

# Concept Generalization in Separable and Integral Stimulus Spaces

Nicolas Davidenko ([ndavidenko@psych.stanford.edu](mailto:ndavidenko@psych.stanford.edu))

Joshua B. Tenenbaum ([jbt@psych.stanford.edu](mailto:jbt@psych.stanford.edu))

Department of Psychology  
Stanford University, Stanford, CA

Many models of human concept learning are built around a hypothesis space of possible concepts with an associated probability distribution (Tenenbaum, 1999). These hypothesis spaces are difficult to describe, even in the case of two-dimensional stimulus spaces. Garner (1974) distinguished two types of stimulus spaces: separable (where similarity judgments follow a city-block metric) and integral (where they follow a Euclidean metric). To explain why generalization contours from one exemplar are diamonds in separable spaces and circular in integral spaces, Shepard (1987) suggested two corresponding hypothesis spaces: the space of axis-aligned rectangles (for separable spaces), and the space of circular discs (for integral spaces). However, the generalization contours from one exemplar do not uniquely determine a hypothesis space (e.g. the hypothesis space of squares under all possible rotations produces the same generalization contours as the hypothesis space of circular discs). In this work, we attempt to constrain the choice of hypothesis spaces by analyzing concept generalization from multiple exemplars. Our preliminary findings are generally consistent with Shepard's formulation except for one significant difference: people show correlational concept generalization even in separable spaces.

## Method

Fourteen subjects (ages 15 to 49) participated in a two-part experiment. Each part included 6 trials, and the order of the parts was counterbalanced across subjects. In each trial in part I (concept generalization in a separable space), subjects observed on a computer screen a group of five stimuli said to be "representative of a larger set." The stimuli were circles of variable size with radial lines of variable orientation. The parameters were chosen to fall on line segments embedded in the Size-Orientation space and were either axis-aligned (varying only in one dimension) or correlational (varying in both dimensions simultaneously). The five exemplars were evenly spread in the stimulus space. After observing the exemplars, subjects rated each of eight test items according to the perceived probability that it belonged to the same set represented by the exemplars, on a 0-10 scale. The test items were placed either on the linear extension of the concept or perpendicular to it. Part II (concept generalization in an integral space) followed the same design except that the stimuli were discs varying in two integral color dimensions: saturation and brightness.

## Results and Discussion

Figure 1 shows the predictions using Shepard's hypothesis spaces of the 50% generalization contours for axis-aligned and correlational stimulus sets (dark dots).

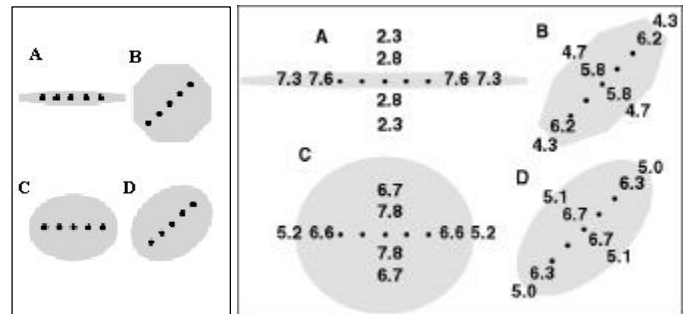


Figure 1

Figure 2

Generalization in a separable space (top row) is orientation-dependent; in particular, adaptation to the one-dimensional extent of the concept occurs only when the concept is axis-aligned (A). In an integral space (C and D), concept orientation is irrelevant.

Figure 2 summarizes the experimental data. Probability ratings of the test items (averaged across subjects) are shown as numbers. The contour regions were obtained by extrapolating the 50% probability boundary. In the separable space (top row), linear adaptation occurs more noticeably in the axis-aligned concept (A), but also occurs to some extent in the correlational concept (B), contrary to an axis-aligned rectangular hypothesis space. This finding suggests that the hypothesis space for separable dimension stimulus spaces is more complex than originally formulated by Shepard, perhaps including rectangular regions of all possible orientations weighted by some prior probability distribution. We are currently exploring this possibility through human experiments and mathematical modeling.

## References

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