Evidence for the Cognitive Role of Space on an Algebraic Production Task

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Introduction

Landy & Goldstone (submitted) argue that although sentential notations are traditionally contrasted with diagrammatic notations (e.g., Stenning, 2002), syntax in algebra may be directly interpreted through perceptual grouping mechanisms (Koffka, 1935). However, the effect of grouping on syntax perception may be a purely "front-end" phenomenon, affecting performance but irrelevant to underlying mathematical competence.

We tested this possibility by exploring spontaneous grouping in a production task. If perceptual groupings formed a deep part of syntax, reasoners might replicate that grouping in physical space when producing equations. We predict, therefore that terms around equals signs will be more widely spaced than around operators, and that spaces around additions will be larger than those around multiplications, *when both appear in the same expression*.

Experiment & Results

39 participants read a page of word equations and were asked to write the corresponding Arabic-numeral equations. Physical spacing was not mentioned. Participants were not asked to compute any actual values, only to write the expressions. Half of the 32 expressions contained only one operator type (+ or *), half contained both + and * on one side of an equality. Spaces between the insides of the numerals on either side of +, *, and = signs were measured.

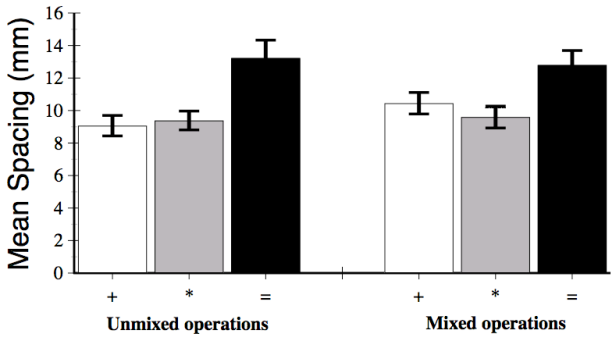

Figure 1: Results from the experiment. Participants produced wide spaces around equals signs. The interaction between addition/multiplication signs and operator 'mixing' is also significant, as can be seen in figure 2.

According to a t-test, '=' spacings were larger than '+' spaces ($t=-5.61$, $p<0.0001$). By a 2-way within-participant ANOVA, there was a significant effect of operator structure (mixed vs. single-op; $F(1,890)=22.1$, $p<0.001$), and a significant interaction between operator structure and operator ($F(1,890) = 21.39$, $p<0.001$).

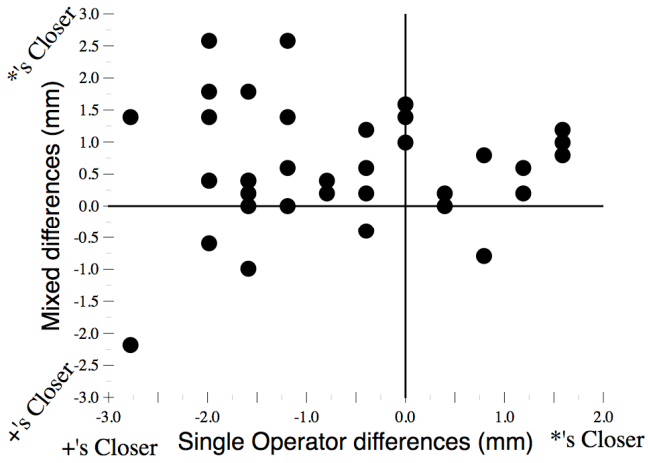

Figure 2: Difference between addition and multiplication spacings on single-operator (vertical axis) and mixed-operator (horizontal axis) equations.

Conclusions

Spacing productions were consistent with the possibility that grouping plays a deep cognitive role in mathematical syntax processing, and are inconsistent with an account that attributes spacing effects to a perceptual 'front-end'. Together with evidence that order of precedence is processed more easily when grouping and syntax are aligned, these result indicate that physical and semantic groups are aligned throughout mathematical processing.

Acknowledgments

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References

Koffka, K (1935). *Principles of Gestalt Psychology*. New York: Harcourt Brace Jovanovich.

Landy, D. & Goldstone, R. (submitted). How abstract is symbolic thought?

Stenning, K (2002). *Seeing Reason*: image and language in learning how to think. Oxford University Press: Oxford.