Linear Separability and Manifestations of Abstract Category Structures

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Abstract

Experimental evidence thus far has been overwhelmingly against the idea that linear separability is intrinsically important in category learning. This paper tries to shed new light on this old problem and shows conditions under which linear separability promotes learning.

After the seminal study of Medin & Schwanenflugel (1981) on the role of linear separability (LS) in categorization, it has been repeatedly reported that LS has virtually no positive effect on category learning especially when within- and between- category similarities are controlled. In this paper, after formally defining LS and linear discriminant function with research review, we introduce several properties of LS that have been unnoticed in the past studies. Then 2 experiments are reported that showed LS learning advantages followed by discussion.

Definition and Research History

Suppose there are two Categories A and B comprised of *m* instances each and instance *i* is represented by a *p*-dimensional row vector: $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{ip})$ A binary dimension is often called a feature in this paper. Linear discriminant function (LDF) *f* is a linear function:

$$f(\mathbf{x}_i) = \omega_1 x_{i1} + \omega_2 x_{i2} + ... + \omega_p x_{ip} + c, \quad \text{such that}$$

if $f(\mathbf{x}_i) \geq 0$ *then* $i \in$ *Category A else* $i \in$ *Category B*, where $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_p)$ is a weight vector and *c* is a constant. If an LDF exists Categories A and B are linear separable.

The LDF approach has been a topic of intense debate (Ashby & Maddox, 2005.) There are many studies which directly compared learning rates of LS and NLS (not linear separable) categories. Shepard and Chang (1963) compared the difficulty of supervised classification learning for 6 category structures, and found that LS categories were easier to learn. They suggested that "the easy classifications tend to differ from the difficult ones in that their points can be roughly partitioned into the two subclasses simply by drawing a straight line through the two-dimensional space" (p.102). Ashby and his colleagues (Ashby & Gott 1988; Ashby & Maddox 1990, 1992) found a consistent advantage for LS categories over NLS ones. Wattenmaker et al. (1986) and Wattenmaker (1995) also found some positive effects of LS. Specifically, strong interactions between category structures and activated domain theories were found and they were attributed to the coding of stimulus properties induced by theories or themes. They concluded that when theories are available, the compatibility between theory and category structure will determine the ease of learning, which gave an impetus to the theory-based category learning models (Murphy & Medin, 1985). The idea of theory-based category, however, tends to underestimate the effect of structures in the environment (Malt, 1995) because people do not generally use theories or themes.

It is fair to say that experimental evidence thus far is overwhelmingly against the idea that LS is essential in category learning; Medin and Schwanenflugel (1981), Kemler-Nelson (1984), Nakamura (1985), Wattenmaker et al. (1986), and Wattenmaker (1995) reported that LS had virtually no positive effect *per se* on category learning when within- and between- category similarities are controlled. Smith, Murray, and Minda (1997), who encourage prototype model, also failed to provide evidence. But they raised several key questions about how experimental stimuli are generated. In particular, their demonstration of poor differentiation of NLS categories in the universe of category structures is quite insightful. More recently Blair and Home (2001) have found an LS advantage, but their experimental setting is different from that of the present study.

Some Hidden Properties of Linear Separability

In the research of categorization, there are several conceptual and procedural problems that might have disturbed the detection of LS effects.

Reconstruction of Feature Space Consider the binary, abstract category structure shown in Table 1a. Category structures of this type are used in numerous studies, and an experimenter will assign somewhat arbitrary concrete values (on a nominal, ordinal, interval, or ratio scale) to the 0's and 1's to generate stimulus manifestations (Table 1b). A key assumption in such studies is that participants will and can reconstruct the experimenter-defined abstract structure from the experienced category instances, whereas in ignorance of the experimenter's feature value assignment people should assign new abstract binary values to the manifested values in the reconstruction process and thus the reconstructed abstract structure may look like Table 1c. Moreover, in order to control possible interactions between category structure and feature manifestation, the concrete feature values are ordinarily assigned randomly between subjects. As such, a participant might reconstruct a category structure that is a double transform of the original abstract structure.

Interpretation of Features Since we cannot numerically add a *circle* and *whiteness*, the construction of an LDF requires inter-dimensional *additivity* (including subtractive-ty) of the abstract values. If the binary features in Table 1c are interpreted as nominal or logical variables, their addition is unnatural and logical operations should be used instead. If the values are interpreted as *numerical*, however, arithmetic operations can be used and LDFs can be computed. Furthermore, since we are reluctant to compare, say, body weight with annual income, dimensional *homogeneity* will promote LDF construction together with additivity.

Table 1 An Abstract Category Structure (Medin and Schwanenflugel(1981) Experiment 1, LS structure) and a flow of typical categorization experiment

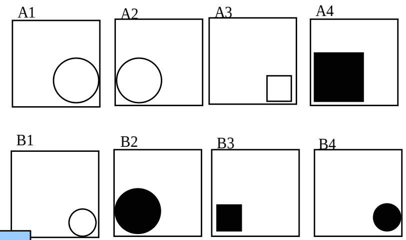
(a) Experimenter Defined Structure

	Dim 1	Dim 2	Dim 3	Dim 4
Category A	1	0	1	1
	1	0	1	0
	1	1	0	1
	0	1	1	0
Category B	1	0	0	1
	0	0	1	0
	0	1	0	0
	0	0	0	1

Random Assignment of Concrete Values

Dim 1	Dim 2	Dim 3	Dim 4
1=white	1=square	1=big	1=right
0=black	0=circle	0=small	0=left

(b) An Experimental Manifestation



(c) Reconstructed Structure

	Dim 1	Dim 2	Dim 3	Dim 4
Category A	1	1	0	1
	1	1	0	0
	1	0	1	1
	0	0	0	0
Category B	1	1	1	1
	0	1	0	0
	0	0	1	0
	0	1	1	1

Reconstruction of Abstract structure

Ease of Weight Computation In Table 1a we can easily find an LDF with weights $\omega_1=\omega_2=\omega_3=1,\ \omega_4=0$:

If dim1+ dim2 + dim3 $\geq$ *2 then Category A else B,* (1)

while in Table 1c it is not easy to find an LDF:

If dim1 - dim2 - dim3 $\geq$ *0 then Category A else B.* (2)

It seems unlikely that the computational cost is the same for all LDFs: *f(x)=3.7dim1-.9dim2+.2dim3+.4* should be harder to compute than *g(x) =dim1+dim2+dim3+0*. In an extreme situation where one dimension, say Dim2, is sufficient to construct an LDF with null weights to irrelevant dimensions, we have: *If dim2 >c then Category A else B*, which is often called a *rule* and Dim2 is ordinarily called a defining dimension. If an LDF happens to be: *If dim1 - dim2 >0 then Category A else B,* this LDF is identical to a relational property "*larger than*": *dim1> dim2*. This example shows that LS can capture relational properties, contrary to the assertion of previous studies (e.g., Medin & Schwanenflugel (1981)). "Larger than" relational property reduces to a pair of defining features in binary cases. In sum, any category structure that is partitioned by either a rule or a type of simple relational property should be LS (but not vice versa), and this fact supports the idea that LS should play some role in categorization.

Within- and Between- Category Variation LS and NLS categories are different in coherence and separation measured by within- and between- category distances or similarities (e.g., Smith et al, 1997; Blair & Homa, 2001). Define Total- , Within-, and Between- category squared distances as

$$TSD\begin{pmatrix}\text{Total}\\ \text{Squared}\\ \text{Distances}\end{pmatrix}=\sum_{i\in A\cup B}\sum_{j\in A\cup B}d_{ij}^2=\sum_{i\in A\cup B}\sum_{j\in A\cup B}\sum_{k}(x_{ik}-x_{jk})^2$$

$$WSD\begin{pmatrix}\text{Within}\\ \text{Squared}\\ \text{Distances}\end{pmatrix}=\sum_{i\in A}\sum_{j\in A}d_{ij}^2+\sum_{i\in B}\sum_{j\in B}d_{ij}^2,\quad BSD\begin{pmatrix}\text{Between}\\ \text{Squared}\\ \text{Distances}\end{pmatrix}=\sum_{i\in A}\sum_{j\in B}d_{ij}^2$$

then it is easy to find a relation: *TSD=WSD+2BSD.* (3)

WSD measures category coherence, and *BSD* category separation. Equation (3) shows that with *TSD* fixed, *WSD* and *BSD* are negatively correlated, as are category coherence and separation. An alternative way of defining category coherence and separation is through the total- within-, and between- category variances as in ANOVA: Define *TV*, *WV*, and *BV* as follows:

$$TV=\sum_{i\in A\cup B}\sum_{k}(x_{ik}-\bar{x}_k)^2$$

$$WV=\sum_{i\in A}\sum_{k}(x_{ik}-\bar{x}_{Ak})^2+\sum_{i\in B}\sum_{k}(x_{ik}-\bar{x}_{Bk})^2$$

$$BV=\sum_{i\in A}\sum_{k}(\bar{x}_{Ak}-\bar{x}_k)^2+\sum_{i\in B}\sum_{k}(\bar{x}_{Bk}-\bar{x}_k)^2$$

where

$$\bar{x}_k=\sum_{i\in A\cup B}x_{ik}/2m,\quad \bar{x}_{Ak}=\sum_{i\in A}x_{ik}/m,\quad \bar{x}_{Bk}=\sum_{i\in B}x_{ik}/m.$$

TV is psychologically interpretable as the diversity of the overall configuration of instances, *WV* as the within-category variation, and *BV* as a measure of separation or contrast between two categories. We have an interesting relation similar to (3), namely,

TV=WV+BV (4)

which again shows a negative correlation between coherence and separation. With additional work relations:

$TSD=4mTV,\quad WSD=2mWV,\quad BSD=mWV+2mBV$

can be derived and we find that :

TSD=WSD+2BSD=4mTV=4mBV+4mWV.

As an index of category separation, Correlation Ratio:

$\eta^2=BV/TV=(2BSD-WSD)/TSD$ (5)

is used in the sequel, which has the merit of permitting both binary and continuous dimensions. This index ranges over [0, 1], with larger values indicating a better separation of categories. For binary dimensions *Structural Ratio* (additive version) is often used and we have the relation between the two indexes:

$$SR=\frac{\sum\text{within similarity}}{\sum\text{between similarity}}=\frac{\sum_{i\in A}\sum_{j\in A}s_{ij}+\sum_{i\in B}\sum_{j\in B}s_{ij}}{2\sum_{i\in A}\sum_{j\in B}s_{ij}}=\frac{2mp-2TV(1-\eta^2)}{2mp-TV(1+\eta^2)}$$

Although it is not impossible to set up LS and NLS structures with equated η^2, in many cases LS categories have smaller *WV* and thus larger *BV*, while NLS categories have larger *WV* and smaller *BV*. It seems, therefore, that NLS categories are less easily discriminated and will have slower learning rates. A similar argument holds for the relationship among *TSD*, *WSD*, and *BSD* as well.

Given the properties pointed out in this section, the question of whether LS *per se* is advantageous to learning under some conditions still deserves investigation. The present study tries to reexamine the effects of LS by arranging conditions in which several of the following requirements are satisfied. R1) Reconstruction of the feature space is a transparent process for subjects and the reconstructed abstract structures are not too diverse. R2) Interpretation of reconstructed features is unambiguous. In particular, the features are interpreted as continuous and additive (homogeneous) with constant dimensional polarity. R3) LDF weights are simple, and promote relational properties (e.g., "larger than") or rules. The three requirements are all designed to promote the LS advantages. Finally, R4) within- and between- category variations are controlled in terms of η^2, respecting the tradition in this field.

Experiment 1a

Using the LS structure in Table 1a and NLS one in Table 2a, a set of bar charts was generated as shown in Table 2b (only the NLS set of bar charts are shown.) Relating bar lengths to dimension values satisfies R1, R2, and R3, and the category structure complies with R4.

In the original study of Medin and Schwanenflugel (1981), binary qualitative features were used and the LS and NLS structures were equalized in terms of η^2=.143. In the present experiment 0 referred to a short bar and 1 referred to a long bar and no random assignment of concrete values were performed, that is, the subjects experienced the same set of manifested stimuli. Evidently, if subjects construct an LDF similar to (1) the categories can be learned perfectly. The goal of Experiment 1a is to contrast the effect of manifestation of dimensions with the original experiment in which LS category was no easier to learn. Since the possibility remains that subjects interpret the bar length as a discrete feature, *noisy* conditions were arranged where bar length was fluctuated with small random numbers ranging from -9 to 9 to discourage the subjects from interpreting the dimensions as binary. Noisy conditions will also discourage memorization strategies. There were 4 conditions combining Factors A and B (Factor A: LS vs. NLS, Factor B: no-noise vs. noisy). The subjects were 112 students of Waseda University.

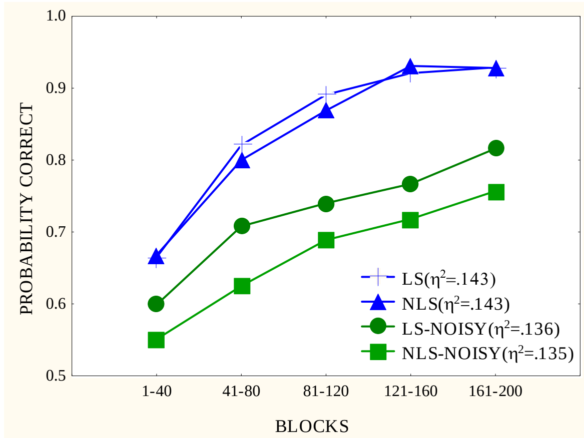


Figure 1 Learning Curves of Experiment 1a

Procedure To each condition 28 subjects were randomly assigned. Twenty-five randomized runs were used to make a total of 200 stimulus presentations. The experiment was run individually and was subject-paced. After a subject sat in front of a CRT screen, the instruction was appeared on the screen. Then a stimulus was presented one at a time and his/her task was to classify the stimulus into either Category A or Category B by pushing two response keys. An immediate feedback was given and then the next stimulus was presented. Proceeding in this way, a total of 200 presentations were made. The labeling of categories and response keys were randomly assigned across subjects.

Results and Discussion The learning curves for the four conditions are shown in Figure 1. There was an interaction between category structure and noise. In no-noise conditions the categories were easier to master than in noisy conditions but the LS categories were easier to learn only in the noisy condition. Logistic regression analysis revealed that the LS effect was significant ($\chi^2(1)$ =20.38, p<.01), the effect of noise was significant ($\chi^2(1)$ =727.49, p<.01), and the interaction between them were significant ($\chi^2(1)$ =10.90, P<.01). Post hoc test by Tukey method revealed that there was no significant difference between the LS no-noise and NLS no-noise conditions across blocks.

The values of η^2 for the four conditions were virtually the same and thus the LS gain in the noisy condition is not attributable to the category variability: The results showed evidence that there are situations in which LS without favorable properties (large η^2, relational properties, and rules) can promote category learning. One common property of previous studies such as Medin and Schwanenflugel (1981) that found no positive LS effect is that the dimensions were binary. Consistent with these studies, the binary valued dimensions in the no-noise condition did not produce an LS advantage, whereas in the noisy conditions where the random perturbation suggested to the subjects that the dimensions were continuous, the LS structure was easier to master.

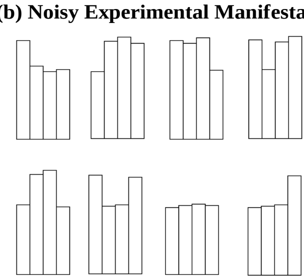
Table 2 An Abstract Category Structure
Medin and Schwanenflugel(1981) Experiment 1, NLS structure

(a) Experimenter Defined Structure

	Dim 1	Dim 2	Dim 3	Dim 4
Category A	1	0	0	0
	0	1	1	1
	1	1	1	0
	1	0	1	1
Category B	0	1	1	0
	1	0	0	1
	0	0	0	0
	0	0	0	1

1 -> long bar (145 dots)
0 -> short bar (100 dots)
The base is 20 dots

(b) Noisy Experimental Manifestation



Why, then, do binary dimensions impair learning of LS categories? As mentioned previously, the concept of LS in general (and LDFs in particular) is meaningful in a continuous space, because summation and multiplication are valid only when the values are on an interval or on a ratio scale. It follows that, for a summation strategy to be feasible, the feature values should be *interpreted* as continuous even if they can only take discrete values. In ordinary situations, however, binary features are interpreted as nominal values, preventing multiplication and summation and rendering the concept of LS irrelevant.

Experiment 1b

It seems that previous studies assume that there are no differences in the ease of setting up LDFs. In prototype models, in particular, subjects should always set up LDFs. Although Experiment 1a produced some evidence that LS promotes learning, the LDFs to be derived have only positive, unit weights (see equation (1)) and thus are easy to compute. In Experiment 1b a variation of the original abstract structure was used to introduce negative weights, making the LDF computation more difficult. Even if subjects are able to use LDFs in some situations, they might fail to utilize negative weights and the LS effect may vanish.

Category Structure The two conditions compared were LS (Table 1c) and NLS (Table 3) structures with noise. The LS structure was arranged by reversing the abstract values of dim2 and dim3 in Table 1a. The LDF thus takes the form of equation (2). The NLS structure was arranged by first reversing the values of dim2 and dim3 in Table 2a and changing the order of dimensions to dim3, dim4, dim1, dim2. Note further that 7 out of 8 stimuli in both structures are common, which may control memorization effects.

Subjects and Procedure The subjects were 32 students of Waseda University and they were randomly assigned to the two equal-sized conditions. None of these subjects participated in Experiment 1a. The general procedure was identical to that of Experiment 1a.

Results and Discussion Learning curves are depicted in Figure 2. Evidently, no difference was found between the LS and NLS conditions ($\chi^2(1)$=.3572, P>.5). The results indicate that an LDF with negative weights did not promote category learning, and suggest that the ease of LDF computation would affect LS category learning.

Table 3 NLS Abstract Category Structures in Experiment 1b

LS structure is shown in Table 1c

NLS

	Dim 1	Dim 2	Dim 3	Dim 4
Category A	1	0	1	1
	0	1	0	0
	0	0	1	0
	0	1	1	1
Category B	0	0	0	0
	1	1	1	1
	1	0	0	1
	1	1	0	1

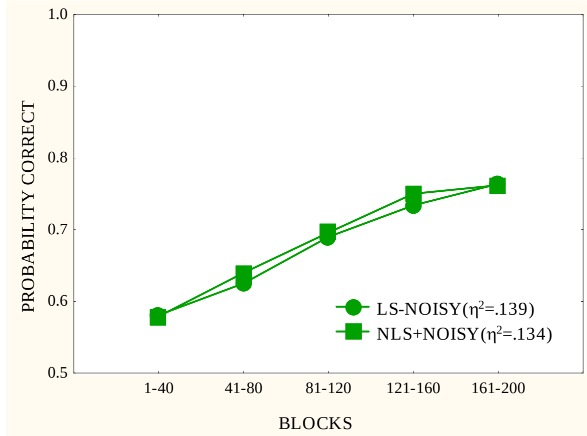


Figure 2 Learning Curves of Experiment 1b

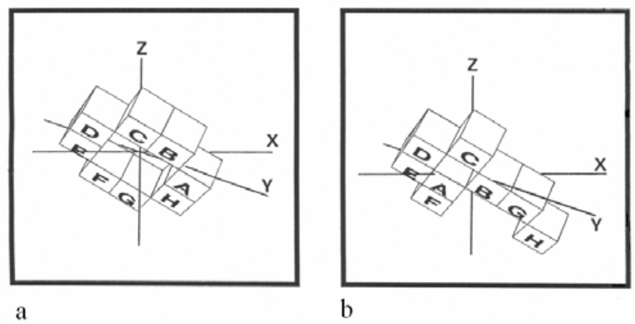


Figure 3 Stimulus Arrangement in Experiment 2

Experiment 2

In Experiment 2, using bar charts with three bars as stimuli, we highlight the effects of one dimensional rules and "longer than" relational properties (RP) that are sufficient conditions for LS. Specifically, we contrast the LS+RULE and LS+RP category structures to two NLS structures, one with the matched-η^2 and the other with a small η^2.

Category Structure The numerical values of the experimental stimuli with three bars x, y, and z are summarized in Table 4. The position of each stimulus in the stimulus space is represented in Figures 3a and 3b, where each axis corresponds to one of the bar lengths and the center (centroid) of each cube corresponds to the coordinates in Table 4. Two factors were considered. Factor A, concerned with structural differences, includes four conditions: LS with a relational property (LS+RP), LS with a defining dimension (LS + RULE), NLS with a small η^2 (NLS-small-η^2), and NLS with a large η^2 that matches the η^2 of LS + RP and LS + RULE conditions (NLS-matched-η^2). Factor B comprised the no-noise and noisy conditions as in Experiment 1. The no-noise condition is explained first.

No-noise Conditions In this condition, bar charts corresponding to the centroids of the cubes in Figures 3a and 3b are presented to subjects. Category structures are defined by how each cube is associated with a category label (Table 4.) In the LS+RP condition (LS with a relational property), cubes A, B, C, and D in Figure 3a were combined to make

Category 1, and E, F, G, and H to make Category 0: The category structure has a “longer than” relational property : *if z > y then Category 1 else 0.* In the LS+RULE condition cubes A, B, G, and H in Figure 3a were combined to make Category 1 and C, D, E, and F Category 0, where Dimension x is the defining dimension (Table 4.) In the NLS-small-η^2 condition, A, B, E and F were combined to make Category 1 and the others were combined to make Category 0. Finally, in the NLS-matched-η^2 condition, A, B, G and H of Figure 3b were combined to make Category 1 and the others were combined to make Category 0. The LS+RP, LS+RULE, and NLS-matched-η^2 conditions have the same η^2 value, while the NLS-small-η^2 condition have a much smaller value of η^2 but have the same set of instances as those of the LS+RP and LS+RULE conditions.

Noisy Conditions While in no-noise conditions only the centroids of the cubes were used, in noisy conditions random samples from inside the cubes were presented to the subjects as perturbation of controids. The side length of the cube was 60 dots. This procedure is equivalent to adding a uniform noise term whose boundary is the cube surface to the cube centroid.

Procedure The subjects were 128 undergraduate and graduate students of Waseda University. No subject participated in Experiment 1. The subjects were randomly assigned to one of the eight equal-sized experimental conditions. The general procedure was identical to that of Experiment 1 and 25 randomized runs were used to make a total of 200 stimulus presentations.

Results The learning curves for 8 conditions are shown in Figures 4 and 5. Overall they show that the LS categories were easier to learn than NLS ones even when η^2 values were equated.

The Analysis of No-noise Condition Logistic regression analysis revealed that the variable of category type (LS+RP, LS+RULE, NLS-small-η^2, and NLS-matched-η^2) produced different learning rates (χ^2 (3)=452.42, P<.001). Multiple comparison between the four category types revealed that all the differences were significant. Because the differences between the NLS-matched-η^2 and LS+RULE conditions ($q(4, \infty)$=5.25, p<.01) and between the NLS-matched-η^2 and LS+RP conditions ($q(4, \infty)$=21.08, p<.01) were significant, we obtained evidence that LS categories with a “longer than” relational property and those with a defining dimension are easier to learn even when η^2 values are equated. Furthermore the results were consistent with the idea that η^2 affects performance: the two LS categories were easier to learn than the NLS-small-η^2 category (LS+RP and NLS-small-η^2, $q(4, \infty)$=29.20, P<.01 ; LS+RULE and NLS-small-η^2, $q(4,\infty)$=15.59, P<.01). Interestingly, the difference between the LS+RP and LS+RULE conditions was also significant ($q(4,\infty)$=16.95, P<.01) indicating that the “longer than” property was more advantageous to category learning than the defining dimension in the present experiment. The difference between the LS+RULE and NLS-matched-η^2 was, however, not too compelling. In particular, there was no difference between them in the third, fourth and fifth blocks ($q(20, \infty)$=1.63, P>.05 ; $q(20, \infty)$=1.38, P>.05 ; $q(20, \infty)$=0, P>.05, respectively)

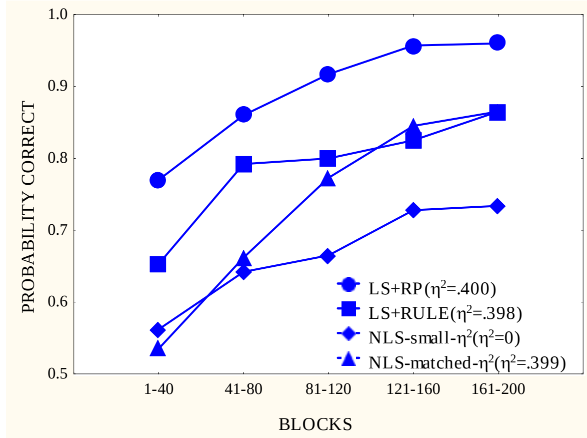


Figure 4 Results of Experiment 2 no-noise conditions

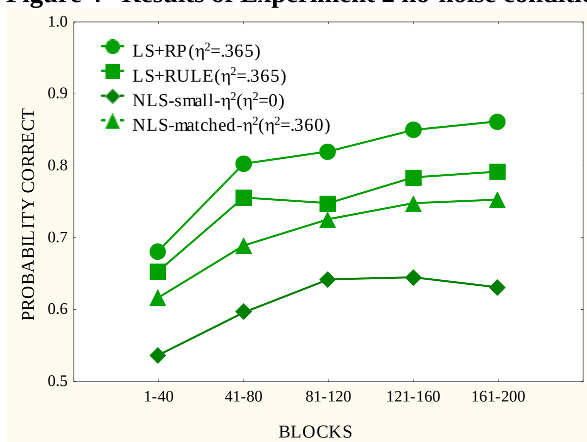


Figure 5 Results of Experiment 2 noisy conditions

The Analysis of Noisy Conditions The learning curves and statistical results were very similar overall to those of the no-noise conditions. Logistic regression analysis revealed that the effect of category structure on learning rates was significant (χ^2 (3)=317.38, P<.001) and multiple comparison revealed that the differences between the four conditions were all

TABLE 4 Numerical Values of Cube Controids and Category Structures

Category Structure of Figure 3a

Cube	Dimensional Values of Cube Centroid			Category Structure		
	x	y	z	LS+RP	LS+RULE	NLS-small-η^2
A	120	65	76	1	1	1
B	101	54	112	1	1	1
C	86	69	133	1	0	0
D	75	110	139	1	0	0
E	80	135	124	0	0	1
F	99	146	88	0	0	1
G	114	131	67	0	1	0
H	125	90	61	0	1	0

Category Structure of Figure 3

Cube	Dimensional Values of Cube Centroid			Category Structure
	x	y	z	NLS-matched-η^2
A	95	120	103	1
B	105	80	97	1
C	86	69	133	0
D	75	110	139	0
E	80	135	124	0
F	99	146	88	0
G	120	65	76	1
H	140	75	39	1

Notes Relational property : if z>y then category 1 else 0 exists in LS+RP
x is a defining deminsion in LS+RULE

reliable : Again the differences between the NLS-matched-η^2 and LS+RULE conditions ($q(4,\infty)=5.15$, $P<.01$) and between the NLS-matched-η^2 and LS+RP conditions ($q(4,\infty)=13.10$, $P<.01$) were both significant. The LS categories with a "longer than" property and with a defining dimension were easier to learn than the NLS-small-η^2 category (LS+RP and NLS-small-η^2, $q(4, \infty)=29.05$, $P<.01$; LS+RULE and NLS-small-η^2, $q(4, \infty)=16.60$, $P<.01$). Finally, the difference between the LS+RP and LS+RULE conditions was again significant ($q(4, \infty)=8.09$, $P<.01$).

Joint analysis of noisy and no-noise conditions. Logistic regression analysis showed that the effect of category type ($\chi^2(3)=627.66$, $p<.001$) and the effect of noise ($\chi^2(1)=158.39$, $p<.001$) were both significant.

Summary and Discussion Essentially, the results showed that the differences between LS+RP and LS+RULE, LS+RULE and NLS-matched-η^2, and NLS-matched-η^2 and NLS-small-η^2 were all reliable and clearly the LS+RP condition was the easiest to learn. Therefore we can state the following: (1) We obtained evidence that "longer than" relational properties and defining dimensions, which can be derived from LS but never from NLS, both promote category learning. This will be so even when contrast NLS category structures have almost the same η^2. (2) At the same time, the results suggested that relational properties could be more effective than defining dimensions. (3) The difference between the LS+RULE and NLS-matched-η^2 conditions without noise would not be too compelling.

These results clearly indicate that some types of LS category are easier to learn when the four requirements described at the end of introduction are satisfied. A next question is whether this is also the case in the absence of such beneficial properties. The results of Experiment 1b and numerous existent studies strongly suggest that the LS advantage will vanish.

In this experiment the "longer than" relational property was more effective than the defining dimension, which may show that people are more sensitive to emerging relational properties than they are to parent dimensions at least under some conditions. The apparent fact that bar charts are used to compare values supports the idea that the bar chart stimuli would have promoted dimensional comparisons and helped subjects to find relational properties. Another interpretation is, because the separation of values on Dimension x in the LS+RULE condition was small (there was only 2 dot difference); the defining dimension did not have perfect validity especially in the noisy condition in which random noise on the original dimensional values made the difference quite imperceptible.

General Discussion

This paper does not claim that LS should be beneficial in all situations and thus contributes little to the revival of prototype theory. This paper does deny the assertion, however, that LS has nothing to do with categorization. It is highly plausible that high structural coherence, relational properties, and rules will promote LS category learning. One reason why previous studies failed to demonstrate positive effects of LS is that they used qualitative and/or heterogeneous dimensions; under such situations, LS categories would not be able to enjoy the benefits of relational properties and rules.

The generality of the present results should be examined further, paying close attention to the scale type of dimensions, because the use of homogeneous dimensions would have promoted the evaluation of relative lengths of the bars. As in the numerous previous studies, if we had used heterogeneous and qualitative dimensions instead, the results would have been greatly different. Because there is no logical reason that stimulus dimensions should be homogeneous, it is safe to say for the present that LS is advantageous to category learning only when dimensions are homogeneous and/or directly comparable inducing relational properties and rules. Conversely, because there is no necessity that stimulus should be comprised of binary features and in view of the plain fact that many dimensions of natural categories are continuous, further studies contrasting binary and continuous dimensions will be needed.

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