The Psychology of Su Doku Problems

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Introduction

Three experiments addressed the questions: what tactics do individuals use to solve Su Doku puzzles, and what determines the difficulty of using these tactics? A typical Su Doku consists of an array of 9x9 cells; the array is divided into nine boxes of 3x3 cells. Some of the cells in the array already contain digits. The task is to fill in all the empty cells so that each row, column, and box contains each of the digits 1 to 9 once and only once. The puzzles can be solved using pure deduction.

Both simple and advanced tactics exist for Su Dokus, and Figure 1 illustrates them. Simple tactics call for one-step deductions of definite digits. X must be 5, because 5 is already in the columns and row that intersect all the empty cells in the box containing X. Advanced tactics call for multi-step deductions of both possible and definite digits, and the listing of possible digits for cells. The only possible digits for Y are 4, 6, 9, but the only possible digits for the two empty cells in the row above Y are 6, 9. Hence, Y cannot be either of these two digits, and so it must be 4.

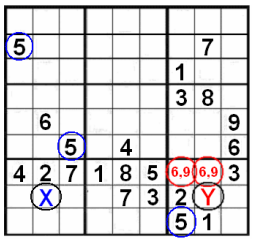

Figure 1: An illustration of a simple and an advanced tactic.

Experiment 1

Experiment 1 tested whether naïve individuals were more likely to use simple tactics in which they deduced definite digits, as opposed to advanced tactics in which they deduced possible digits. Ten participants had to infer the values of as many empty cells as possible in 15 minutes on each of three Su Dokus. They solved 81% of digits with simple tactics, but only 7% of digits with advanced tactics. This difference was reliable (Wilcoxon's T=0.0, z=3.5, p<<.001).

Experiment 2

Simple tactics differ in difficulty. To find X in Figure 1, you need to consider four constraints: the row below it, its two neighboring columns, and its box. It is possible to manipulate the number of constraints on the value of a digit, i.e., its *relational complexity* (RC, see Halford, Wilson, & Philips, 1998). The higher a cell's RC, the harder it should be to infer its value. Eighteen participants had four minutes to find the value of target cells with RC's of 2, 3, 4, and 5. The results supported the prediction: the percentages of correct solutions were 74, 61, 47, and 58, respectively (Page's L=4330, z=3.1, p<<.001), and the latencies showed a similar trend. The slight increase in performance for an RC value of 5 is probably attributable to salience of four digits of the same value in the array.

Experiment 3

Advanced tactics call for the inference of possible digits, and such digits also differ in RC. Hence, it is also possible to test the difficulty of advanced tactics as a function of RC. Ten participants had four minutes to infer the value of each of three target cells that could be solved only by using three sorts of advanced tactics with varying RC (low/medium/high). The percentages correct were 70, 30, and 25, respectively (Wilcoxon's T=7.0, z=2.1, p<.025), and the corresponding latencies showed the same trend.

Discussion

Experiment 1 showed that naïve individuals used simple tactics when they first tackled Su Dokus, but with experience, they do develop more advanced tactics. This development is contrary to theories that posit just a single deterministic strategy for deductive reasoning (e.g., Rips, 1994). Experiments 2 and 3 showed that relational complexity is likely to account for the difficulty of both simple and advanced tactics. More broadly, Su Dokus show that individuals are capable of making deductions without explicit instruction to do so (cf. Evans & Over, 1996). They can make deductions from multiply quantified premises, and do so without relying on probabilities (cf. Oaksford & Chater, 1998).

References

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