Process of explanation and representation in a counterintuitive problem of probabilities

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Abstract

Starting with the version "Card game" (Tubau and Alonso, 2003) of the counterintuitive problem of probabilities Monty Hall, a version of the problem was designed in which there is a "Help-sentence" and a "Double question" that allow a shallow increase in the proportion of correct responses. The experiments tries to confirm to what extent the increase in the percentage of switches in the First illusory question is due to an associative effect. There are a small percentage of participants who seem to take advantage of these helps and yet keep their correct answer until the end of the questionnaire. It is suggested that there is some sort of non linguistic representation in such participants that could be either more associative or more numerical in nature, and some future research to explore this idea further is proposed.

Keywords: self-explanation, implicit knowledge, reasoning, inference, scheme, mental representation.

Introduction

This study analyses different explanations that participants gave to an illusory question that they answered correctly. It argues that there is an implicit knowledge behind the justifications that most of the participants give for their answer to this illusory question. An example from the reasoning field of the presence of this kind of knowledge could be the subcomponents studied by Chi, Bassok, Lewis, Reimann & Glaser (1989), Chi & van Lehn (1991), Renkl, (1997). There, the example of a physics text (newtonian mechanics) shows how many elements are actually involved (rather than explicitly written) in our presumably good educational texts. It would be interesting to continue in this direction and explore the psychological representation that relies on that.

The Monty Hall Dilemma: version "card game"

The Monty Hall dilemma is the adaptation of an American television gameshow from the 1960's, which involves a contestant guessing which of 3 doors hides a prize. The contestant initially chooses one of 3 doors, which remains closed, and the presenter, who knows what is behind each of the doors, opens one of the other two remaining doors, showing that that one doesn't contain a prize. The question that the contestant has to answer is whether he would rather choose to stick with his initially chosen door, or whether he would like to change to the other door that the presenter has not opened yet.

Tubau and Alonso (2003) presented this dilemma in the form of a card game (see Appendix for details of the game).

The initial probability of each card being correct is the same, 1/3. Once the informant has showed one of his cards (situation of elimination), we tend to make the illusory inference that "if there are only two cards, the probability of each of them of being the ace is the same (1/2)", when the probabilities are in fact the same as they were at the beginning: for the decision-maker, p (ace) = 1/3, and for the informant, p (ace) = 2/3.

A possible explanation for such an illusory response could draw from a heuristic of similarity, where in situations in which there are two alternatives, we think the probabilities are equal.

We will see that there are several ways of arriving at the correct response to the dilemma without resorting to the application of the theorem of Bayes.

A simple and completely correct explanation for the participants' continued belief in the probabilities after the elimination appeals to the invariability of the probabilities of the initial sets. The elimination *is not* informative from the point of view of the probabilities of the initial sets: since the beginning we have known that at least 1 of the 2 cards held by the informant is different from the ace; the elimination *is* informative from the point of view of the individual probabilities of the cards (since the cards of the informant change from having equal probabilities of 1/3 to having probabilities of 0 and 2/3), but these do not affect the probabilities of the initial sets.

Necessary and sufficient information for the correct resolution of the dilemma

It is important to realise that the key point in the explanation of the dilemma is the situation of elimination. Any of the factors that facilitate the resolution of the dilemma in fact facilitate the comprehension of the implications of the situation of elimination. It is at this moment that it is necessary to bear in mind that the performance of the informant is conditioned, and this is relevant to their response. Girotto & Gonzalez (2005) suggests that, "the

crux of the problem is that these combinations appear as irrelevant to most individuals, so that they resort to more basic probabilities".

Let us consider, furthermore, that to overcome the strong illusory inference, it is not enough only to consider the variety of possible cases of performance of the informant (that is to say, to think that "if John has the ace and the 7, he will always show the 7, whereas if he has the 7 and the 8 he will show half of the time the 7 and half of the time the 8), but *it is necessary also* to consider the globality (or generic sum) of these and to compare the number of times in which there is the ace and the 7 and the number of times in which there is the 7 and the 8. For in the case where the informant has the 7, for example, the reasoning would be as follows:

Table 1

"Help sentence"	generic case	specifying for a natural number
If the informant has the ace and 7, he will always show 7.	n times	n = 2
If he has 7 and 8, he will show half of the time the 7.	n/2 times	n/2 = 1
Therefore,	*Every 3n/2 times that he shows 7, n will be with the ace and n/2 with 8.*	*Every 3 times that he shows 7, 2 of them will be with the ace and 1 with the 8.*

Therefore, if the informant shows the 7, it will be with the ace more times than with the 8; we have explored this reasoning in several questionnaires (see Tubau, 2008).

Experiment

A version of the problem was designed (see Appendix for details of the game) and the experiment was separated among 3 conditions; they were administered at different consecutive stages.

The aim of Condition A is to see if, in order to overcome the illusory inference, it is sufficient for the participants to be given a "Help-sentence" that shows to them the ways in which the informant might perform (following the line suggested by Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni (1999), Girotto and González (2005)); this condition wants to see if, having only been given part of the necessary information, they are able to deduce the rest. With this aim, as well as the questions that are normally administered –Decision and Probability-, a "Previous question" was included, with the aim of confirming the most immediate effect of such a "Help-sentence". If the presentation of possibilities is sufficient, there will be global progress in the results.

In the conditions B and C, a "Double question" was designed in order to help participants to answer correctly the dilemma and asking them for a justification for their responses. If the aid of the Help-sentence and the Double question allow the use of reasoning that becomes explicit, then the justification will be clear and complete enough and the results solid; if the help is basically associative (such as Schul and Mayo's 2003 comment: "it might not be surprising that the experiential mode is used for processing highly complex narratives"), then the results will be unclear and the quality of the justifications insufficient.

Method

Material. See Appendix.

In condition B, question "(a)" will always be correctly answered if the statement was correctly comprehended. Question "(b)", which in fact already implies the illusory inference, is the one that theoretically could imply serious problems; more specifically, it could be similarly difficult to the two questions (Decision and Probability) that finally were considered in order to evaluate the correct resolution of the dilemma.

In Condition C, instead of the Help sentence, the complete explanation given for participants in condition B was included: i.e., the correct response, followed by the connective 'because' (there is plenty of literature supporting its help and processing[1]), and followed by an explanation of the different ways in which the informant could behave: "John is more likely to be hiding the ace because if John has the ace and the 7, he will always show the 7, if he has the ace and the 8 he will always show the 8, and if he has the 7 and the 8 he will show half of the time the 7 and half of the time the 8."

Participants. 22 students of the Universitat of Barcelona participated in exchange for course credit (one of the participants was eliminated due to previous familiarity with the problem); in condition B, 52 students; in condition C, 58 students of the University of Balearic Islands participated.

Procedure. The texts / questionnaires were administrated to groups of 4 participants. The time taken to complete the questionnaire was 15 minutes (none of the participants took any longer). It was emphasised that they should read the sentences with special attention.

[1] In relation to the processing of causal inferences in expository texts, see Noordman, Vonk, & Kempff (1992) and Singer, Harkness, & Stewart (1997).

Results

Analyses of switches

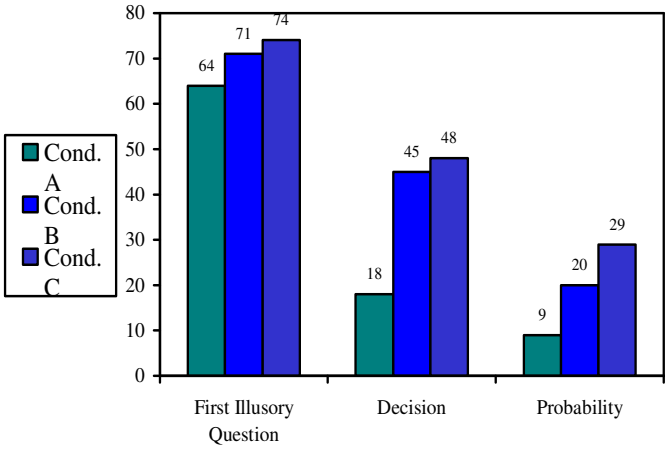

Figure 1: Percentage of participants who reasoned correctly in Conditions A, B and C. The 'First illusory question' is, in Condition A, the Previous question and, in Conditions B and C, the second part of the Double question.

In the condition A, in the questions of Decision and of Probability, participants switched their choices 18% and 9% of the time respectively. Looking at the results obtained in the Previous question (First illusory question), significant differences between the Previous question and the question of Decision (64% vs. 18%; x^2 (1, N=44) = 9.40, p<.005) were found; there was no significant difference between the questions of Decision and Probability.

The difference between the questions of condition A and condition B was significant in the Decision question (18% vs. 45%; x^2 (1, N=73) = 4.78, p<.05); it was significant neither in the Probability question nor in the First illusory question.

In the condition B, by seeing the difference between questions, significant differences were found between the First illusory question and the Decision question (71% vs. 45%; x^2 (1, N=103) = 7.19, p<.01); significant differences comparing the Decision question and the Probability question were also found (45% vs. 20%; x^2 (1, N=102) = 7.57, p<.01). In the condition C, looking at the differences between questions, there were obtained 48% and 29% of switches (correct response) in the questions of Decision and of Probability, respectively; there were significant differences (48% vs. 29%; x^2 (1, N=112) = 4.57, p<.05); in the First illusory question, significant differences between this question and Decision were found (74% vs. 48%; x^2 (1, N=114) = 8.08, p<.005).

Looking at the differences between conditions for a same question, significant differences in the Decision question between condition A and condition C were also found (18% vs. 48%; x^2 (1, N=78) = 5.95, p<.01); by contrast to condition B, almost significant differences in the Probability question between condition A and condition C (29% vs. 9%; x^2 (1, N=78) = 3.38, p=.06) were found.

Analyses of justifications

Most of the participants (60% of the 53 participants) answered, and even justified, correctly the Double question (i.e., answered correctly both parts of the Double question). In condition C, again most of the participants (74% of the 58 participants) answered the Double question correctly, and justified it correctly.

The answers that contained a verbal justification were analysed and classified by the following criteria:

Table 2. Classification of the justifications of the participants (conditions B and C)

Types of justification[2]	Examples of justifications (of the participants)
C: Repetition of the consideration of the *Cases* of the Help-sentence	"It's more probable that John shows the ace because if he has the ace and the 7 he always shows the 7 [and if he has the 7 and the 8 he only shows it half the time]"
N: Consideration (explicit) of a consecutive *Number* of trials	"Because if they play several times and always show the 7 it would be that hides the ace."
E: Double sense (*Equivalency*) of the relationship between the cards that John has and the card that he shows	"If John shows the 7 it's more probable that he has the ace because if John has the ace and the 7 it's more probable that shows the 7." "Because the 7 and ace always go together."
S: Invariability of the probabilities of the initial *Sets*	"Because John always has 2 cards"
H: John *Hides* the ace.	"The ace, because he is forced to hide it." "Because he hides the 8 only half of the time whereas he always hides the ace."

[2] Considering only the correct responses ('Inc' expresses 'Incorrect Responses')

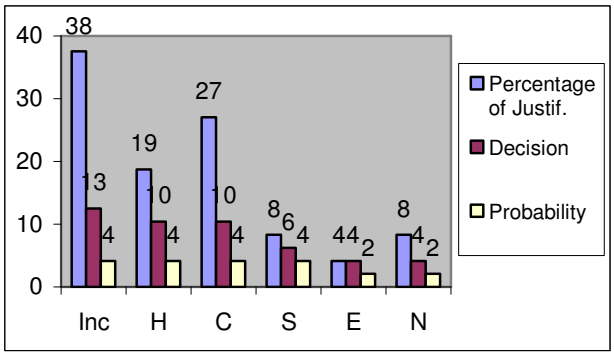

Figure 2. Type of justification and correct responses (%) in the Decision and Probability questions for Condition B

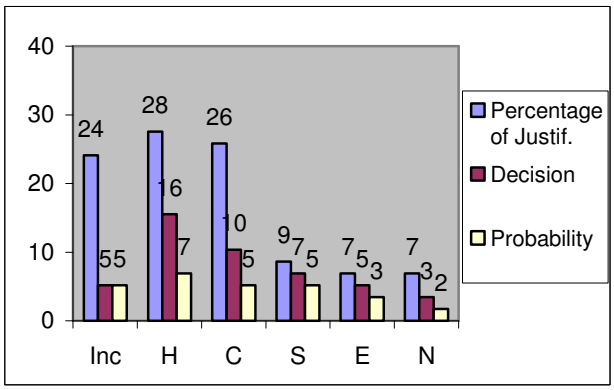

Figure 3. Type of justification and correct responses (%) in the Decision and Probability questions for Condition C

Comparing the justifications of the conditions B and C, there was an increase in the condition C of the type of justifications 'H' (not being significant). There was difference between the type of justification and correct responses to the 'Decision' and 'Probability' questions, being the correctness of both the Decision and the Probability questions more predictable from the type of justifications "S" (perhaps more creative) and "E".

Discussion

As was expected, in the Decision question (in which the participant was only given the option of switching or sticking with the initially selected card), better results than in the Probability question (in which the exact probabilities were also asked) were found in both the B and C conditions (following the tendency that there is more difficulty expressing the answers by exact numbers rather than relative probabilities).

In the condition A, it can observed that the participants' knowledge of the possibilities was not sufficient for the task, despite being complete (i.e., differentiating the possibilities as a function of the informant's behavior); having an understanding of the possibilities of performance of the informant was not sufficient for the participants to answer the questions of Decision and Probability correctly. Nevertheless it has been seen that they do answer correctly the Previous question.

The results show a surprisingly high percentage of switches in the Previous question and a strong decrease in the later questions, despite all of them inviting the same illusory inference. In condition C, the results, as the increase of the justifications of type "H" shows, seem to confirm the strength of the illusion and they seem to confirm the associative help of the Help-sentence and the Double question.

How do the participants use the information from the Help-sentence to answer the Previous question (condition A) correctly? Let us note the possibility that, taking in account the statement of the Help-sentence and the Previous question, most of the participants had highly activated in memory the pair "ace, 7", in such a way that it facilitated their answering of the Previous question. However, taking into account the increase in percentage of switches in the later questions in comparison to the classical versions (with results lower than the 10% and the 5% in the questions of Decision and Probability, respectively), it may be that there is a more complex associative effect that could stimulate a more completed elaboration of the Help-sentence.

In condition B and C, looking at the classification of the justifications, it can observed that what most of the participants do (approximately 60% -comprised of the "C", "H" and "E" types of justification) is literally repeat the consideration of the cases they got from the Help-sentence. It may be interesting to focus in on these justifications (the repetition of the help sentence), because they were the most common ones.

Analysing the *quality* of the justifications that the participants gave for their answers to the illusory question that they answered correctly, it can be seen that their justifications were mostly incomplete and insufficient. This may be for several reasons:
(1) a possible logical formalisation of the justification shows that the implied reasoning is complex enough to need a more elaborate explanation[3];
(2) the low results obtained when the *explanans*, which gives the explanation, appears alone (the description of the possibilities; condition A);

[3] One possible formalisation of what is said by the participants could be:
Let H_1 = <have ace and 7>, and H_2 = <have 7 and 8> be two exclusive events (with the probability of the intersection of both events being zero) which are equally probable. Then, we have the following relationship:
$p(D|H_1) > p(D|H_2)$ => $p(H_1|D) > p(H_2|D)$.

(3) the inconsistency in the results (high percentage of correct choices in the first question, then falling on the following questions –though still implying the same illusory inference-);
(4) one would expect that the participants considered, on a consecutive number of trials, the sum of the cases and the different preeminence of the ace in each of them (see Table 1).

The line of reasoning that they seem to take is: "if whenever John has the ace and the 7, he shows the 7 (and only half of the times when he has the 7 and the 8), this implies that, if John shows the 7, it is more likely that he will have the ace and the 7 (than the 7 and the 8)". Such reasoning is, in this case, correct (it's true because the previous probabilities are equal), but it is not obvious.

In what follows, from the observation of the justifications, two possible ways will be commented in which the participants who literally repeat the hint from the Help-sentence could be reasoning in order to correctly answer the first illusory question and yet keep their correct answer until the end of the questionnaire.

(1) Bias between <showing the 7> and <having the ace and the 7>.

The double question, together with the Help-sentence, can stimulate a bias or association between <showing the 7> and <having the 7 and the ace>, which can be called an associative or "matching bias" (Evans, 2003). Especially in Experiment 2, with the Double question, the way the questions are formulated might lead the participant to see (in a associative form) *a double sense in the relation of probabilities* appearing in the help texts, in such a way that they don't merely understand that (a)" If John has the ace and the 7 is more likely that he shows the 7" but also that they *affirm* the inverse sense, (b) "if John shows the 7, it's more likely that he has the ace and the 7 (than the 7 and 8)". Another way (see Tubau, 2008 for a graphic schema) to express the double sense would be:

Table 3

Initial sentence	Inverse sense
"if John has ace and 7, it is more probable that he shows the 7 "	"if John shows 7, it is more probable that he has ace and 7 "
"if John has 7 and 8, it is less probable that he shows the 7 "	"if John shows 7, it is less probable that he has 7 and 8"

(2) Consideration of the sum of the cases.

The help sentences can stimulate that, thanks to the explanation of the different possibilities of the informant, a generic sum of cases is presupposed in which they observe that, most of the times, it's shown the ace (see Table 1); see also Gigerenzer & Hoffrage (1995) for a similar argument.

These types of reasoning may be processed by means of numbers, but may be also processed in a more abstract or imaginistic form (Kosslyn, 1980) using, for example, graphic schemata.

General discussion

In future experiments these kinds of representations using protocol analyses could be explored with a delay, in different conditions, of a few hours and two days; a complementary methodology of task recognition could be used; it seems difficult to choose, however, which could be the best item (or sentence) to present in the recognition task. It could be interesting also to explore if some analogous problems can be found in which there was a similar probabilistic structure and where it could be see which are the justifications given for the participants. Finally, the role of WM capacity in the kind of justifications (type and correctness) could be explored; following the suggestion of two systems of reasoning (Evans, 2003), it could be explored whether the WM capacity have any influence so that participants have access to any sort of heuristic and begin to justify incorrectly (because of its bad application) the second part of the Double question.

ANNEX

Questionnaires

"Problem 3 cards"

Mary and John have 3 cards: the ace, 7 and 8 of diamonds and they play to the following game: Mary chooses a card and keeps it without seeing it. John keeps the other two.

Imagine that John has to show one card apart from the ace to Mary. So, John has to inspect his cards and show one non-ace card to Mary.

[5]If John has the ace and the 7, he will always show the 7, if he has the ace and the 8 he will always show the 8 and if he has the 7 and the 8 he will show half of the time the 7 and half of the time the 8.

[6]a. Who is more probable that hides has the ace? / Who is more likely to hide the ace? b. Why?

[7]a. When is it more likely that John shows the 7: (1) when he has the ace and 7 or (2) when he has 7 and 8?

b. If John shows a card with the 7, which card will he most likely be hiding? The ace or the 8?

Justify your response to the question b.

Once John has showed a card that is not the ace, Mary can choose between sticking with her initial card (which she has still not seen) and changing it for the one that John is still hiding.

[8]What should Mary do if she wants to draw the ace as many times as possible? Why?

a) Switch the card b) Stick with her initial card c) Either of the above. The probabilities are the same.

[9]a) If Mary sticks with the initial card, what is the probability of her having the ace? b) If Mary switches the card, what is the probability of her having the ace? c) Why?

[5] Help-sentence: Phrase that explains the different possibilities of the informant

[6] The "Previous Question" is only presented in Condition A (it would be the 1st Illusory Question).

[7] The "Double question" is only presented in Conditions B and C (the 1st Illusory Question would be the second part, (b), of the Double question)

[8]DECISION

[9]PROBABILITY

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Acknowledgments

I am grateful to Elisabet Tubau; I am also grateful to the anonymous reviewers. Preparation of this manuscript was supported by the 'Programme for scholarship of further study abroad "Sa Nostra"'.