Did Language Give Us Numbers?
Symbolic Thinking and the Emergence of Systematic Numerical Cognition

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Abstract

What role does language play in the development of numerical cognition? In the present paper I argue that the evolution of symbolic thinking (as a basis for language) laid the grounds for the emergence of a systematic concept of number. This concept is grounded in the notion of an infinite sequence and encompasses number assignments that can focus on cardinal aspects ('three pencils'), ordinal aspects ('the third runner'), and even nominal aspects ('bus #3'). I show that these number assignments are based on a specific association of relational structures, and that it is the human language faculty that provides a cognitive paradigm for such an association, suggesting that language played a pivotal role in the evolution of systematic numerical cognition.

Introduction

Over the last decades, results from several disciplines relating to cognitive science (in particular from psycholinguistics, developmental psychology, cognitive ethology, and cognitive neuroscience) have shed new light on the relationship between language and numerical cognition.

On the one hand, the acquisition of some aspects of mathematical knowledge seems to be linked to the number words of a language. Psychological and neurological studies suggest that the representation of memorised mathematical knowledge such as multiplication tables and its application in mental calculation is closely linked to the language it was originally learned in (cf. Dehaene, 1997).

In addition, cross-linguistic studies on the acquisition of number words have shown that the structure of a number word sequence can have an impact on children's mathematical performance:[1] a highly regular and transparent number word sequence makes it easier for children to grasp multiplicative and additive relationships between numbers and to correlate them with Arabic numerals, than a sequence that contains opaque elements.

For instance in the Chinese number word sequence, as opposed to the one in English, the underlying decimal structure is always transparent in complex number words (for instance, the Chinese counterparts for English 'ten – eleven – twelve – thirteen – fourteen – ... – twenty' have the form 'ten – ten-one – ten-two – ten-three – ten-four – ... – two-ten'). In accordance with this linguistic difference, Chinese children were shown to have a better grasp of the base ten structure of their number system and performed initially better in arithmetic tasks than their American counterparts.

On the other hand, converging evidence from developmental psychology and cognitive ethology has revealed numerical capacities that seem to be independent of language. Preverbal infants as well as higher animals were shown to be able to grasp small numerosities (the cardinality of small sets) and perform simple arithmetic operations on them.[2] Evidence from lesion and brain-imaging studies indicates that a specific brain region, the inferior parietal cortex, might be associated with this ability.[3]

This suggests that, while some later aspects of mathematical cognition might be influenced by linguistic factors, we also possess a biologically determined concept of cardinality: a concept of numerical quantities and their inter-relations that is independent of the acquisition of a specific language, and independent of the human language faculty in general.

Does this mean that our concept of *number* is independent of language? In this paper, I will argue that it is not. I will argue that language contributed to numerical cognition in a fundamental way: in the history of our species the emergence of language as a mental faculty opened the way for systematic numerical cognition. Symbolic thinking as the basis of language provided a cognitive pattern that enabled humans to make the step from primitive quantitative reasoning to a generalised concept of number, a concept that is not restricted to cardinality, but allows us to employ numbers to identify cardinal as well as ordinal and even nominal relationships between empirical objects.

To develop this claim, I will first spell out the relationship between numbers and cardinality and show that it is crucial for our understanding of the cognitive

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[1] Cf., for instance, Miura et al. (1993) and Ho & Fuson (1998) for Asian (Chinese, Korean and Japanese) versus US-American (English-speaking) and European (British, French and Swedish) first-graders and kindergarteners.

[2] Cf. Wynn (1998) for a detailed discussion of the evidence from infants and new-borns; Butterworth (1999) and Dehaene (1997) for overviews of numerosity concepts in human infants and animals.

[3] Cf. Dehaene, Dehaene-Lambertz & Cohen (1998).

number domain not to focus on cardinality alone. I will then introduce a unified notion of number assignments that brings together cardinal, ordinal and nominal aspects. On this basis, I analyse structural parallels between number assignments and symbolic reference that suggest that language provides a cognitive pattern for systematic number assignments.

Numbers and Cardinality

One of the aspects that make numbers so interesting is their enormous flexibility. A quality like colour, for instance, can only be conceived for visual objects, so that we have the notion of a red flower, but not the notion of a red thought. In contrast to that, there seem to be no restrictions on the objects numbers can apply to. In his 'Essay Concerning Human Understanding', John Locke put it this way: "[...] number applies itself to men, angels, actions, thoughts; everything that either doth exist, or can be imagined." (Locke 1690, Book II, Ch.XVI, §1).

This refers to contexts where numbers identify the cardinality of a set: they tell us how many men or actions etc. there are in the set. This number assignment works for any sets of objects, imagined or existent, no matter what qualities they might have otherwise; the only criterion is here that the objects must be distinct in order to be quantified.[4]

Frege (1884) regarded this flexibility as an indication for the intimate relationship between numbers and thought: "The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the existent, not only the intuitable, but everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought?" (Frege 1884, §14).

However, this is only one respect in which numbers are flexible. Not only can we assign them to objects of all kinds, we can also assign them to objects in ways that are so diverse that on first sight, they seem not to be related at all. Of these number assignments, the one that relates to the cardinality of sets is probably the first that comes to mind, but it is by no means the only way we can assign numbers to objects.

The same number, say 3, can be used to give the cardinality of pencils on my desk ('three pencils'); to indicate, together with a measure unit, the amount of wine needed for a dinner with friends ('three litres of wine'); it can tell us the rank of a runner in a Marathon race ('the third runner'); or identify the bus that goes to the opera ('bus #3' / 'the #3 bus').[5]

[4] This criterion on objects can be reflected in language by the distinction of count nouns versus mass nouns, as their designations (cf. also Wiese & Piñango, this volume).

[5] As the examples in brackets illustrate, these different usages of numbers establish different contexts for number words that have to be mastered in first language acquisition. Cf. Fuson & Hall (1983) for a study of the acquisition process.

We can subsume our different usages of numbers under three kinds of number assignments: cardinal, ordinal, and nominal assignments (cf. Wiese, 1997).

Cardinal number assignments are denoted by expressions like 'three pencils' or 'three litres of wine', where 'three' is an answer to 'How many?'. In cardinal assignments, the number identifies the cardinality of a set, e.g. a set of pencils or a set of measure units that identify a certain volume (in our example, litres).

In *ordinal number assignments*, the number applies to an element of a sequence. For instance in the Marathon example, 3 indicates the rank of a particular person within the sequence of runners (the third runner).

We encounter *nominal number assignments* in the form of house numbers, in subway and bus systems, in the numbering of football players, or in telephone numbers. What these cases have in common is the fact that the numbers identify objects within a set: in nominal assignments, numbers are used as readily available (and inexhaustable) proper names. So rather than thinking of names like 'Mike' or 'Lucy' for buses, we assign them numbers when we want to identify them (for instance, 'bus #3'), and similarly, we assign numbers to houses in a street or to the members of a football team.

Hence, numbers are flexible tools that can be used in a wide variety of contexts, where they identify different properties of objects. Of these properties, cardinality is only one instance – it is *a* property that we can identify with numbers, but it is not necessarily more closely connected to numbers than other properties that can also be identified in number assignments (that is, the rank of an object in a sequence, or the identity of an object within a set). Figure 1 illustrates this view:

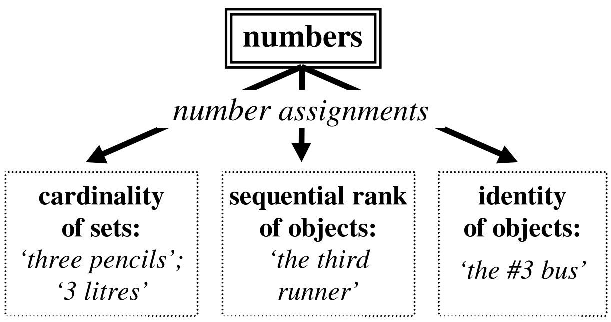

Figure 1: Numbers as flexible tools –
Integration of cardinality into the number domain

This approach, then, integrates cardinality into a broader view of the number domain. It distinguishes numbers and cardinality by characterising numbers as elementary tools that are not necessarily linked up with cardinality, but can equally bear on cardinal aspects, ordinal aspects, or nominal aspects in application, when employed in the different kinds of number assignments.

A Unified Approach to Number Assignments

What is it that makes numbers so flexible, how are their different usages related to each other? To answer this question, let us have a closer look at the different ways numbers apply to objects, that is cardinal, ordinal and nominal number assignments.

A theory that gives us a handle on these different types of number assignments is the Representational Theory of Measurement (henceforth, RTM).[6] This theory, which has been highly influential within philosophy and experimental psychology, is concerned with the features that make a number assignment[7] significant; it aims to establish the criteria that make sure that the number we assign to an object does in fact tell us something about the property we want to identify.

In the present section I will employ the machinery of this theory to a somewhat different purpose, interpreting the RTM as a unified framework for number assignments. This framework allows us to lay down the constitutive features of meaningful number assignments, the features that underly a systematic concept of numbers and of the relations which they identify between empirical objects.

In a preliminary approach, we can identify an assignment of numbers to objects as meaningful when certain relations between the numbers represent relations between the objects. Figure 2 gives an example:

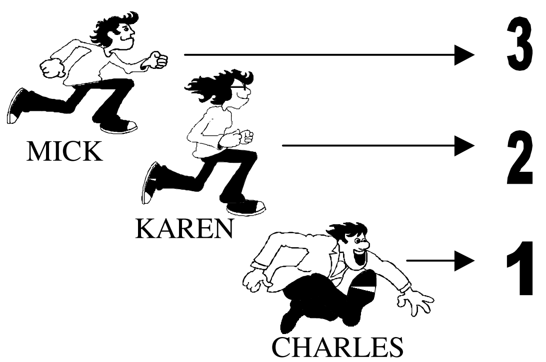

Figure 2: A meaningful number assignment: Numerical ranking of runners in a race

In this instance of number assignments, numbers have been assigned to participants in a race, such that the '<' relation between the numbers represents the ordering of the runners by the relation 'is faster than': Charles, as the fastest runner, received the smallest number, 1; Mick, who is slowest, received the largest number, 3, and Karen, who is faster than Mick and slower than Charles, got a number that is smaller than Mick's and larger than Charles's, namely 2. This way the ordering of the runners as 'Charles is faster than Karen who is faster than Mick' is reflected by the ordering of the numbers that they received: '1 < 2 < 3'.

The general features that make a number assignment meaningful can be captured by two requirements. The first requirement is that we regard the objects and the numbers only insofar as they form *relational structures*, that is, sets of elements that stand in specific relationships to each other. The two relational structures are distinguished as *numerical relational structure* (the relational structure constituted by the numbers) and *empirical relational structure* (the one established by the objects).

Accordingly, in the runner example we regarded the runners not as unrelated individuals, but treated them as elements of a particular sequence. The empirical relational structure is here constituted by the relation 'is faster than'. The relation between the numbers that we focused on was '<' ('lesser than'). All other relations that might hold between the objects (for example, the relative age of the runners) or between the numbers (for example, odd numbers versus even numbers), are ignored for the purposes of number assignment.

The second requirement for the number assignment is that the correlation between numbers and objects constitutes a *homomorphic* mapping, one that not only correlates the elements of the two relational structures, but also preserves the relevant relations between them.

In our example, the homomorphism associates the relation 'runs faster than' from the empirical relational structure (the sequence of runners) with the '<' relation in our numerical relational structure (the numbers). So for instance from the fact that one runner received the number 2 and another one got the number 3, one can deduce that the first runner was faster than the second one, because 2 < 3.

The interesting aspect for our discussion is now that this implies that number assignments are essentially links between relations: it is not so much the correlation between individual objects and individual numbers that counts, but the association of relations that hold between the empirical objects with relations that hold between the numbers.

As a result, ew can now analyse the different kinds of number assignments as instances of a unified pattern: they are constituted by a homomorphic mapping between two relational structures; a mapping that associates, in each case, a particular numerical relation with a relation between empirical objects:[8]

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[6] Cf. Krantz et al. (1971), Narens (1985), Roberts (1979).

[7] The RTM uses the term 'measurement' (instead of 'number assignments') here. This terminology is slightly at odds with our pre-theoretical usage, where 'measurement' refers only to a particular class of cardinal number assignments (those identifying empirical properties like weight, volume or temperature), but excludes ordinal and nominal number assignments, which are included under the RTM notion of 'measurement'. In the present paper, I therefore use the more intuitive term '(meaningful) number assignments'.

[8] For a detailed discussion and formalisation of the different kinds of number assignments cf. Wiese (2001).

In *cardinal number assignments*, the empirical objects are sets. A number *n* identifies the cardinality of a set *s* (*n* tells us how many elements *s* has). The mapping associates the numerical relation '>' with the empirical relation 'has more elements than'. The number assignment is meaningful if and only if a one-to-one-correlation between the numbers from 1 to *n* and the elements of *s* is possible. For instance, when we assign the number 3 to a set of pencils, this number assignment can be regarded as a meaningful cardinal number assignment when it is possible to link up each pencil with a different number from 1 to 3. (We employ this verification procedure in counting routines.)

In *ordinal number assignments* (like the one in Figure 2), the empirical objects are not sets, but individual elements of a sequence. A number *n* identifies the rank of an object within a sequence *s*. For this task, we focus on the sequential order of numbers. The homomorphism that constitutes our number assignment associates the numerical relation '<' (or '>', respectively) with the relative ranks of the objects within *s* (for instance, the relative ranks of runners as established by the relation 'is faster than' in Figure 2). The number assignment is meaningful if and only if objects receive higher and lower numbers with respect to their higher and lower positions within *s*.

In *nominal number assignments*, the empirical objects are elements of a set (for example the bus lines in a city), and the numbers are used as labels: a number *n* identifies an object within a set *s*. The mapping associates the numerical relation '=' (or '≠') with the empirical relation 'is (non-)identical with'. The numerical statement is meaningful if and only if distinct objects always receive distinct numbers.

What these different number assignments have in common is the translation of relational structures. In cardinal, ordinal, and nominal number assignments alike, a relation between empirical objects is associated with a relation that holds between the numbers. It is this translation of relational structures that constitutes number assignments, and by doing so, lays the ground for systematic numerical cognition.

How did this principle evolve? In the following section I argue that the translation of relational structures as a cognitive pattern might have its origins in the emergence of symbolic thinking. I will argue that it is symbolic thinking, as a basis for the human language faculty, that made this pattern available to the human mind, and this way enabled us to develop a systematic number concept.

The Contribution of Language to the Emergence of a Systematic Number Concept

According to an account of language evolution as developed in Deacon (1997), the main step in the emergence of human language (as opposed to animal communication systems) is the development of a symbolic system; in a process of co-evolution of language and the brain, the adaptation of our brain to symbolic thinking gave rise to the emergence of the linguistic faculty we have today. To understand the significance of this view for our investigation into numbers and language, it is crucial to understand what Deacon means by *symbolic reference* here.

Following a semiotic taxonomy as introduced by Charles Peirce, Deacon distinguishes three kinds of signs: icons, indices and symbols. In *iconic* reference the sign shares some features with its referent, it is similar to the object it refers to (such as the icon ♿ that refers to a wheel-chair user). In *indexical* reference the sign is related to the object by a physical or temporal relation; it occurs together with its referent (for instance, tears could be interpreted as an index for grief).

In *symbolic* reference, the link between sign and object is established by convention, as in the case of human languages. The critical similarity, the similarity between symbols and their referents, emerges on a higher level, namely on that of the system. Symbols are always part of a system, and they refer to objects not as individual tokens, but with respect to their position in that system. In the case of symbols, reference shifts from individual signs and individual objects to relations between signs and relations between objects; it shifts from the token to the system.

Under this account, symbolic reference as the basis of human languages is crucially a link between relations (sign-sign and object-object), not between individuals (signs and objects). It is the relations between words that reference is based on.

These can be linear relations like the order of words in a sentence, or hierarchical relations like 'object of' or 'subject of', which mark the relations between a verb and its complements. For instance in the sentence "The dog bites the rat." one can identify the dog as the attacker and the rat as the victim, because the noun phrase 'the dog' comes before the verb, which is the position for the subject in English, and 'the rat' comes after the verb, in object position, and the noun phrases in these positions denote the Agent (attacker) and the Patient (victim) of the 'biting'-action, respectively.

So the connection one makes is between (a) symbolic relations like 'The words *the dog* come before the word *bites*' (linear) or 'The noun phrase *the dog* is subject of the verb *bite*' (hierarchical) and (b) relations between referents, namely 'The dog is the Agent in the biting-event'; and similarly for the rat:

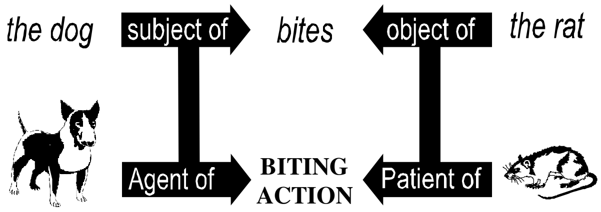

Figure 3: Symbolic reference as an association of relations

According to Jackendoff (1999), in the course of language evolution direct relationships between the linear order of words and their referents are replaced by links that are mediated by complex hierarchical syntax. Here, this would mean that the focus of symbolic relationships shifts from linear to hierarchical relations: on an early stage in the evolution of language, *linear relations* between symbols as evident in speech ('comes before / after') would directly be associated with hierarchical relations between referents ('Agent of / 'Patient of'), whereas on a later stage we would have (syntactic) *hierarchical relations* between symbols, like 'subject of / object of', which can be linked up with hierarchical relations between referents.

In both cases, it is the relationships that are associated, rather than individual symbols and individual referents. Unlike in iconic and indexical reference, in symbolic reference we pick out an object indirectly, relying on links that connect relationships between symbols (such as 'comes before / comes after', or 'subject of' / object of') with relationships between objects (such as 'Agent of / Patient of'). This is what symbolic reference is ultimately about: it is a connection between signs and referents that focuses on relationships.

This means that symbolic reference is constituted by a mapping between *relational structures*: we regard the symbols and their referents only insofar as they are part of a system whose elements stand in specific relations to each other; the association of symbols and their referents is determined by the respective relations that hold between them.

This is a phenomenon very similar to the one we encountered in the case of number assignments. As Figure 4 illustrates (for two of the runners from Figure 2 above), number assignments are based on links between relations, too: in number assignments we associate numerical relations with relations between empirical objects, just as in language we associate *symbolic* relations with relations between objects.

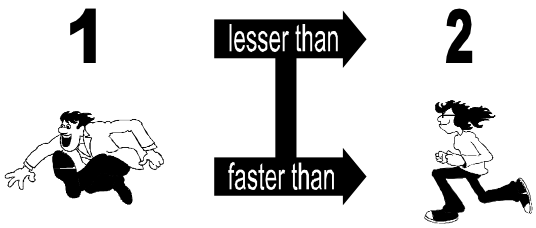

Figure 4: Number assignment as an association of relations

When we assign numbers to empirical objects, the links we establish are not between individual numbers and individual objects, but between a numerical relational structure and an empirical relational structure. And when we assign symbols to their referents, the links we establish are not between individual signs and individual objects, but between a relational structure of signs and a relational structure of the objects that they refer to.

This means that we can identify the same pattern in number assignments and in symbolic reference: in number assignments a numerical relational structure is correlated with an empirical relational structure; in symbolic reference a 'symbolic relational structure' is correlated with an empirical relational structure.

In both cases, the links between individual tokens (a number and an object, or a symbol and its referent) are based on their respective positions in the system, they are constituted by links between relations (numerical relations and empirical relations, or symbolic relations and relations between referents). Figure 5 illustrates these parallels:

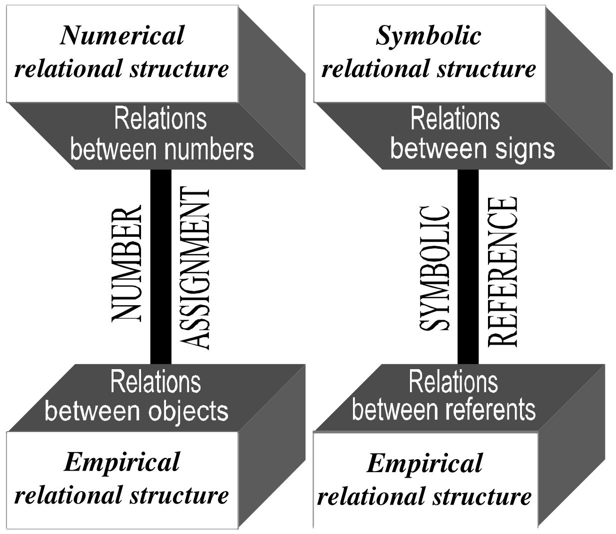

Figure 5: Translation of relational structures in number assignments and symbolic reference

This puts number assignments in a close association with the symbolic reference that lies at the core of our linguistic capacity, and shows us a way how systematic numerical cognition could have evolved in the human mind: in the development of our species the evolution of symbolic thinking in the emergence of language might have enabled us to grasp the logic of number assignments.

Once we passed the symbolic threshold, a paradigm was set for the systematic correlation of relational structures, and could be applied in the number assignments that underlie our numerical concepts. This way symbolic thinking prepared the way for systematic numerical cognition.

Under this approach, we can account for the capacity to systematically assign numbers to objects by a relatively small evolutionary step. According to this account, the use of numerical relational structures did not develop from scratch, but could build on already existing cognitive patterns that had evolved as part of symbolic cognition – a re-usage that makes a lot of sense in terms of evolutionary economy.

At the same time, language gave us a handle on infinity. The phrases we can potentially generate in a language represent a discrete infinity: from a set of primitive elements – the lexical items of our language – we can generate an infinite number of complex constructions by means of combinatorial rules. In the words of Steven Pinker: "In a discrete combinatorial system like language, there can be an unlimited number of completely distinct combinations with an infinite range of properties." (Pinker 1994, 84).

It is these combinatorial rules that constitute the infiniteness of number word sequences. The sequences of words we employ for counting ('one, two, three, …') are open-ended because of the generative rules governing the construction of complex elements. Through number words, language provides us with the notion of an infinite sequence.[9]

Note that it is the possesion of the *language faculty*, the emergence of language as a mental faculty in the history of our species, that is crucial here, not the successful and complete acquisition of a particular language in individual development. This also means that acquired or innate impairments of the language capacity do not necessarily affect our ability to grasp number assignments, as long as the basic linguistic capacity is still intact (including the association of relational structures by homomorphic mappings).

And let me emphasise again that this does not mean that without language, we would have no concept of properties like cardinality or rank that we identify with numbers. As the above-mentioned evidence from animal studies and studies with human infants shows, the emergence of our number concept could draw on prelinguistic capacities we share with other species, for instance our grasp of cardinality as a property of sets.

Language has been crucial in integrating these early concepts into a systematic number concept, one that is based on an infinite sequence of numerical tools that can be used to identify empirical properties via a correlation of relational structures.

Under this notion, numerical cognition as well as language can be regarded as genuinely human; as mental faculties that are not merely of greater complexity (than, say, animal communication systems and numerosity concepts) and grounded in a higher general-purpose intelligence, but qualitatively different and specific to the human mind.

[9] Cf. Hurford (1987) for a detailed analysis of number words; Wiese (1997; 2001) for the status of number word sequences within language and numerical cognition. In Wiese (2001, ch.4) I show that linguistic generativity (and therefore infinity) could be passed on to numerical cognition via counting sequences, and that this transfer could take place not only in individual cognitive development – as for instance assumed by Bloom (1994) –, but also in hominid evolution.

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