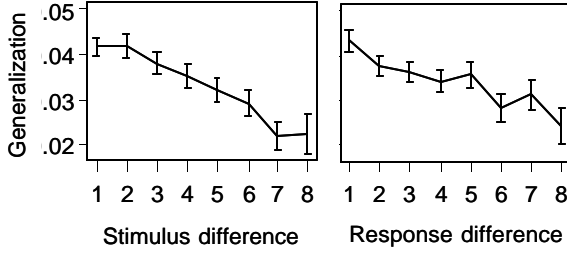


Figure 3. Generalization gradients for stimuli (numerals) and responses (letters) in Experiment 2.

stimuli, it was uncertain whether perceptual contrast effects would be observed. Therefore, this experiment served primarily as a test of the decision portion of the model.

Method

72 undergraduates were assigned stimulus-response maps such that, for any two stimuli, S_1 and S_2 , and responses, R_1 and R_2 , exactly one subject had S_1 mapped to R_1 and S_2 mapped to R_2 . The stimulus on each trial was a numeral 1-9 presented in the center of a monitor. The responses, letters A-I, were arranged in a circle (2.54 cm radius) around the stimulus, in a different random configuration on each trial. The subject used a mouse to click on a response, and was then shown the correct answer. This feedback remained for 750 ms. Trials were grouped into blocks of 36, containing four repetitions of each stimulus. The experiment ended when a block was completed with at most two errors.

Results and Discussion

The response-confusion matrix was estimated by computing $P(R = j | f(S) = i)$ for all responses i and j , where $f(S)$ is the correct response for the current stimulus. The stimulus-confusion matrix was estimated by computing $P(R = f(j) | S = i)$ for all stimuli i and j . Generalization gradients were obtained from the confusion matrices by averaging over all pairs of stimuli or responses differing by the same number of steps. These differences were determined by the standard numeric and alphabetic orderings (123..., ABC...). As Figure 3 shows, these gradients are remarkably regular, even though the psychological similarity structure of numbers and letters is likely more complex than just that induced by their orderings.

Unfortunately, conditioning the confusion matrices on the previous stimulus and feedback breaks the symmetry from the counterbalancing of stimulus-response maps. Therefore sequential effects were evaluated using a formal modeling approach, in which the RG model was combined with the standard Luce-Shepard identification-choice model (Luce, 1963; Shepard, 1957). This model assumes processing occurs in three stages: stimulus generalization, stimulus-response mapping, and response generalization and selection. The first stage was modified to accommodate perceptual recency by assuming that stimulus generalization is based not on S_n but on Ψ_n as given by Equation 1. Here s represents any stimulus, and $A(s)$ is its activation.

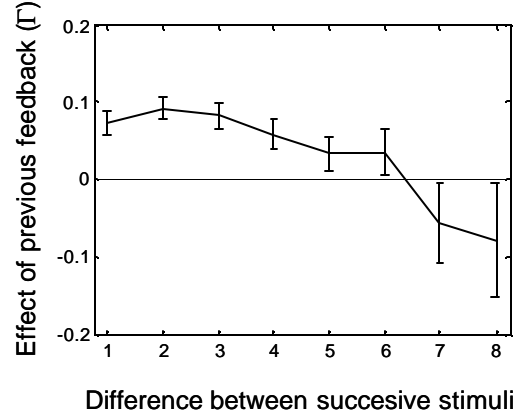


Figure 4. Effect of previous feedback on current response as function of stimulus similarity, from fit of nonparametric model to Experiment 2.

$$A(s) \propto \text{sim}(s, \Psi_n) \quad (3)$$

The stimulus-response mapping stage involves retrieving the correct response for each stimulus, or else guessing with probability g . This determines the activation of responses.¹

$$A(f(s)) = g + (1-g) \cdot A(s) \quad (4)$$

In the response stage of the Luce-Shepard model, activation is generalized among responses according to similarity. The effect of reinforcement from the previous trial was incorporated by assuming initial activations are biased towards the previous feedback by the same formula as Equation 2. This leads to the following response rule (where Luce-Shepard has just $\text{sim}[R, r]$ for the final term).

$$P(R) \propto \sum_r A(r) \cdot \text{sim}[R, r + \beta \cdot \text{sim}(S_n, S_{n-1}) \cdot (F_{n-1} - r)] \quad (5)$$

Similarity was assumed to be an exponential function of distance (Shepard, 1987), $\text{sim}(x, y) = \exp(-\alpha \cdot |x - y|)$, where α equals α_{stim} for stimuli and α_{resp} for responses. Distance was again determined by the numeric and alphabetic orderings, as a simple working assumption. Thus there were three parameters for the base identification model – similarity parameters (α_{stim} , α_{resp}) and guessing probability (g) – and two more for sequential effects: perceptual assimilation or contrast (ϵ) and reinforcement effect (β). The principal predictions to be tested were $\epsilon < 0$ and $\beta > 0$.

The best-fitting value of β was .117, indicating a positive bias of the response towards the previous feedback. Comparison to a model with β fixed to 0 showed this effect is highly reliable ($\chi^2_1 = 45.64$, $p < 10^{-10}$). As a more rigorous test of the dependence of the previous-feedback effect on similarity, the model was refit with the $\beta \cdot \text{sim}(S_n$,

¹ The internal variables $A(s)$ and $A(r)$ can be interpreted either as probabilities of which value is activated or as distributed representations, without change in the formalism or predictions. This allows evaluation of sequential effects with minimal theoretical commitment regarding the architecture underlying the task.

S_{n-1}) term in Equation 5 replaced by a nonparametric function, $\Gamma(|S_n - S_{n-1}|)$, allowing a different free parameter for each possible distance. The best-fitting values of Γ are plotted in Figure 4 which shows a clear decrease in the effect of the previous feedback with increasing distance between successive stimuli. Comparison to a model in which Γ was constant showed this relationship to be reliable ($\chi^2_7 = 16.44, p < .05$). However, the nonparametric model did not fit significantly better than the original model of Equation 5 ($\chi^2_7 = 7.87, p > .1$), indicating similarity provides an adequate fit to the pattern in Figure 4. This supports the core principle of the RG model, that previous reinforcement is generalized to the current trial to a degree determined by the similarity between successive stimuli.

The best-fitting value of c was .005, which was not reliably different from zero ($\chi^2_1 = .008, p > .5$). Therefore there was no evidence for an effect of the previous stimulus on the perception of the current stimulus. Given that the stimuli were symbolic (numerals), this is not necessarily a concern for the model. We are currently piloting a version of this experiment using more perceptual stimuli to determine whether a contrast effect is observed in that case.

General Discussion

Sequential effects in learning provide important clues to how stimuli are represented and the decision process used to identify them. The reinforcement-and-generalization (RG) model proposed here posits that separate sequential effects operate at both of these levels. The perception of the current stimulus is contrasted away from the previous stimulus, and the current response is biased towards the previous feedback to a degree determined by the similarity between successive stimuli. The latter effect derives from fundamental principles of reinforcement learning and generalization (e.g., Sutton & Barto, 1998).

An important next step in testing the RG model is to identify factors that separately influence the two recency effects. One factor that seems to influence perceptual contrast is the nature of the stimuli. Contrast was observed for the perceptual stimuli in Experiment 1 but not for the symbolic stimuli in Experiment 2. One factor that has been seen to influence the reinforcement effect (decisional recency) is selective attention. Stimulus differences on a task-irrelevant dimension do not attenuate the reinforcement effect as much as differences on the relevant dimension (Jones et al., 2005), consistent with established effects of selective attention on similarity (e.g., Tversky, 1977).

The RG model is not a complete model of identification, but an account of sequential effects (in this task and others). However, it may shed light on other phenomena in this domain. For example, one consistent finding is the *bow effect*, whereby discrimination is better between stimuli near the ends of the range than near the middle (Murdock, 1960). Another is the *spacing effect*, whereby discrimination between two fixed stimuli is worse if the spacing between other stimuli is increased. Many explanations for these

phenomena rely on assumptions about stimulus representations, for example that they are based on position relative to the endpoints of the stimulus range (Parducci, 1965). These theories lead to specific predictions regarding similarity, which can be tested with the RG model by using the decisional recency effect as an index of similarity. Thus the present theory offers a useful window onto how stimulus representations adapt to the task at hand.

References

- DeCarlo LT, & Cross DV (1990). Sequential effects in magnitude scaling: Models and theory. *J Exp Psychol Gen*, 119, 375–396.
- Estes WK (1957). Theory of learning with constant, variable, or contingent probabilities of reinforcement. *Psychometrika*, 22, 113–132.
- Garner, W. R. (1953). An informational analysis of absolute judgments of loudness. *J Exp Psychol*, 46, 373–380.
- Holland MK, & Lockhead GR (1968). Sequential effects in judgments of loudness. *Percept Psychophys*, 3, 409–414.
- Jones M (2009). Identifying category representations through sequential effects in learning. *In prep*.
- Jones M, Love BC, & Maddox WT (2006). Recency as a window to generalization: Separating decisional and perceptual sequential effects in categorization. *J Exp Psychol Learn*, 32, 316–332.
- Jones M, Maddox WT, & Love BC (2005). Stimulus generalization in category learning. *Proc 27th Meeting Cognitive Science Society*, 1066–1071.
- Luce RD (1963). Detection and recognition. In RD Luce, RR Bush, & E Galanter (Eds.) *Handbook of Mathematical Psychology, I*. New York: Wiley.
- Luce RD, & Green DM (1974). Response ratio hypothesis for magnitude estimation. *J Math Psychol*, 11, 1–14.
- Murdock BB (1960). The distinctiveness of stimuli. *Psych Rev*, 67, 16–31.
- Parducci A (1965). Category judgment: A range-frequency model. *Psych Rev*, 72, 407–418.
- Petzold P (1981). Distance effects on sequential dependencies in categorical judgments. *J Exp Psychol Human*, 7, 1371–1385.
- Sakamoto Y, Jones M, & Love BC (2008). Putting the psychology back into psychological models: Mechanistic vs. rational approaches. *Mem Cognition*, 36, 1057–1065.
- Sekuler R, & Blake R (1994). *Perception*. New York: McGraw-Hill.
- Shepard RN (1957). Stimulus and response generalization: A stochastic model relating generalization to distance in psychological space. *Psychometrika*, 22, 325–345.
- Shepard RN (1987). Toward a universal law of generalization for psychological science. *Science*, 237, 1317–1323.
- Sutton RS, & Barto AG (1998). *Reinforcement Learning: An Introduction*. Cambridge, MA: MIT Press.
- Stewart N, Brown GDA, & Chater N (2005). Absolute identification by relative judgment. *Psych Rev*, 112, 881–911.
- Tversky A (1977). Features of similarity. *Psych Rev*, 84, 327–352.