Queuing Network Modeling of Reaction Time, Response Accuracy, and Stimulus-Lateralized Readiness Potential Onset Time in a Dual Task

Changxu Wu (changxuw@umich.edu)
Department of Industrial and Operations Engineering, University of Michigan,
1205 Beal Avenue, Ann Arbor, MI 48105 USA

Yili Liu (yililiu@umich.edu)
Department of Industrial and Operations Engineering, University of Michigan,
1205 Beal Avenue, Ann Arbor, MI 48105 USA

Abstract

Reaction time, response accuracy and psychophysiological measures such as lateralized readiness potential (LRP) have been used extensively to study information processing in dual tasks. To model these three dependent variables in a dual task, we propose a new mathematical modeling approach—a queuing network approach based on queuing network theory of human performance (Liu, 1996, 1997) and current discoveries in neuroimaging studies. This modeling approach is composed of a queuing network architecture of information processing in the brain and a set of mathematical equations in quantifying the three dependent variables in the dual task. This modeling approach can be used to account for information processing in both spatial and temporal dimensions and it provides a coherent and quantitative linkage between the neural signals (LRP) and behavioral data in the dual task. Despite its relative simplicity, this queuing network modeling approach is useful to quantify and predict behavioral performance and important aspects of the macroscopic electrical activity of the brain. Further development and extension of the current modeling approach are discussed.
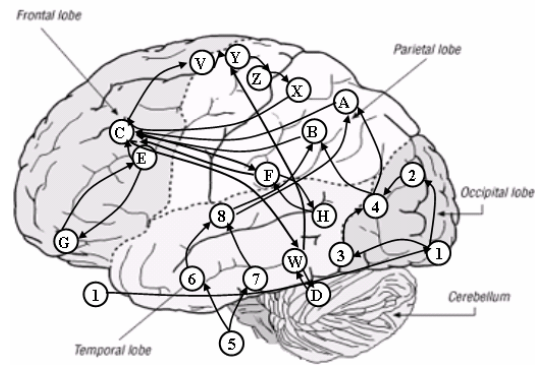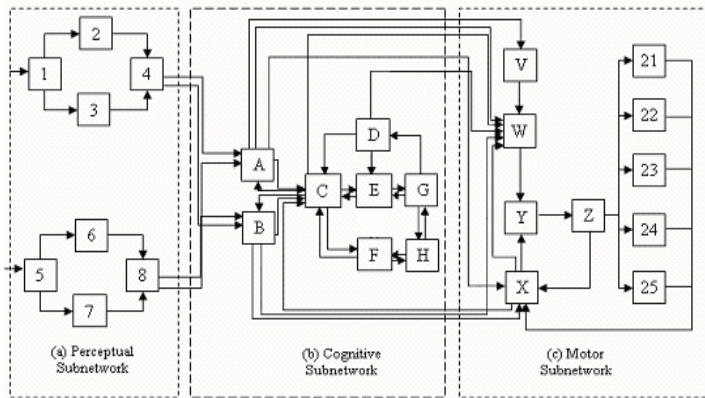
Introduction

With the development of electrophysiological techniques, there is an emerging body of experimental studies which measure reaction times (RT), response accuracy, and event-related potentials (ERP) simultaneously to study one of the basic questions in cognitive science—human cognition in dual task situations (Sangals, Ross, & Sommer, 2004; Sommer, Leuthold, & Schubert, 2001). Compared to the traditional behavioral experimental studies, these studies are able to provide more information about the temporal properties of cognitive processes, reflecting basic cognitive information processing in the dual task. Among various ERP techniques, a brain potential extracted from ERP—lateralized readiness potential (LRP), has become a useful and powerful supplemental measurement to reaction time and response accuracy, since it not only shares ERP's excellent temporal resolution but also reflects the information processing in the brain beyond the measurements of overt behavior.

LRP measures response activation or preparation at the level of the cerebral motor cortex (Coles, 1996). More specifically, LRP reflects motor preparation taking place within the premotor area or the primary motor cortex (H. Leuthold & Jentzsch, 2001; Ulrich, Leuthold, & Sommer, 1998). The start of this motor preparation is reflected by the LRP onset time, which is identified as the most important aspect of the potential (Mordkoff & Gianaros, 2000). A frequently-used onset time of LRP is called stimulus-synchronized LRP (S-LRP onset time or S-LRP interval). S-LRP onset time is measured as the interval between the arrival of stimulus and the emergence of LRP (negative going potentials), reflecting premotoric processes (Sommer et al., 2001).

Besides the experimental studies of LRP and other dependent variables, several mathematical models have been successfully established, focusing on modeling ERP/EEG: building on a lumped-parameter model, Jansen and Rit (1995) developed a computational model to produce EEG rhythms. Based on Jansen and Rit's model, a neural mass model (David & Friston, 2003) assumes that the behavior of a population of neurons can be approximated using several state variables (e.g. firing rates). The model reproduces brain signals within the oscillatory regime by simply changing population kinetics. However, few mathematical models have been built to model all of the three dependent variables simultaneously and quantify them in dual task situations.

In this paper, we propose a queuing network modeling approach as a new mathematical modeling method to quantify reaction time, response accuracy and S-LRP onset time simultaneously in dual task situations. First, we introduce the platform of this modeling approach—a queuing network architecture of information processing in the brain, representing the major brain regions and their connections as a network. Second, based on this network platform, a set of mathematical equations is developed to quantify the three dependent variables. Third, the modeling results are presented and validated with the results in an experimental study. Finally, we discuss the implication of the modeling approach and its further extensions to model the experimental results of other electrophysiological studies.

Figure 1: The general structure of the queuing network model (function of each server and corresponding brain areas)

Perceptual Subnetwork	Cognitive Subnetwork	Motor Subnetwork
1. Common visual processing (eyes, lateral geniculate nucleus, superior colliculus, primary and secondary visual cortex) 2. Visual recognition (dorsal system) 3. Visual location (ventral system) 4. Visual recognition and location integration (distributed parallel area including the connections among V3 , V4 and V5, superior frontal sulcus, and inferior frontal gyrus) 5. Common auditory processing (middle and inner ear) 6. Auditory recognition (area from dorsal and ventral cochlear nuclei to the inferior colliculus) 7. Auditory location (area from ventral cochlear nucleus to the superior olivary complex) 8. Auditory recognition and location integration (primary auditory cortex and planum temporale)	A. Visuospatial sketchpad (right-hemisphere posterior parietal cortex) B. Phonological loop (left-hemisphere posterior parietal cortex) C. Central executive (dorsolateral prefrontal cortex (DLPFC), anterior-dorsal prefrontal cortex (ADPFC) and middle frontal gyrus (GFm)) D. Long-term procedural memory (striatal and cerebellar systems) E. Performance monitor (anterior cingulate cortex) F. Complex cognitive function: decision, calculation, anticipation of stimulus in simple reaction etc. (intraparietal sulcus (IPS), the superior frontal gyrus (SFS), the inferior frontal gyrus (GFi), the inferior parietal cortex and the ventrolateral frontal cortex, the intraparietal sulcus and the superior parietal gyrus) G. Goal initiation (orbitofrontal region and amygdala complex) H. Long-term declarative & spatial memory (hippocampus and diencephalons)	V. Sensorimotor integration (premotor cortex) W. Motor program retrieval (basal ganglia) X. Feedback information collection (somosensoy cortex) Y. Motor program assembling and error detecting (supplementary motor area (SMA) and the pre-SMA) Z. Sending information to body parts (primary motor cortex) 21-25: Body parts: eye, mouth, left hand, right hand, foot

Queuing Network Architecture

To model human performance and electrophysiological signal of the brain, the queuing network modeling approach regards the human cognition system as a queuing network based on several similarities between them. First, ample research evidence has shown that major brain areas with certain information processing functions are localized and connected with each other in the brain cortex via neural pathways (Bear & Connor, 2001; Smith et al., 1998; Roland, 1993; Faw, 2003), which is highly similar to a queuing network of servers that can process entities traveling through the routes serially or/and in parallel depending on specific network arrangements. Therefore, brain regions with similar functions can be regarded as servers and neural pathways connecting them are treated as routes in the queuing network (see Figure 1). Second, it has been discovered that information processed in the brain are coded in spike trains (Rieke, Warland, R.S., & Bialek, 1997); depending on different tasks and learning stages, the to-be-processed information represented by these spike trains sometimes are processed by the brain regions (servers) immediately; sometimes they have to be maintained in certain regions to wait for the previous spike trains being processed (E. E. Smith & Jonides, 1998; Taylor et al., 2000). Hence, these spike trains can be represented as entities in the queuing network naturally and entities are processed in the network by certain queuing process as an analogy to represent the waiting and maintaining process of spike trains.

In modeling human performance, computational models based on queuing networks have successfully integrated a large number of mathematical models in response time (Liu, 1996) and in multitask performance (Liu, 1997) as special cases of queuing networks. Queuing network modeling approach has been successfully used to generate human behavior in real time, including simple and choice reaction time, driver performance and transcription typing (Liu, Feyen & Tsimhoni, in press; Wu & Liu, 2004a).

In modeling brain imaging pattern, previous work in queuing network modeling was focused on modeling the dynamic connectivities among brain regions. Wu and Liu (2004b) successfully modeled how brain imagining patterns change with different learning stages and different stimuli to be processed. These connectivities of brain regions were modeled as dynamic changes of routing probability

(probability of entities enter one of multiple routes) in the queuing network during the learning process.

Modeling of the Dependent Variables

In the following we describe our use of the queuing network modeling approach to model reaction time, response accuracy, and S-LRP in a representative dual task experimental study. To model this task, it is necessary to determine the route of entities in the network first since all tasks do not necessarily activate the same network routes. Reaction time can be modeled by the time for the entities to traverse through the routes; response accuracy and the S-LRP onset time are modeled by the processing of entities in the network.

Target Experiment to be Modeled

Among the dual task studies which measured the 3 dependent variables, a representative experimental study (Sommer et al., 2001) is selected to be modeled. In their experiment, task 1 (T1) was an auditory-manual choice reaction task: low and high tones were presented to the subjects who were asked to make manual responses with their index or middle fingers of left and right hand. Task 2 (T2) was a visual-manual reaction task including two conditions: one condition was a two-choice reaction task—corresponding to letter "X" or "O", subjects made manual responses with the middle or index fingers of the left and right hand; the other condition was a simple reaction task in which letters required a response with one of the fingers. The delay between the presentation of the stimulus of T1 and T2 is called stimulus onset asynchrony (SOA).

Route of Entities

The route of entities in the queuing network is determined based on previous queuing network modeling work in modeling the connectivity of brain regions (Wu and Liu, 2004b): in general, depending on the task to be performed, servers whose function is related to the target task are included in the route of entities. In task 1, entities representing the auditory stimulus enter the auditory perceptual subnetwork first (server 5->6/7->8) (see Figure 1); then, they are transmitted into the cognitive subnetwork including server B, C, and F for making the phonological judgment. After that, they travel to the motor subnetwork (server W, Y, Z and hand server) for retrieving motor programs, assembling the motor programs, and initiating the motor response. According to the functions and connections of these brain regions, the route of task 1 and 2 are:
T1: 5 ->6/7->8-> B-> C->F->C->W->Y->Z-> Hand
T2: 1 ->2/3->4-> A-> C->F->C->W->Y->Z-> Hand

In the simple reaction condition of task 2, route of entities is selected according to the functions of the severs and the physiological study of Kansaku et al. (2004):
1 ->2/3->4->A->C->F->C->V->Z-> Hand

Mathematical Modeling of Reaction Time

Choice Reaction Time Independent of the SOA conditions, the response time of T1 (RT1) can be predicted by the sum

of servers' processing time in the route of entities of T1 since no previous entities occupy any of the servers in the route (see T1 in Figure 2 and Equation 1).
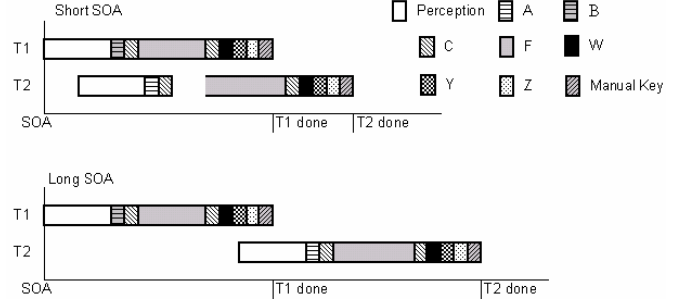

Figure 2: Illustration of the RT1 and RT2 (choice reaction time)

$$E(RT1)=T_{1,AP}+T_{1,B}+T_{1,C}+T_{1,F}+T_{1,C}+T_{1,Y}+T_{1,W}+T_{1,Z}+T_{1,K} \quad (1)$$

where, $T_{1,AP}$ is the processing time of the auditory perceptual subnetwork; $T_{1,B}$, $T_{1,C}$, $T_{1,F}$, $T_{1,Y}$, $T_{1,W}$, $T_{1,Z}$, and $T_{1,K}$ represents the processing time of server B, C, F, Y, W, Z and Hand, respectively.

In the choice reaction condition, the response time of T2 (RT2) depends on the comparison between a) the difference between SOA and the time point when entities of T1 exit server F ($T_{1,AP}+ T_{1,B} +T_{1,C} +T_{1,F} -SOA$) and b) the duration of the processing time before entities of T2 enter server F (the sum of processing time at the perceptual subnetwork, server B and C, i.e. $T_{2,VP}+T_{2,A}+T_{2,C}$) (see Equation 2)

$$E(RT2) (choice\ reaction)=$$
$$max(T_{1,AP}+T_{1,B}+T_{1,C} +T_{1,F} -SOA,\ T_{2,VP}+T_{2,A}+T_{2,C}) \quad (2)$$
$$+ T_{2,F}+ T_{2,C}+ T_{2,Y}+ T_{2,W}+ T_{2,Z}+ T_{2,K}$$

Simple Reaction Time RT2 in simple reaction condition is modeled in Appendix 1 (see Equation 6 and Equation 16 in Appendix 1).

Mathematical Modeling of Response Accuracy

The expected response accuracy (P_c) is estimated according to the difference between SOA and the sum of $T_{2,C}$, $T_{2,V}$, $T_{2,Z}$, $T_{2,K}$, and T_{Fst} (see Appendix 2).

Mathematical Modeling of S-LRP

Simple Reaction Time Since LRP reflects motor preparation within the premotor area (server V) or the primary motor cortex (server Z) and server V is in the route of simple reaction task, the arrival time of entities into server V is regarded as the LRP onset time in this simple reaction time situation. Based on Figure 6 in Appendix, the time that entities enter server V (V_{st}) can be estimated in two conditions depending on the value of t_a (t_a= 0, short SOA conditions in Figure 6; t_a >0, long SOA conditions in Figure 6, see Equation 3)

$$V_{st}= \begin{cases} Max(T_{1,AP} + T_{1,B} +T_{1,C} +T_{1,F}, & t_a=0 \\ \quad SOA+ T_{2,VP}+T_{2,A}+T_{2,C})+T_{2,F}+T_{2,C} & \\ & \\ T_{Fst} +t_a+T_{2,C} & t_a>0 \end{cases} \quad (3)$$

Since V_{st} starts from the arrival of S1 and S-LRP onset time (S-LRP) starts from the arrival of S2 (Sommer et al., 2001), S-LRP equals V_{st} - SOA, i.e.:

$$S\text{-}LRP = \begin{cases} Max(T_{1,AP}+T_{1,B}+T_{1,C,}+T_{1,F,} & t_a=0 \\ SOA+ T_{2,VP} +T_{2,A}+T_{2,C})+T_{2,F}+T_{2,C}\text{-}SOA & \\ T_{Fst} +t_a+T_{2,C}\text{-}SOA & t_a>0 \end{cases} \quad (4)$$

Choice Reaction Time LRP reflects motor preparation in the regions represented by server V or Z and server Z is in the route of the choice reaction time condition of T2, therefore, the arrival time of entities at server Z is the expected S-LRP onset time (*S-LRP*) (see Equation 5).

$$S\text{-}LRP = max(T_{1,AP}+T_{1,B}+T_{1,C} +T_{1,F} -SOA, T_{2,VP} \quad (5)$$
$$+T_{2,A}+T_{2,C}) + T_{2,F}+ T_{2,C}+ T_{2,Y}+ T_{2,W}$$

Modeling Results and its Validation

Using the equations derived in the previous sections, the predicted results of the dependent variables are presented and validated with the target experiment results. The value of parameters of these equations is set based on a classic cognitive modeling study (Byrne & Anderson, 2001) (see Appendix 3).

Figure 3 showed the modeling results in comparison with experimental results. The R square of the model is .84 and the RMS=53.9 ms.
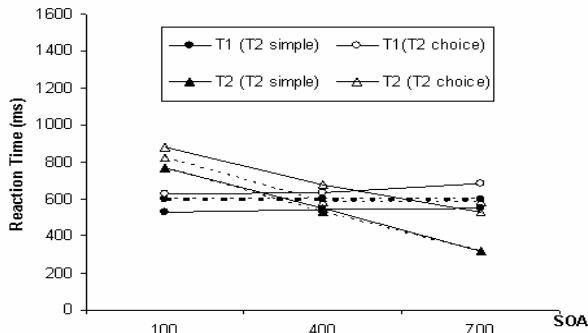

Figure 3: The reaction time in the study of Sommer et al. (2001) (solid lines) along with the queuing network modeling results (dashed lines)

The modeling results of response accuracy in comparison with the experiment result are shown in Figure 4. The R square between the prediction and the experiment result is 0.99 with RMS=.037. Moreover, it is found that at SOA=700 ms, the percentage of negative RT2 is 15% which is consistent with the Sommer et al.'s (2001) experimental results (16%).
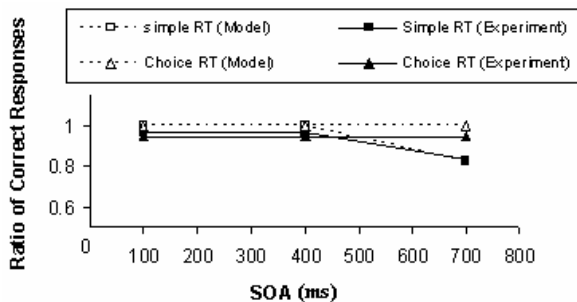

Figure 4: Response accuracy in the study of Sommer et al. (2001) (solid lines) along with the queuing network modeling results (dashed lines)

In addition, the expected pattern of S-LRP exhibits the similar pattern with the experimental results (see Figure 5 for the comparison of the S-LRP onset time between the prediction of the model and the experimental results, R square=.96; RMS=127.5 ms).
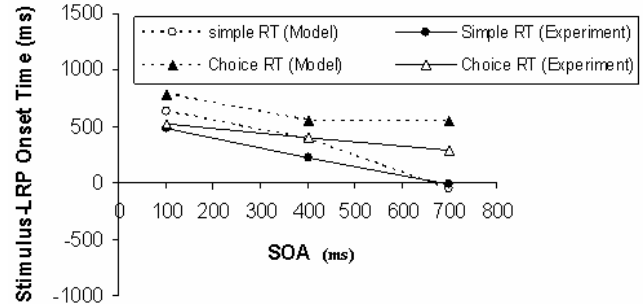

Figure 5: The S-LRP onset time in the study of Sommer et al. (2001) (solid lines) along with the queuing network modeling results (dashed lines)

Discussion

We described a queuing network modeling approach to model reaction time, response accuracy and S-LRP onset time simultaneously in the dual task situation. Based on the queuing network architecture as a platform, a set of mathematical formulas is built which successfully modeled the three dependent variables coherently with analytical solution and very few free parameters.

The queuing network modeling approach is able to model information processing in the brain in both spatial and temporal dimensions. This modeling approach incorporates the queuing network architecture which covers a wide range of brain regions. This feature provides the model a platform to predict fMRI BOLD signal (blood oxygenation level-dependent) of several major brain regions in fMRI studies (Wu & Liu, 2004b), reflecting the spatial location of information processing in the brain. Combining with the current work, this modeling approach has the potential to model experimental results of both fMRI studies with spatial accuracy and ERP studies with temporal accuracy.

The current work extends the advantages of this modeling approach in modeling reaction time (Liu, 1997) by unifying the neural signals and behavioral data. The model's prediction is not only consistent with the external behavior of the subjects, but also in line with the electrophysiological measurements. The current modeling approach provides at least an alternative way to quantify the external behavioral data and to some extent explain how they are generated by the internal information processing in the brain.

Moreover, this modeling approach provides a parsimonious and accurate quantification of the S-LRP onset time and behavioral data, since all of the dependent variables are modeled by the analytical solutions and only two free parameters are used in the modeling process.

We are extending the current model approach to model a wider range of behavioral and physiological measurements in experimental studies including P3 components in ERP.

Overall, the queuing network modeling approach is a useful modeling method to predict the behavioral performance and electrophysiological phenomena of the cognitive system.

Acknowledgments

Reference

Bear, M. F., Connors, B. W., & Paradiso, M. A. (2001). *Neuroscience: exploring the brain* (8th ed.). Baltimore, MD: Lippincott Williams & Wilkins.

Byrne, M. D., & Anderson, J. R. (2001). Serial modules in parallel: The psychological refractory period and perfect time-sharing. *Psychological Review, 108*(4), 847-869.

Coles, M. G. H. (1996). The lateralized readiness potential: Past, present, and future. *Psychophysiology, 33*, S3-S3.

David, O., & Friston, K. J. (2003). A neural mass model for MEG/EEG: coupling and neuronal dynamics. *Neuroimage, 20*, 1743-1755.

Eisler, H. (1976). Experiments on subjective duration1968-1975 : a collection of power function exponents. *Psychological Bulletin, 83*, 1154-1171.

Faw, B. (2003). Pre-frontal executive committee for perception, working memory, attention, long-term memory, motor control, and thinking: A tutorial review. *Consciousness and Cognition, 12*(1), 83-139.

Jansen, B. H., & Rit, V. G. (1995). Electroencephalogram and Visual-Evoked Potential Generation in a Mathematical-Model of Coupled Cortical Columns. *Biological Cybernetics, 73*(4), 357-366.

Kansaku, K., Hanakawa, T., Wu, T., & Hallett, M. (2004). A shared neural network for simple reaction time.*Neuroimage, 22*(2), 904-911.

Leuthold, H., & Jentzsch, I. (2001). Neural correlates of advance movement preparation: A dipole source analysis approach. *Cognitive Brain Research, 12*, 207-224.

Liu, Y., Feyen, R., & Tsimhoni, O. (in press). Queueing Network-Model Human Processor (QN-MHP): A Computational Architecture for Multitask Performance. *ACM Transaction on Human Computer Interaction*.

Liu, Y. L. (1996). Queuing network modeling of elementary mental processes. Psychological *Review, 103*(1), 116-136.

Liu, Y. L. (1997). Queuing network modeling of human performance of concurrent spatial and verbal tasks. *IEEE Transactions on Systems Man and Cybernetics Part a-Systems and Humans, 27*(2), 195-207.

Mordkoff, J. T., & Gianaros, P. J. (2000). Detecting the onset of the lateralized readiness potential: A comparison of available methods and procedures. *Psychophysiology, 37*(3), 349-360.

Rieke, F., Warland, D., R.S., R., & Bialek, W. (1997). *Spikes: Exploring the Neural Code (Computational Neuroscience)*: MIT Press.

Roland, P. E. (1993). *Brain activation*. New York, NY: Wiley-Liss.

Sangals, J., Ross, L., & Sommer, W. (2004). Time pressure effects on information processing in overlapping tasks: evidence from the lateralized readiness potential. *Acta Psychologica, 117*(3), 275-294.

Schubotz, R. I., & von Cramon, D. Y. (2001). Functional organization of the lateral premotor cortex: fMRI reveals different regions activated by anticipation of object properties, location and speed. *Cognitive Brain Research, 11*(1), 97-112.

Smith, E. E., & Jonides, J. (1998). Neuroimaging analyses of human working memory. *Proc. Natl. Acad. Sci. USA, 95*, 12061-12068.

Sommer, W., Leuthold, H., & Schubert, T. (2001). Multiple bottlenecks in information processing? An electrophysiological examination. *Psychonomic Bulletin & Review, 8*(1), 81-88.

Taylor, J., Horwitzc, B., Shaha, N. J., Fellenzb, W. A., Mueller-Gaertnera, H.-W., & Krausee, J. B. (2000). Decomposing memory: functional assignments and brain traffic in paired word associate learning. *Neural Networks, 13*, 923-940.

Ulrich, R., Leuthold, H., & Sommer, W. (1998). Motor programming of response force and movement direction. *Psychophysiology, 35*, 721-728.

Wearden, J. H., Edwards, H., Fakhri, M., & Percival, A. (1998). Why "Sounds Are Judged Longer Than Lights": Application of a Model of the Internal Clock in Humans. *The Quarterly Journal of Experimental Psychology, 51B*(2), 97-120.

Wu, C., & Liu, Y. (2004a). *Modeling human transcription typing with queuing network-model human processor.* Paper presented at the Proceedings of the 48th Annual Meeting of Human Factors and Ergonomics Society, New Orleans, Louisiana, USA.

Wu, C., & Liu, Y. (2004b). *Modeling Psychological Refractory Period (PRP) and Practice Effect on PRP with Queuing Networks and Reinforcement Learning Algorithms.* Paper presented at the Proceedings of the 6th International Conference on Cognitive Modeling (ICCM-2004), Pittsburgh, PA, USA.
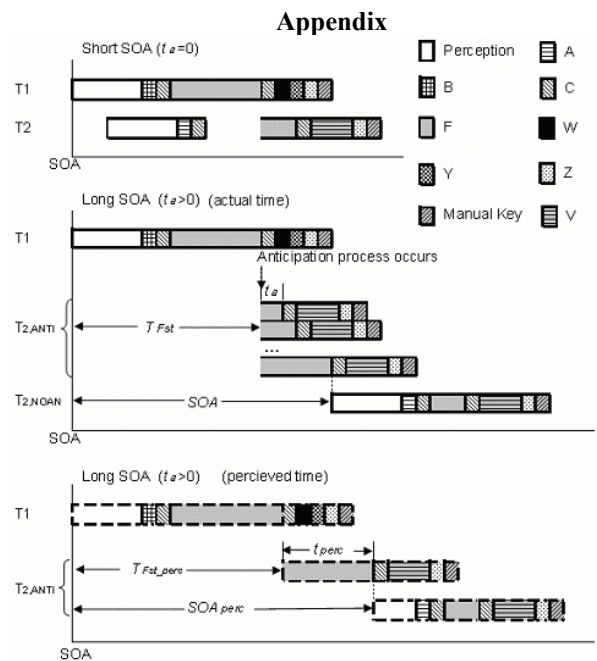
Appendix

Figure 6. Modeling mechanisms of the expected RT2 under the simple reaction time condition. t_a is the duration between when server F starts the anticipation process and when entities of S2 arrives at the perceptual subnetwork; T_{Fst} is the time point when server F starts its anticipation process $T_{Fst} = T_{1,AP} + T_{1,B} + T_{1,C} + T_{1,F}$.

1. Modeling of Expected RT2 (Simple Reaction Time)

There are two conditions in modeling the expected RT2 at the simple reaction time condition. At short SOA conditions (entities of T2 arrive at server F before server F starts its anticipation process, $t_a=0$, see Figure 6), entities of T2 have to wait until entities of T1 leave server F; after entities of T1 leave server F, entities of T2 will enter server F immediately. Since server F is occupied by the entities of T2 (subjects are busy in performing judgment of T2), the anticipation process is not occurred in this condition. At long SOA conditions, server F starts its anticipation process before entities of T2 arrive at server F. The mathematical models of RT2 (simple reaction condition) are constructed based on these two conditions in the following sections.

Short SOA Condition ($t_a=0$) Under the short SOA condition of T2 (simple reaction condition), the expected RT2 is also modeled with the same form of equation in the choice reaction condition of RT2 except the motor subnetwork's servers are replaced by the servers involved in the simple reaction time (see Equation 6 and Figure 6).

$$E(RT_2|t_a=0) = max(T_{1,AP}+T_{1,B}+T_{1,C}+T_{1,F}-SOA, T_{2,VP}+ \tag{6}$$
$$T_{1,A}+T_{2,C}) + T_{2,F}+ T_{2,C}+ T_{2,V}+ T_{2,Z}+ T_{2,K}$$

Long SOA Condition ($t_a>0$)

1) Quantification of the Anticipation Process

The anticipation process (R2 is made without seeing S2) at server F is quantified by the following mechanisms in time perception. According to the function of server F timing the perceptual anticipation of a sensory event in simple reaction time task (Schubotz, et al. 2001): the longer server F anticipating S2 (defined as perceived waiting time, t_{perc}), the higher probability (defined as p) to trig motor response without seeing S2 (anticipation process): $p = t_{perc}/T_{perc}$, where, T_{perc} is the duration between when the anticipation process starts and when the probability that subjects make the motor response equal to 1.

Based on several psychophysical researches in studying the relationship between perceived waiting time (t_{perc}) and actual waiting time (t_a) in very short time periods, there has been considerable support for a psychophysical law for perceptual duration described by a power function following the Steven's power law (Eisler, 1976). Thus,

$$t_{perc} = kt_a^{\beta} \tag{7}$$

where, t_a is the duration between when server F starts the anticipation process and when S2 arrives at the perceptual subnetwork. k and β are the parameters in Steven's power law (Wearden, et al., 1998). Since $p = t_{perc}/T_{perc}$, we have:

$$p = kt_a^{\beta}/T_{perc} \tag{8}$$

Moreover, since p is defined as the probability that the response of T2 is made with the anticipation process (R2 is made without seeing S2), there are two conditions in which expected RT2 is modeled: RT2 with or without the anticipation process.

2) Expected RT2 with the Anticipation Process ($RT_{2,ANTI}$)

Based on Figure 6, we have

$$RT_{2,ANTI} = T_{Fst} + t_a + T_{2,C} + T_{2,V} + T_{2,Z} + T_{2,K} - SOA \tag{9}$$

From Equation 7, t_a can be rewritten into:

$$t_a = (t_{perc}/k)^{1/\beta} \tag{10}$$

Moreover, since subjects end their waiting process of S2 when they perceive the time reaches the perceived SOA, the perceived the waiting time (t_{perc}) equals the perceived SOA (SOA_{perc}) minus the perceived T_{Fst} (T_{Fst_perc}), i.e.:

$$t_{perc} = max(SOA_{perc} - T_{Fst\_perc}, 0) \tag{11}$$

where, SOA_{perc} and T_{Fst_perc} can be derived from Equation 7, thus:

$$t_{perc} = max (kSOA^{\beta} - k T_{Fst}^{\beta}, 0) \tag{12}$$

Combining Equation 9, 10, and 12, results in:

$$t_a = max[(SOA^{\beta} - T_{Fst}^{\beta})^{1/\beta}, 0] \tag{13}$$

$$RT_{2,ANTI} = T_{Fst} + max[(SOA^{\beta} - T_{Fst}^{\beta})^{1/\beta}, 0] \tag{14}$$
$$+ T_{2,C} + T_{2,V} + T_{2,Z} + T_{2,K} - SOA$$

3) Expected RT2 without the Anticipation Process ($RT_{2,NOAN}$)

Under the condition that there is no anticipation, the expected RT2 ($RT_{2,NOAN}$) is modeled with the same form of equation in the choice reaction condition except the motor subnetwork's servers are replaced by the servers involved in the simple reaction time (see Equation 15 and Figure 6).

$$RT_{2,NOAN} = max(T_{1,AP}+T_{1,B}+T_{1,C}+T_{1,F}-SOA, T_{2,VP}+ \tag{15}$$
$$T_{1,A}+T_{2,C}) + T_{2,F}+ T_{2,C}+ T_{2,V}+ T_{2,Z}+ T_{2,K}$$

Hence, the expected RT2 in long SOA conditions ($t_a>0$) can be quantified by Equation 16:

$$E(RT_2|t_a>0) = pRT_{2,ANTI} + (1-p)RT_{2,NOAN} \tag{16}$$

2. Mathematical Modeling of Response Accuracy of RT2

In simple reaction condition of RT2, the response accuracy is 1 minus the probability of negative RT2 (P_n) (RT2<0 means that the R2 occurs prior to the arrival of S2 (time=SOA)). Based on Figure 6, the interval between the arrival of S1 (time=0) and R2 is $T_{Fst}+t_a+T_{2,C}+T_{2,V}+T_{2,Z}+T_{2,K}$. Supposing $u=T_{2,C}+T_{2,V}+T_{2,Z}+T_{2,K}$, result in, $P_n = P\{RT_2<0\} = P\{t_a < SOA-u-T_{Fst}\}$. Since t_a ranges from 0 to SOA- T_{FSt} (t_a ends when S2 arrives according to its definition), the probability of the RT2<0 (P_n) is:

$$P_n = \int_0^{SOA-u-T_{Fst}} 1/(SOA - T_{Fst}) dt_a \tag{17}$$

Solving this equation, probability of correct response ($P_c=1-P_n$) is:

$$P_c = \begin{cases} \dfrac{T_{2,C}+T_{2,V}+T_{2,Z}+T_{2,K}}{SOA - T_{Fst}} & \begin{matrix} SOA \geq T_{2,C}+T_{2,V} \\ +T_{2,Z}+T_{2,K}+T_{Fst} \end{matrix} \\ 1 & SOA < T_{2,C}+T_{2,V}+T_{2,Z}+T_{2,K}+T_{Fst} \end{cases} \tag{18}$$

3. Parameter Setting in the Modeling Process

The parameter setting method in this article follows the parameter setting method in a classic cognitive modeling study (Byrne & Anderson, 2001): two free parameters (processing time of server F and T_{perc}) are adjusted at long SOA conditions to fit the modeling results with experimental results; then without changing their value, these parameters are used in the model to produce the modeling results at short SOA conditions. Therefore, there is no free parameter to fit the experimental result at short SOA conditions. All of the three dependent variables are modeled based on the same set of parameters' values in Table 1. Except the value of the free parameters, key closure time (Byrne & Anderson, 2001), k and β (Wearden et al., 1998), the value of all the other parameters come from the same modeling approach which models a wide range of human performance in various tasks (Liu, et al, in press).

Table 1: Parameters used in the modeling process

Parameter	Value	Parameter	Value
$T_{1,AP}$	126 ms	$T_{2,V}$	24 ms
$T_{2,VP}$	126 ms	$T_{1,W}, T_{2,W}$	24 ms
$T_{2,A}, T_{2,B}$	18 ms	$T_{1,Y}, T_{2,Y}$	24 ms
$T_{1,C}, T_{2,C}$	18 ms	$T_{1,Z}, T_{2,Z}$	24 ms
$T_{1,F}$	338 ms	$T_{1,X}, T_{2,X}$	24 ms
$T_{2,F}$ (choice RT)	324 ms	T_k	10 ms
$T_{2,F}$ (simple RT)	293 ms	k	2.1
T_{perc}	570 ms	β	.93