Asymmetrical Analogical Transfer in Insight Problem Solving

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Abstract

We report three experiments that examine transfer between structurally and superficially similar insight problems. Experiment 1 showed spontaneous positive transfer from a difficult insight problem to an easier one, but not vice versa. Experiment 2 replicated this asymmetry when participants were given an explicit hint as to the relatedness of the problems. Experiment 3 introduced physical props to reduce problem difficulty, but failed to promote transfer from easier to more difficult problems. This transfer asymmetry is consistent with the proposal that the same processes are used to solve insight and non-insight problems, and offers a new focus for modeling in theories of analogical transfer.

Introduction

Most people have had the "aha" experience, when they suddenly see the solution to a problem that, until then, has eluded them. Psychologists study this phenomenon as "insight problem solving", typically from one or the other of two broad theoretical perspectives. One approach, the "nothing special" position (e.g., Newell, Shaw, and Simon, 1964), considers that insight problem solving taps into the same processes as problem solving more generally, and involves no emergent properties. The other, the "special process" view (e.g., Davidson, 1996), considers that insight is qualitatively different from other forms of problem solving.

A common finding in general problem solving research is that practice on one problem may have a positive or negative effect on performance on another problem. It has frequently been observed that facilitative "transfer" is greater when the practice problem is the more difficult of the two than when it is the simpler (Cook, 1937; Kotovsky, Hayes & Simon, 1985; Reed, Ernest & Banerji, 1974). Asymmetric transfer effects have been observed many times in standard problem solving (Bassok, 1990; Bassok & Holyoak, 1989; Reed & Evans, 1987). Explanations for the effect typically invoke the concept of searching the problem space: positive transfer may arise from difficult to easier problems because solving the more difficult problem first forces a deeper search and greater consequent understanding of the problem space. At the same time, transfer may fail from more simple problems because the problem space of a difficult target problem may be too complex to allow mappings to be made from the easier source problem (Reed, Dempster & Ettinger, 1985).

However, it is less clear if asymmetric transfer also occurs in insight problem solving. If insight problem solving involves "nothing special" and functions through the same set of processes as conventional problem solving, then it would be reasonable to expect asymmetrical transfer effects to occur, under appropriate conditions. If it turned out that they did *not*, then the result would be inconsistent with the "nothing special" view. If, on the other hand, insight involves different processes, asymmetrical transfer may not be characteristic of them. In fact, in one special process view of insight, we would expect asymmetrical transfer *not* to occur. This is the representational change position, which holds that, once representational change has taken place, it should persist and transfer to subsequent problems that require the same insight (Knoblich, Ohlsson, Haider & Rhenius, 1999).

One area of research where transfer of insight-type problem solving has been studied is that of analogical transfer, when knowledge of one problem is used to solve a similar one (Gick & Holyoak, 1980). However, the results do not clearly answer whether asymmetrical transfer occurs. Indeed, this does not appear to be a question addressed by current theories of analogical transfer (e.g., Gentner, 1983; Keane, Ledgeway & Duff, 1994; Hummel & Holyoak, 1997; Hofstadter, 2001), presumably since the finding has not been clearly established. One limiting factor is that analogical transfer rarely occurs spontaneously in the absence of hints, directions, or other cues (Bassok & Holyoak, 1989; Needham & Begg, 1991). Another limitation is that people frequently respond to irrelevant surface similarities of problems more than to structural similarities relevant to the solution (Bassok & Holyoak, 1989; Novick, 1988). Spontaneous positive analogical transfer appears to be facilitated when source and target problems share *both* surface and structural features.

The purpose of the present research was to seek a clear answer to the issue of whether asymmetrical transfer occurs in insight problem solving. To test for this, two requirements had to be met. First, we had to identify two insight problems with sufficient surface and structural similarities to meet the boundary conditions for transfer. Second, the problems had to be of clearly different levels of difficulty.

We initially selected for study the Four-Tree problem (Metcalfe & Wiebe, 1987), and the Four-Coin problem (Ormerod et al., 2002). The Four-Tree problem asks how a gardener can plant four trees so that the trees are equidistant. A correct answer is to plant three trees round the base of a mound with the fourth tree on top. The Four-Coin problem asks how four identical coins can be arranged so that each one touches each of the other three. A correct answer is to lay 3 of the coins flat with their edges touching and to stack the fourth on top. The problems are superficially similar in that they each involve arranging four objects to be in relative positions that meet similar constraints. The problems also have deeper structural similarities, in that they involve placing three objects in a triangular arrangement and locating the fourth above them. Thus the two problems appear to meet the first requirement, above, of surface and structural similarity.

The second requirement was that the two problems should differ in difficulty. Unpublished data from 40 participants given 10 minutes to attempt a solution indicated that the Four-coin problem was of moderate difficulty, with approximately 80% solving within the time limit (see Figure 1). Although we have been unable to find comparable results for the Four-Tree problem, what empirical evidence there is indicates that it is likely to be more difficult than this (Metcalfe & Wiebe, 1987).

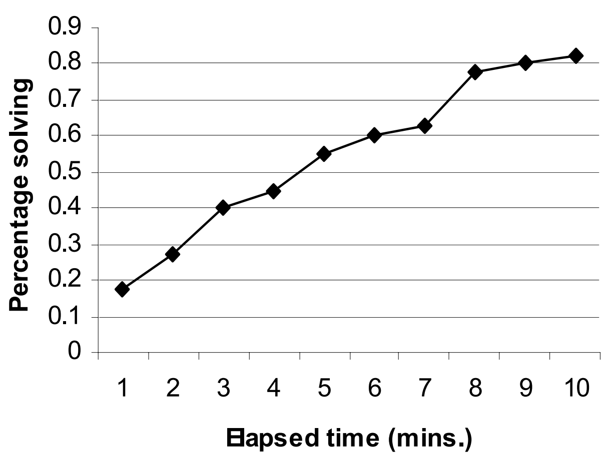

Figure 1: Cumulative percentage of 40 participants solving the Four-Coin problem in successive one minute blocks.

Experiment 1

The purpose of the first experiment was to provide an initial test for the presence of asymmetrical transfer with insight problems. If there is such an effect, then receiving the more difficult tree problem (and solution) first should facilitate performance on the subsequent coin problem, whereas receiving the coin problem first should have no, or lesser, effect on subsequent tree performance.

Method

Participants. The participants were 18 student volunteers. Participants were randomly assigned to either the tree-coin condition or the coin-tree condition, with 9 in each.

Materials and Procedure. Testing was conducted in a group setting. Participants received a booklet containing the two problems on separate pages, in the order prescribed by experimental condition, separated by a sheet containing the answer to the first problem followed by two sheets containing filler items (two versions of an unrelated Traveling Salesperson problem in which participants were required to draw minimal tours around sets of nodes – see Chronicle, MacGregor & Ormerod, in press). Participants were instructed that as soon as they thought they had found a solution, to note down the time from the digital timer that was prominently displayed, followed by a written or diagrammatic description of their solution. They were then to raise their hand so that the solution could be checked for accuracy by an experimenter. If incorrect, they continued, if correct, they were asked to wait for further instructions before turning the page. Five minutes were allowed for each problem, separated by a gap of one minute in which the solution to the first problem was read by participant, followed by a further six minutes for completion of the filler items.

Results and discussion

Overall, 15 of the 18 participants (83%) solved the coin problem and 0 (0%) the tree problem. The difference was significant, by the Wilcoxon matched-pairs test, confirming that the latter problem was the more difficult of the two ($Z=3.87$, $p<.001$). Six of the 9 participants who received the coin problem prior to the tree problem solved (67%), whereas all 9 who received it after the tree problem were successful (100%).

For the coin problem, mean time to solution was 208s for those receiving it first and 95s for those receiving it second. For the tree problem, mean solution times were 300s in both order conditions (participants failing to solve were assigned the maximum time of 300s). Analysis of solution time data was conducted using a mixed analysis of variance, with Problem Type as a Within factor and order of presentation as the Between factor. The results indicated significant effects of Problem Type (Four-Coin or Four-Tree), Order (coin first or tree first) and their interaction. For the effect of Problem Type, $F(1,16)=39.34$, $Mse=5045$, $p<.001$, and of Order, $F(1,16)=5.74$, $Mse=5045$, $p<.05$. For the interaction effect, $F(1,115)=5.74$, $Mse=5045$, $p<.05$.

The results were useful in a number of respects. First, they confirmed that the two problems were similar enough for transfer to occur. Second, they indicated that the Four-Tree problem was the more difficult of the two, by a substantial margin. Third, they provided initial support for the hypothesis of asymmetrical insight transfer based on differences in problem difficulty. At the same time, the experiment had several limitations. One was the small number of participants. Another was that participants were not made explicitly aware of the relatedness of the two problems. Thus, failure to transfer from the Four-Trees to the Four-Coins problem may have arisen because participants may not have detected similarities between the two problems. Experiment 2 was conducted to address these limitations.

Experiment 2

The second experiment repeated the first, but with a much larger pool of participants. Also, in an attempt to facilitate solutions to the tree problem, participants were explicitly instructed to use the solution to the first problem as a clue to solving the second.

Method

Participants. The participants were 127 volunteers visiting Lancaster University. Participants were randomly assigned to either the tree-coin condition or the coin-tree condition, with 60 in the former and 67 in the latter.

Materials and Procedure. Testing was conducted in a group setting. Participants received a booklet containing the two problems on separate pages, in the order prescribed by experimental condition, separated by a sheet containing the answer to the first problem and filler items as in Experiment 1. The statement of the second problem included a hint, that the solution to the first problem may provide a useful clue to finding the solution to the second. Participants were instructed that, as soon as they thought they had found a solution, to note down the time from the digital timer that was prominently displayed, followed by a description of their solution. They were then to raise their hand so that the solution could be checked for accuracy by an experimenter. If incorrect, they continued, if correct, they were asked to wait for further instructions before turning the page. Three minutes were allowed for each problem.

Results and discussion

Ten protocols were excluded from analysis because no time data were recorded or the solution provided was ambiguous. This left 61 participants in the coin-tree condition and 56 in the tree-coin. Overall, 50 (43%) of participants solved the Four-Coin problem, 4 (3%) the Four-Tree problem. The difference was significant by the Wilcoxon matched-pairs test, confirming that the latter problem was the more difficult of the two ($Z=6.64, p<.001$). Solution rates for the Four-Coin problem were lower than those of Experiment 1. This difference was unexpected since the effect of a hint would normally be to raise solution rates to the second problem (Gick & Holyoak, 1980). However, the poorer performance in Experiment 2 is likely to have arisen because, due to constraints on the length of the testing session, participants were only allowed three minutes to solve each problem compared with five minutes in Experiment 1.

For the Four-Coin problem, the percentages solving (and the mean times to solution) were 30% (160s) when first and 57% (102s) when second. For the Four-Tree problem, percent solving and mean solution times were 4% (176s) when first and 3% (177s) when second (participants failing to solve were assigned the maximum time of 180s). Analysis of solution time data was conducted using a mixed analysis of variance, with Problem Type as a Within factor and order of presentation as the Between factor. The results indicated significant effects of Problem Type (coin or tree), Order (coin first or tree first) and their interaction. For the effect of Problem Type, $F(1,115)=74.53$, $Mse=1629$, $p<.001$, and of Order, $F(1,115)=23.48$, $Mse=2150$, $p<.001$. For the interaction effect, $F(1,115)=29.19$, $Mse=1629$, $p<.001$.

The results were similar to those of the first experiment in showing a strongly asymmetric transfer effect, with positive transfer from the Four-Tree to Four-Coin problem, but not from Four-Coin to Four-Tree. It appears that the hint to use the solution to the first problem as an aid in solving the second had little or no effect. The ineffectiveness of an explicit hint to analogize is surprising in the light of the results of Gick & Holyoak (1980) and others, where typically rates of positive transfer increase by up to a factor of five in the presence of such a hint. However, performance on the tree problem remained virtually at floor in both order conditions. It would be desirable to have a higher number of solutions, to identify with more certainty that no transfer was taking place with the tree problem. Therefore, we conducted a third experiment to try to increase the solution rate.

Experiment 3

The strategy of the third experiment was to generate more solutions to the tree problem. To do so, we introduced several procedural changes. The previous experiments were conducted in a group setting, with the strong possibility that the distracting presence of other participants may have influenced attention and performance. While there is no reason why this would have differentially affected the two experimental conditions, it could have resulted in generally depressed performance levels. Experiment 3 tested participants individually in a controlled laboratory setting. In addition, instead of a paper-and-pencil approach, it allowed participants to manipulate physical objects. In the coin condition, these were four hexagonal metal tiles placed on a table. In the tree condition we supplied bamboo rods as the “trees”, and provided a sandbox and miniature shovel for “planting”. It seemed possible that these might act as a hint to construct the mound required for solution. Previous research has shown that providing physical props can enhance insight (Murray, 2004), and generally it has been shown that providing devices to enable the externalization of problem constraints aids problem-solving performance (Zhang, 1997).

Method

Participants. The participants were 41 student volunteers randomly assigned to either the tree-coin condition or the coin-tree condition, resulting in 21 in the former and 20 in the latter condition.

Materials and Procedure. Testing was conducted individually as part of a larger study on insight problem solving. Following the first problem and its solution, participants worked on a variety of verbal problems for approximately 20 minutes before receiving the second problem. The instructions and procedure for the coin problem were the same as in the previous experiments, except that participants were given four metal hexagonal tiles to work with. For the tree problem, participants were provided with 4 short bamboo rods to represent the trees, and a small sandbox and a miniature spade for "planting" them. Five minutes were allowed for each problem. Participants were instructed that a correct solution did not require that the 4 rods be exactly the same distances apart, only that the solution should be correct in principle.

Results and discussion

Success rates for the coin problem were 100% in both conditions. For the tree problem, 11 of the 41 participants solved it overall (27%). The difference was significant, by the Wilcoxon matched-pairs test, confirming the latter problem to be the more difficult of the two, $Z=5.48$, $p<.001$. For the tree problem, solution rates were similar whether presented before the coin problem (29%), or after (25%).

For the coin problem, the mean times to solution were 48.50 s when first and 41.76 when second. For the tree problem, the corresponding mean solution times were 278.43 s when first and 274.40 when second (participants failing to solve were assigned the maximum time of 300 s). Analysis of solution time data was conducted using a mixed analysis of variance, with Problem Type as a Within factor and order of presentation as the Between factor. The results indicated a significant effect of Problem Type only, $F(1,39)=469.42$, $Mse=2334.65$, $p<.001$.

The experiment succeeded in increasing tree problem performance above floor levels, to a success rate of 27% overall. However, even with a success rate where transfer effects should be discernible, none were observed here, suggesting that the failure to find positive transfer effects in Experiment 1, from the Four-Tree to the Four-Coin problem, was not an artifact of floor level performance. The result increases confidence in the conclusion that the asymmetric transfer effect of Experiment 1 was the result of differences in problem difficulty.

General Discussion

If the "nothing special" view of insight is correct, then we would expect to observe asymmetric transfer effects between insight problems as well as between non-insight problems. We conducted three experiments, and the first revealed such an effect under the conditions where it has been found with non-insight problems. The second experiment replicated the effect, and showed that it arises even when participants are given an explicit hint as to the relatedness of the problems. The third experiment helped to rule out that the asymmetry of transfer stemmed from a floor effect in the more difficult of the two insight problems.

Previously, we have reported results with a variety of insight and non-insight problems, indicating that solution processes in both are characterized by goal-directed search and the application of a criterion for monitoring progress, which operates to select among alternative moves those that are evaluated as making satisfactory progress towards the goal (Chronicle, MacGregor & Ormerod, 2004; MacGregor, Ormerod & Chronicle, 2001; Ormerod, MacGregor & Chronicle, 2002). The present results are consistent with these processes, in demonstrating that insight problems can exhibit a similar asymmetric transfer effect previously observed with non-insight problems, where transfer is more successful going from the more difficult to the simpler problem than vice versa.

The result does not appear to be consistent with Representational Change Theory, which holds that once a representational change has taken place in the solution of an insight problem, then it should transfer to all similar problems (Knoblich et al, 1999). It could be argued that for a representational change to transfer, the person has to generate the initial solution, rather than simply being shown it. A similar argument has been advanced for the notable difficulty that people have in reproducing the solution to the nine-dot problem (Dominowski &Dallob, 1996). On the other hand, there is evidence that generating an insight solution is not critical to retention. We have found that, when shown the stacking solution to a variant of the present coin problem, people are able to reproduce the solution, in some cases after a considerable time lapse (Ormerod et al, 2002).

Spontaneous analogical transfer has been difficult to observe, and most cases of successful transfer has required intervention in the form of instructions or hints (Needham & Begg, 1991). The results of Experiment 1 are rare in this respect, by showing spontaneous analogical transfer from the tree problem to the coin problem. Previous research has suggested that failure of spontaneous analogical transfer results from failure to recall the analog experience, rather than failure to see its relevance (Perfetto, Bransford & Franks, 1983). Analogs that involve insight experiences may have an advantage in this respect, in that the solution principles of simple insight problems appear to be highly memorable (Dominowski & Dallob, 1996; Knoblich, Ohlsson, Haider & Rhenius, 1999; Ormerod, MacGregor & Chronicle, 2002). In addition, several studies have reported that solution cues which create a momentary puzzlement or "aha" are more effective in prompting solutions than declarative cues with the same informational content (Auble, Franks & Soraci, 1979; Lockhart, Lamon & Gick, 1988).

We suggest that differences in the size of problem space between the Four-Trees and Four-Coins problems underlie both differences in difficulty and also the observed asymmetry in transfer. The Four-Trees problem is more difficult than the Four-Coins problem because it has a contextually richer description. This in turn creates a larger space of move alternatives that must be searched. Moreover, some readily accessible moves (e.g., planting trees at the corners of a square layout) appear to make considerable progress towards the goal state: they come close, though not close enough. In contrast, the Four-Coins problem is knowledge-lean and allows few move alternatives that make apparent progress. As a consequence, participants experience *criterion failure* earlier with this problem, that is, a failure to find any moves that make sufficient progress towards the goal to warrant further consideration. As a consequence, participants widen the problem space, that is, they seek move alternatives that are not explicitly available within the initial problem representation. This allows the discovery of new types of move (e.g., using three dimensions).

In the present case, successful transfer may have occurred because the solution to the previous tree problem was readily available to be accessed while our participants worked on the coin problem. Our theoretical position suggests that, on experiencing criterion failure in the coin problem, the problem solver would expand or restructure the initial problem space. This restructuring could implicitly or explicitly be guided by the readily accessible experience with the tree problem.

At the same time, if insight solutions are memorable, then why does the same facilitation not occur in the reverse direction? The same theoretical position offers an explanation. The tree problem is more complex than the coin problem and, we propose, has a larger initial problem space, within which the solver can search for longer while maintaining a sense of satisfactory progress. Thus, while the coin solution should potentially be just as accessible to the participants working on the tree solution as the reverse, they are not similarly prompted to access it. We have previously shown that individuals are more likely to make use of solution-relevant hints (e.g., a stacked pair of coins in an eight-coin variant of the Four-Coin problem) once they have experienced criterion failure, a position also consistent with the opportunistic-assimilation hypothesis of Seifert, Meyer, Davidson, Patalano & Yaniv (1995). The same principle may explain why participants did not transfer from Four-Coin to Four-Tree problems even when given an explicit hint to do so: unless and until they experience criterion failure with the Four-Tree problem (i.e., they run out of things to try that seem to make progress), a hint to make use of a previous problem's solution might seem to be irrelevant or erroneous advice.

Finally, and as a speculation, the asymmetry presents a potentially interesting test case for theories and computational models of analogical problem-solving. Mapping components across analogical problems may involve matching superficial similarities (e.g., Ross, 1989) structural alignments (Gentner, 1983) or elements of both (Hummel & Holyoak, 1997). None of these theories dictate preferential access to attributes and relations in one problem rather than another during mapping. Whatever the substrate of the mapping process, given the observed asymmetry it seems likely that features of analogs are not accessed with equal ease in all cases. Some analogs may be more analogous than others.

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