A Strong Schema Can Interfere with Learning: The Case of Children's Typical Addition Schema

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Abstract

This study investigated whether children's schema for typical addition interferes with their ability to learn about mathematical equivalence. In a pretest, elementary school children (1) solved a set of math equivalence problems (e.g., 3 + 4 + 5 = 3 + __), (2) reconstructed equivalence problems after viewing them briefly, and (3) provided definitions of the equal sign. Children were categorized according to the number of measures (out of 3) on which they exhibited the typical addition schema. Children then received one of four interventions that presented new information about equivalence problems. Finally, children completed a posttest similar to the pretest. From pretest to posttest, children who exhibited the addition schema on all three measures were the least likely to change their strategy for solving the problems, followed by children who exhibited the schema on two or one of the measures. All of the children who did not exhibit the schema on any of the three measures changed. It is important to note that all children used incorrect strategies at pretest, so it was the addition schema in particular that was associated with change resistance. Thus, a strong schema can interfere with learning. Furthermore, children's addition schema may put them at risk for difficulties in learning higher-level mathematics.

Some behaviors and ideas are particularly difficult for individuals to acquire, even after numerous attempts have been made by the individuals to learn them or by instructors to teach them. Consider, for example, a woman who has always counted on her fingers when solving addition problems. She does not wish to count on her fingers; in fact, she finds it to be an infuriating habit. However, no matter how hard she tries to memorize the addition facts, she always seems to resort to counting on her fingers. Consider also a father who is trying to learn a second language as an adult. The father is frustrated because his daughter, who is only five years old, is learning so much more quickly than he is. Why is it so difficult for the woman to memorize the addition facts and for the father to learn a second language? One reason may have to do with the organization and strength of the behaviors and ideas they already possess. The woman's knowledge about addition may be organized into a strong schema that interferes with her memorizing of the addition facts, and the father's knowledge about his first language may be organized into a strong schema that interferes with his learning of the second language.

A schema can be defined as a higher-level assembly of knowledge that is "unitized" in that it supersedes its constituent parts and acts as a whole (cf. Smolensky, 1986; see also Hayes-Roth, 1977; Hebb, 1949). Once established, schemata can serve as selective mechanisms that determine how environmental stimuli are encoded, interpreted, and stored in memory. In general, schemata enable fast and efficient processing of environmental stimuli. However, this efficiency can come with a price, especially if a particular schema is inaccurate or lacking in some way.

When a schema is strong, it resists change (e.g., Allport, 1954; Bartlett, 1932; Bruner, 1957; Schutzwohl, 1998). Individuals who have a strong schema have been shown to resist learning new information when it is more specific than (Adelson, 1984; Thorndyke & Hayes-Roth, 1979), not applicable to (Voss, Vesonder, & Spilich, 1980), or discrepant with (Marchant, Robinson, Anderson, & Schadewald, 1991; Markus, 1977) their current schema. Indeed, individuals who have a strong schema often actively resist change by modifying or distorting environmental input so that it corresponds to their schema (Bartlett, 1932; Bruner & Postman, 1949; Guion, Flege, Akahane-Yamada, & Pruitt, 2000; Hannigan & Reinitz, 2001). Importantly, the stronger a schema is, the more resistant it is to changing (see Luchins, 1932).

Although the strength of a schema has clear behavioral consequences, it is not always obvious how to operationalize schema strength apriori. Some theories suggest that strength may be determined by how practiced particular action procedures are (Luchins, 1942), while others suggest that it may be derived from tightly organized perceptual (Flege, Bohn, & Jang, 1997; Intraub & Bodamer, 1993) or conceptual (Wellman & Gelman, 1992) information. These accounts need not oppose one another, though, because any given schema is likely constructed out of various related sub-schemata (Smolensky, 1986). We propose that schema strength depends on the relationship between the three knowledge sources (action procedures, perceptual information, and conceptual information). According to this view, the strongest schema is one in which all three knowledge sources converge.

In the current study, we focused on a particular schema that most elementary school children possess. We refer to the schema as the *typical addition schema*. This schema includes action procedures for solving typical addition problems (e.g., 4 + 5 = __, 9 + 2 + 6 = __), such as counting up all of the numbers in the problem or summing all of the numbers by retrieving addition facts from memory. It also includes particular perceptual patterns, such as the pattern of having "= __" at the end of problem (Baroody & Ginsburg, 1983). The schema also includes a conceptualization of the equal sign as an operational symbol that means "the total" (Kieran, 1981; Rittle-Johnson & Alibali, 1999). When an addition problem is presented, the typical addition schema can be activated as a whole, enabling fast and accurate processing of the problem (McNeil, 2001).

The typical addition schema is adaptive because children need to use their knowledge of addition to learn other math concepts, such as multiplication and division. However, it may come with a price. Specifically, it may interfere with children's ability to learn about novel mathematics problems, such as mathematical equivalence problems, which are problems that have addends on both sides of the equal sign (e.g., 3 + 4 + 5 = 3 + __; Perry, Church & Goldin-Meadow, 1988).

Past work has shown that some children have an especially strong typical addition schema that leads them to distort environmental input about mathematical equivalence to correspond to their schema. Consider the problem 3 + 4 + 5 = 3 + __. In solving this problem, some children add up all the numbers and put 15 in the blank (McNeil & Alibali, 2000). In reconstructing the problem after viewing it briefly, some children reconstruct it in terms of the "= __ at end" perceptual pattern and write "3 + 4 + 5 + 3 = __" (McNeil, 2001). In defining the equal sign in the problem, some children say that it means to "add up all the numbers" (McNeil & Alibali, 2002).

Once children gain more experience with or are instructed about mathematical equivalence problems, they are bound to generate new ways of thinking about the problems. However, these new ways of thinking may be in conflict with their already established typical addition schema, and the two may compete for precedence (cf. Siegler, 1999). Accordingly, we hypothesize that the strength of the typical addition schema should be directly related to children's tendency to resist changes in their thinking after an intervention that provides new information about equivalence problems.

Method

Participants

The sample consisted of 67 third-, fourth-, and fifth-grade children (29 boys and 38 girls), all of whom solved a set of mathematical equivalence problems on the experimental pretest incorrectly. Children attended public or parochial schools in the greater Madison, Wisconsin area.

Measures

Problem Solving The problem solving measure elicited action procedures for solving the problems. It consisted of three mathematical equivalence problems of the form a + b + c = a + __. For each problem the experimenter placed the problem on an easel and said, "Try to solve the problem as best as you can and then put your answer in the blank." After children wrote a solution, the experimenter said, "Can you tell me how you got x?" After explaining each solution, children were asked to rate how certain they were about their "way of doing" the problem on a 7-point scale that ranged from "It's definitely wrong" to "It's definitely right," with "I'm not sure if it's wrong or right" as the midpoint.

Problem Reconstruction The problem reconstruction measure elicited perceptual representations of the problems. Two tasks made up the measure. The first was taken from Rittle-Johnson and Alibali (1999). Children were asked to reconstruct three equivalence problems of the form a + b + c = a + __ after viewing each for five seconds. The second task also included three problems and was a recognition version of the first task. Children were given a sheet of paper face down with seven problems on it. One of the problems was an equivalence problem in its correct form. The other six problems depicted errors children typically make when reconstructing equivalence problems, one of which was the typical addition foil a + b + c + a = __. After viewing an equivalence problem for five seconds, children were instructed to turn the sheet of paper over and find the problem that they just saw.

Equal Sign Definition The equal sign definition measure was used to elicit conceptual understanding of the equal sign. Two tasks made up the measure. Both were taken from Rittle-Johnson and Alibali (1999). Children were first asked to define the equal sign. Then, they were asked to rate the smartness of six fictitious students' definitions as not so smart, kind of smart, or very smart. The definitions were "the answer to the problem," "repeat the numbers," "the end of the problem," "something is equal to another thing," "two amounts are the same," and "the total."

Procedure

Children participated individually in one experimental session that was videotaped. In the pretest, children first completed the problem-solving measure, followed by the problem reconstruction and equal sign definition measures presented in random order. After the pretest, children were randomly assigned to intervention conditions in a 2 (reconstruction intervention or no reconstruction intervention) x 2 (equal sign definition intervention or no equal sign definition intervention) factorial design. During the intervention, children in all four conditions were presented with an equivalence problem that had the correct solution written in the blank (3 + 4 + 5 = 3 + 9). In the control condition, children were shown the correctly solved problem and were told that it was a correctly solved problem. They were then encouraged to think about the problem for one minute. Children in the other three conditions also were presented with the correctly solved problem, were told that it was a correctly solved problem, and were encouraged to think about it. In addition, children who received the reconstruction intervention were encouraged to notice the equal sign in the problem and were asked to point to it. Children who received the equal sign definition intervention were told that the equal sign means "that the things on one side of it have to be the same as the things on the other side of it" and were asked to repeat the definition. All children spent a total of one minute in the intervention. After the intervention, children participated in a posttest in which they first completed the problem reconstruction and equal sign definition measures in random order, followed by the problem-solving measure.

Coding

Problem Solving Problem-solving strategies were coded using a system developed by Perry, Church and Goldin-Meadow (1988). Strategies were assigned based on children's problem solutions and verbal explanations.

Problem Reconstruction Each reconstruction was examined for conceptual errors. Conceptual errors were errors that reflected inaccurate reconstructions of the structure of the equation, such as omitting the equal sign or one of the plus signs. Errors in reconstructing the particular numbers or order of the numbers were not counted as conceptual errors (e.g., for the problem 3 + 4 + 5 = 3 + __, writing 4 + 3 + 5 = 3 + __). Each recognition response was scored as correct or incorrect based on whether children correctly identified the equivalence problem on the sheet provided.

Equal Sign Definition Definitions were coded as expressing the concept of equivalence or not. None of the children gave a definition that expressed the concept of equivalence on the pretest. Children's ratings of each of the fictitious student's definitions of the equal sign were coded. Two points were given for "very smart" ratings, one point was given for "kind of smart" ratings, and zero points were given for "not so smart" ratings. The sum of the ratings for the two definitions "the total" and "the answer to the problem" were subtracted from the sum of the ratings for the two definitions "two amounts are the same" and "something is equal to another thing" to yield a difference score. A positive difference score indicates that definitions expressing the concept of equivalence were rated as smarter than definitions such as "the answer" and "the total."

Typical Addition Schema Children were categorized according to whether they exhibited the typical addition schema on the pretest measures. They were coded as exhibiting the schema on the problem-solving measure if they (1) used the "add-all-the-numbers" strategy on at least two of three equivalence problems (as shown in Table 1) and (2) gave that strategy an average certainty rating greater than four (on the 7-point scale). Recall that ratings of less than four indicate children think their strategy is incorrect. Children who use the add-all-the-numbers strategy but rate it as incorrect are likely using the strategy because they cannot come up with any alternatives (see Siegler, 1983), rather than because they are operating according to a strong schema per se.

Children were coded as exhibiting the schema on the problem reconstruction measure if they showed evidence of converting at least two problems to typical addition problems (either on the reconstruction task or on the recognition task, as shown in Table 1).

Children were coded as exhibiting the schema on the equal sign definition measure if they showed evidence of thinking that the equal sign means “the sum” or “the total.” Children could show this in one of two ways. They could express the idea of adding or totaling in the definition they provided (as shown in Table 1). Or, they could rate the definition "the total" as "very smart."

Children were categorized according to the number of pretest measures (out of three) on which they exhibited the typical addition schema. Thus, children were placed into an overall typical addition schema category of 0, 1, 2, or 3. The number of measures on which the schema was exhibited was considered to be a reflection of schema strength.

Table 1 presents examples of schema-based and non-schema-based responses on each measure. Notice that children’s responses are incorrect whether they exhibit the typical addition schema or not. Moreover, there is no reason to believe that the thinking of children without the typical addition schema is "closer to correct" than is the thinking of children with the schema. For example, defining the equal sign as "the answer" is just as incorrect, if not more so, than defining it as "the total." Thus, outside of the present framework, there is no reason to expect learning differences between children who do or do not exhibit the typical addition schema.

Table 1: Example Schema-based (SB) and Non-schema-based (NSB) Responses for the Problem 3+ 4 + 5 = 3 + __.

	Strategy (Solution || Explanation)	Reconstruction	Equal Sign Definition
SB	15 || "I added 3 plus 4 plus 5 plus 3."	3 + 4 + 5 + 3 = __	"Add up all the numbers together."
	14 || "I added them all up."	3 + 4 + 5 + 3	"The total of the problem."
NSB	4 || "4 comes after 3 in the pattern."	3 + 4 + 5 = 3 =	"Put your answer."
	24 || "3 and 4 and 5, times 2 is 24."	3 + 4 + 5 = + 3 __	"It's like where you end the problem."

Results

Manipulation Check

We examined pretest to posttest changes in children's performance on the problem reconstruction and equal sign definition measures as a check on whether our interventions provided children with new ways of thinking about the equivalence problems, as they were designed to do. A 2 (problem reconstruction intervention or no problem reconstruction intervention) x 2 (equal sign definition intervention or no equal sign definition intervention) ANOVA was performed with pretest to posttest change in number correct on the reconstruction measure (out of 6) as the dependent variable. As expected, the analysis revealed a significant main effect for problem reconstruction intervention, $F(1, 63) = 4.35$. Children who received the problem reconstruction intervention improved their performance on the reconstruction measure from pretest to posttest ($M = +1.78$, $SD = 1.31$) more so than did children who did not receive the intervention ($M = +1.00$, $SD = 1.65$). Neither the main effect for equal sign definition intervention nor the interaction was significant (both $Fs < 1$).

A similar 2 X 2 ANOVA was performed with pretest to posttest change in difference score on the ratings portion of the equal sign definition measure as the dependent variable. Recall that a positive difference score indicates that definitions such as "the same as" and "equal to" were rated as smarter than definitions such as "the answer" and "the total." As expected, the analysis revealed a significant main effect for equal sign definition intervention, $F(1, 63) = 25.62$. Children who received the equal sign definition intervention improved their difference score from pretest to posttest ($M = +1.94$, $SD = 2.0$) more so than did children who did not receive the equal sign definition intervention ($M = -.12$, $SD = 1.23$). Neither the main effect for reconstruction intervention nor the interaction was significant ($F < 1$ for both).

The preceding analyses indicate that the interventions provided new information about the equivalence problems and that children were, in general, able to take in the presented information in its specific form. The main question at hand is how this new information affected the way children solved the equivalence problems. The interventions themselves did not predict pretest to posttest changes in problem solving, $\chi 2$ (3, N = 67) = 3.89. This is not surprising given that we predicted that children would be differentially affected by an intervention depending on the strength of their typical addition schema.

Effects of Addition Schema

Recall that all children solved the problems incorrectly at pretest. Similarly, only three of the children responded correctly to all six problems on the reconstruction measure at pretest, and none of the children provided an equal sign definition that expressed the concept of equivalence at pretest. It is also important to note that children's pretest schema category (0 to 3) was independent of whether they participated in the control intervention or in one of the experimental interventions, $\chi 2$ (3, N= 67) = 3.89. Our main question was whether the strength of the typical addition schema influenced children's tendency to resist changes in the way they solved the equivalence problems after an intervention that provided new ways of thinking about the problems.

Table 2: Number of children in each typical addition schema category who changed or did not change their problem-solving strategy from pretest to posttest.

Number of Pretest Measures Reflecting Addition Schema	Change	No Change
0	5 (100%)	0
1	14 (64%)	8
2	13 (42%)	18
3	1 (11%)	8
$\chi 2$ (3, N = 67) = 12.88		

Children were classified as changing their problem-solving strategy if they solved any of the three, posttest problems using a different strategy than they used to solve the pretest problems. Table 2 displays the number of children who changed or did not change their strategy from pretest to posttest in each of the typical addition schema categories. As shown in the table, children who exhibited the addition schema on all three measures were highly unlikely to change their strategy from pretest to posttest. Again, all children used an incorrect strategy at pretest, so it was the typical addition schema in particular that was associated with change resistance. All of the children who did not exhibit the schema on any of the measures changed their strategy from pretest to posttest. Such a high proportion of change is surprising, given that the intervention lasted only one minute.

Although we were primarily interested in how a strong schema influences change after an intervention, we were also curious about the correctness of children's strategies at posttest. Results were similar when correctness of posttest strategy was used in the analysis in place of pretest to posttest strategy change. Children were classified as having a correct problem-solving strategy if they solved any of the three, posttest problems using a correct strategy. For example, children would be classified as having a correct strategy for the problem 3 + 4 + 5 = 3 + __ if they put the solution "9" in the blank and said that they added 4 plus 5 to get 9. Of the 9 children who exhibited the schema on all three measures, only 1 (11%) used a correct strategy on the posttest. Of the 31 who exhibited it on two measures, 10 (32%) used a correct strategy on the posttest. Of the 22 who exhibited it on one measure, 8 (36%) used a correct strategy on the posttest. All 5 (100%) who did not exhibit the schema on any of the measures used a correct strategy on the posttest. The analysis revealed a significant relationship between the strength of children's addition schema at pretest and whether or not they used a correct strategy on the posttest, $\chi 2$ (3, N = 67) = 11.52.

Discussion

The results of the current study indicate that the strength of the typical addition schema can interfere with children's ability to change their way of solving mathematical equivalence problems after an intervention that provided new ways of thinking about the problems. Moreover, children who did not exhibit the typical addition schema did not merely change their strategy for solving the problems after an intervention, but actually changed to using a correct strategy.

These results complement previous work that has suggested that strong schemata resist change (e.g., Allport, 1954; Bruner, 1957; Luchins, 1932; Schutzwohl, 1998). The present study indicates that individuals who have a strong schema resist changes in their thinking even after an intervention that supplies new ways of thinking. This finding may provide a potential avenue for investigation into individual differences in learning. When a group of students is presented with a particular instruction, why do some fail to learn or change, while others succeed? Although it is only speculation at this point, it may be the case that individuals who have difficulty learning new ideas are less flexible and more resistant to change because they develop strong, inaccurate schemata more readily than do individuals who are precocious learners.

The present study also extends previous research about schemata by introducing a new way of operationalizing schema strength. Some accounts have defined schema-like structures according to well-practiced action patterns (e.g., Luchins, 1942), while others have emphasized perceptual (e.g., Guion et al., 2000) or conceptual (e.g., Hannigan & Reinitz, 2001) information. In the present study, children exhibited the typical addition schema in their action procedures (i.e., problem-solving strategies), perceptual encodings (i.e., problem reconstructions) and conceptual knowledge (i.e., equal sign definitions). Results provide support for the notion that the strongest, most change-resistant schemata are ones in which all three of the knowledge sources converge on the same idea. Thus, if educators wish to build strong, accurate schemata in their students, they should not focus on building up one aspect of knowledge at the expense of the other two.

The current research has additional implications for educators. Specifically, findings suggest that a strong typical addition schema carries a heavy price and may put children at risk for difficulties in later years when algebraic equations become the focus of mathematics instruction. Thus, educators may want to consider expanding and varying the context in which they present the operation of addition and the equal sign so that children are less likely to form an inaccurate schema from their experience. More generally, results suggest that children's existing knowledge can interfere with the ability to learn new information. Thus, educators should be cautious about what they infer about children's abilities based on proficiency with today's topic.

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