Belief in the Hot Hand Improves Performance: A Mathematical Model

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The widely held belief in the "hot hand" in basketball suggests that a player experiencing a streak should be given the next shot. However Gilovich, Vallone & Tversky (1985) found that streaks of hits in basketball shooting were no more likely than chance, so basketball shots are independent events. Thus it has been widely accepted that belief in the hot hand is a fallacy. Starting with the question of what are the goals of basketball players, Burns (2001) argued that the data only demonstrated that the hot hand is invalid as an *individual* cue to *when* a player will hit a shot, not that it is an invalid *allocation* cue for deciding *who* to give the next shot to. Streaks should occur more often for good shooters.

Burns (2001) used computer simulations to show that giving the next shot to players who hit their last shot improved a basketball team's scoring. However these simulations had two weaknesses. First, occasionally the simulations utilizing the hot hand did not outperform those not using it. Although less than 1% of the simulations, they raised questions about the claim that belief in the hot hand would always be expected to help. Second, in order to limit the number of free parameters and make the entire parameter space explorable, some simplifying assumptions had to be made. Although it was possible to argue that these assumptions were not critical, it would be better to demonstrate this directly.

A Markov Model

To address concerns about the simulations I constructed a Markov model of the first two shots in basketball. The first two shots were modeled because at least one shot is necessary before there can be a hot hand, and if scoring is improved in the first two shots it should be improved over any number of shots. My analysis does not however assume that the hot hand is defined by just one hit. The model could be applied to any definition of the hot hand in which it represents a temporary elevation in the probability of giving a player the next shot that is triggered by recent success.

The model has four parameters and represents Player X and Player (or Players) Y. A bias parameter b represents the probability of giving the next shot to Player X, whereas the same probability for Player Y is $1-b$. The bias parameter represents any bias to give the ball to a player (e.g., high shooting percentage, perceived ability, friendship, etc) that is independent of recent success. The model does not incorporate a parameter for belief in a "cold" hand because there is no empirical evidence for this belief.

The model has separate parameters for the shooting percentages for the two players, s_x and s_y for Players X and Y respectively. The hot hand parameter h temporarily elevates the probability of a player being given the next shot after a hit. Thus the probability of Player X being given the next shot after a hit is $b + h(1 - b)$. All parameters have a range of 0.0 to 1.0. The expected number of hits after two shots, calculated by summing the expected outcomes of all 16 possible states is:

$$E(\text{hits after two shots}) = 2(b(s_x - s_y) + s_y) + h(b - b^2)(s_x - s_y)^2$$

The $h(b - b^2)(s_x - s_y)^2$ component is *never* negative, thus belief in the hot hand can never lower the expected number of hits. Any positive value of h will raise the expected outcome so belief in the hot hand increases expected scoring, just as was shown in the simulations. However there are the same two exceptions to this: h will have no effect when $s_x = s_y$ (if there is no difference between players then it does not matter how shots are allocated), and when $b=1$ or $b=0$. This pure strategy of always giving the ball to one player is neither observed (even when it is optimal) nor would be desirable in NBA basketball. Game theory predicts that for most interesting competitive games there is a mixed strategy equilibrium.

Conclusions

This Markov model provides a mathematical proof that belief in the hot hand is beneficial if shots are independent events. In this way it expands on Burns' (2001) simulations. It also makes clear that giving the ball to good shooters and to players experiencing the hot hand are not mutually exclusive strategies. Instead allocating shots between players is a multi-cue decision making task in which both players' base-rates of success over the long term and short-term streaks are valid allocations cues. Giving weight to both of these allocation cues will improve the amount that a team will be expected to score. False beliefs that shots are dependent may be a way to maintain utilization of streaks.

References

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