

# Learning Perceptual Chunks for Problem Decomposition

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## Abstract

How students learn to use diagrammatic representations is an important topic in the design of effective representations for problem solving or conceptual learning, but few good models of their learning exist. In this paper, we explore the learning process with an experiment using AVOW diagrams as a representation for solving problems in electric circuits. We find that the participants decompose each circuit into a similar set of groups when solving the problems. The natural question is whether these groups are an artifact of the visual form of the circuit, or indeed the result of prior learning. We argue that the decompositions are a result of perceptual chunking, and that they are (at least partly) a result of learning. In support of this, we describe a computational model of perceptual learning, CHREST+, and show that it predicts the decomposition of problems evident in the participants' data.

## Introduction

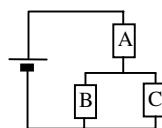
The role of effective representations in supporting or enhancing the conceptual understanding of a student is an important topic within educational psychology (Cheng, 1999b; Larkin & Simon, 1987). However, in spite of the educational interest in effective representations, the manner in which students learn with and about different representations is not well understood. The traditional method of looking for chunks, through timing information (e.g. Chase & Simon, 1973), is hard to apply in problem-solving tasks, as the timing information is associated with the solution, and not directly linked to the problem. We instead use a computational model to match the solutions produced by students in a typical diagrammatic reasoning task, and use the model's learnt associations between problem and solution states to argue that students are using learnt perceptual chunks as a guide to problem decomposition.

This paper describes an example diagrammatic reasoning task, which involves using AVOW diagrams to compute quantities within circuit diagrams. Based on results from an experimental study, we provide some samples of how students tackle a complex problem within this domain, and observe that the problem is decomposed in a consistent form across the students. The natural question is whether these groupings are based upon the students' prior learning, or are merely

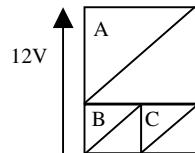
an artifact of the target's visual form. We argue that students do learn these groupings, and that their learning process is explainable in terms of the perceptual chunking theory. In support of this, we trained the CHREST+ model, which is based on the perceptual chunking theory, on the same sequence of circuits as was provided to the human participants. We show that CHREST+'s predicted decomposition of the sample problem matches that used by the participants.

## Computing Unknowns in Circuits

This paper explores how students learn to use diagrammatic representations for problem solving. As an example domain, we use the task of computing unknown quantities in electric circuits using AVOW (Amps Volts Ohms Watts) diagrams, which represent circuits and the domain laws of electricity using diagrams and constraints on their composition. AVOW diagrams are described in Cheng (1998), and Figure 1 provides an example of how AVOW diagrams are constructed and used for problem solving. Essentially, each resistor in an electric circuit is represented as a separate AVOW box. The dimensions of the box are scaled to represent the quantities within a resistor: current (I) is the width of the box, voltage (V) is the height of the box, resistance (r) is the gradient of the box's diagonal, and power (P) is the box's area. Note that the relations between the box's dimensions encapsulate rules of electric circuits. Thus, the gradient of the box's diagonal is its height divided by its width, which restates Ohm's Law,  $r = V/I$ . Similarly, the box's area is its height times its width, which restates the Power Law,  $P = VI$ .



The battery has a voltage of 12V and all three of the resistors have a resistance of 1 ohm. What is the current drawn from the battery?



Each of the AVOW boxes is a square, because each resistor's resistance is 1 ohm. The height of the total diagram is scaled to represent 12V. Hence, the width of the total diagram represents the current, measured to be 8 amps.

Figure 1 : A circuit problem and its AVOW diagram.

Composition of individual boxes is used to represent a circuit of several resistors; the rules for composition preserve the underlying physical laws of electric circuits. In working with this representation, students must first produce an AVOW diagram scaled according to the provided quantities, and the constraints in the diagram ensure that the laws of electricity are followed during its composition. The final AVOW diagram will thus provide information about all other quantities within the circuit, enabling the student to simply measure the appropriate dimension for any unknown quantity. Various studies (Cheng, 1998, 1999a) have shown that AVOW diagrams provide a more effective representation than algebra for learning concepts about electric circuits. An increasingly important element in the design of effective educational material is a better understanding of how humans learn with these representations (Cheng, 1999b). The aim of this paper is to find some indicator in the students' solutions to their underlying learning mechanisms. We achieve this by showing that all participants use a similar decomposition of the circuit problems.

### Observing the Problem Decomposition

A study on the use of AVOW diagrams was performed with six participants (2 with A level physics, 4 without). Each participant received basic instruction in the use of AVOW diagrams, and was then asked to construct appropriate AVOW diagrams for a sequence of electric circuits. Solution diagrams were entered on an electronic sketchpad, which allows diagrams to be constructed on screen using a mouse to place elements such as lines, rectangles and parallel lines of various thicknesses, as well as add textual labels. The computer retains a record of each drawing action with detailed timing information; note, the system provides no support for constructing AVOW diagrams *per se*. After an initial 15 minutes' training session on AVOW diagrams and in using the electronic sketchpad, participants were presented with a graded sequence of 30 problems, ending with complex circuits of up to 12 resistors. After each circuit was attempted, the correct AVOW diagram was shown to the participants. We illustrate here how the participants performed on the last of the 'straight-forward' circuits, illustrated in Figure 2(a) with its target AVOW diagram in Figure 2(b). (The remaining four circuits tested for generalisation to more complex circuit types, such as those requiring 3D layouts, and so are not included.)

Figure 4, at the end of this paper, illustrates in detail the progress of three participants on the sample problem. The graphs show the latency between each of the drawing actions required to complete the solution. Noticeable in these examples, and common to all the participants, is the presence of *peaks*, which separate

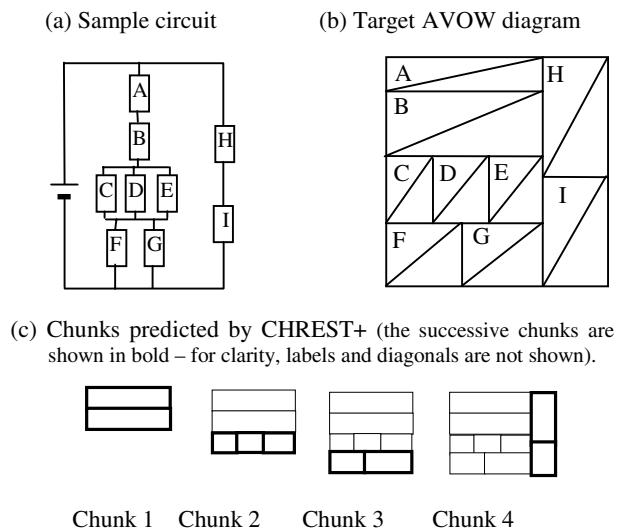


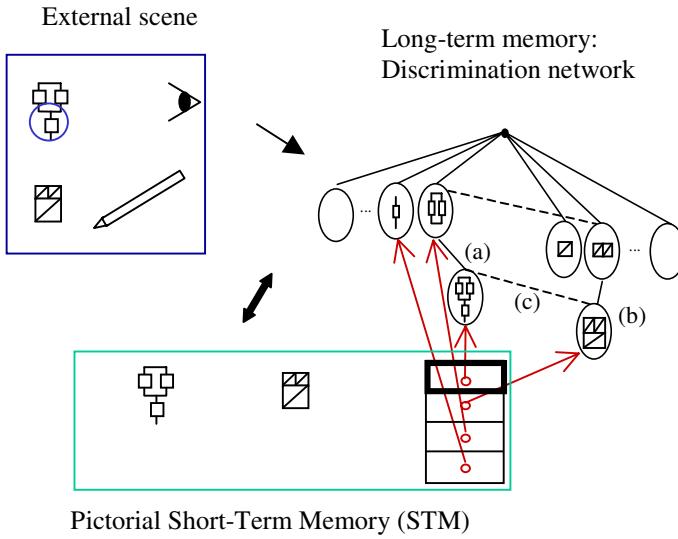
Figure 2 : The sample problem and its solution.

the sequence of actions into a number of stages. Such peaks have also been shown to correlate with meaningful decompositions in other forms of drawing (e.g. Cheng, McFadzean & Copeland, 2001). The figure also illustrates the parts of the AVOW diagram completed during each stage. These stages represent how the participant decomposed the solution.

From this first look at the graphs, we may conclude that the participants are using similar decompositions. These decompositions must be based on features of the target circuit diagram, and the interesting question from the perspective of effective representational design concerns their origin: Are they mere artifacts of the grouping of elements within the circuit, or are they the result of prior training? It is difficult to answer this question directly without some insight into the knowledge which each participant brings to the sample problem. In order to tackle this question, the next section describes a computational model, CHREST+, and shows how it can be used to predict the behavioural characteristics found in the participants' data.

### CHREST+ : Learning Perceptual Chunks

The perceptual chunking theory for human memory has had a long history within cognitive science, and forms the theoretical basis of the EPAM/CHREST family of computational models (for a review, see Gobet *et al.*, in press). Chase and Simon (1973) first proposed how perceptual chunking could be used in a model of problem solving based on EPAM (Elementary Perceiver and Memorizer) (Feigenbaum & Simon, 1984). The EPAM model assumes an input device (e.g. a simulated eye), a short-term memory (STM) for storing intermediate results, and a long-term memory based around a discrimination network containing



### Recognition

The simulated eye is passed over the target stimulus, and the retrieved patterns are sorted through the discrimination network. Matching chunks are retrieved and placed within STM.

### Learning

New nodes are added to the network through a process of *discrimination*. Illustrated at (a) and (b), the new pattern has caused an additional test link and node to be created. *Familiarisation* has then filled in the node image with features from the observed pattern. *Equivalence links* (the dotted lines) are formed when CHREST+ has a pointer to both a circuit chunk and an AVOW chunk within its STM at the same time. Here, the model has just learnt link (c).

Figure 3 : The CHREST+ model. The model includes a simulated eye and pen for interacting with the external environment, a fixed capacity short-term memory, and a long-term memory.

chunks of information. However, EPAM itself was only applied to certain perception and memory phenomena, and not more complex problem-solving domains, in part because of the simplified form of its learning mechanisms. This limitation is corrected in the CHREST (Chunk Hierarchy and REtrieval STructures) model, which includes various extensions to EPAM (Gobet, 1996; Gobet & Simon, 2000).

CHREST+ (Lane, Cheng & Gobet, 2000) has been developed to investigate how a memory of perceptual chunks can be used in problem solving with diagrams; the model is illustrated in Figure 3. Like CHREST, CHREST+ learns a discrimination network of perceptual chunks by scanning circuit and AVOW diagrams with its simulated eye. The network consists of a collection of perceptual chunks, which are stored at nodes in a network, interconnected by test links. Patterns are used to retrieve chunks from the network by sorting them, beginning from the root node, through the network against the tests stored at the test links. Once a pattern reaches a node, learning may occur: if the pattern matches the chunk at the node, then more information can be added from the pattern to the chunk (familiarisation); if the pattern mismatches the chunk, then a new test link is added based on the mismatching features, and a new node is created (discrimination). These processes are illustrated in Figure 3: at point (a), a test link for a single resistor is used to distinguish the collection of three resistors from the two in parallel; this link is added during discrimination, and the contents of the node added during familiarisation.

The process of learning about circuit and AVOW diagrams is illustrated in Figure 3. Because the circuit

and AVOW diagram representations do not overlap, individual nodes within the network will represent either an individual circuit diagram, or an individual AVOW diagram. In consequence, if CHREST+ is to generate AVOW diagrams when presented with a circuit, it must also associate chunks about circuits with chunks about AVOW diagrams. Accordingly, CHREST+ includes an additional learning mechanism for forming *equivalence links*; these are lateral links (Gobet, 1996) connecting two chunks within the discrimination network. An equivalence link is formed when the model is presented with a circuit diagram and its equivalent AVOW diagram. During the process of recognising the two diagrams, separate chunks will be placed into STM, one for the circuit diagram and one for the AVOW diagram. An equivalence link is then formed between the relevant two nodes in the network. Figure 3 illustrates this process, with an equivalence link formed at point (c). As can be seen, particular problems (circuit diagrams) become associated with information about their solution (equivalent AVOW diagrams). Generating an AVOW diagram for a novel circuit diagram then requires the model to locate chunks (sub-networks) within the circuit diagram for which it has an associated AVOW diagram; the AVOW diagrams for these sub-networks may then be drawn, and a further familiar sub-network located. The process by which CHREST+ incorporates its retrieved AVOW diagram into the evolving solution diagram is provided by specific, hand-coded routines – these are akin to the basic training the participants received in AVOW diagrams.

## Predicting the Observed Decompositions

We trained CHREST+ using the same sequence of circuits as the participants. By the time CHREST+ reaches the sample problem, it has learnt a discrimination network of 72 chunks, 42 for circuit diagrams and 30 for AVOW diagrams, with 11 equivalence links. When presented with the sample problem, CHREST+ retrieves four separate chunks whilst constructing its solution; these are illustrated in Figure 2(c). Note that the assumption in CHREST+ that information is contained in encapsulated chunks strongly predicts that problems will be decomposed as familiar chunks. Also, because CHREST+'s chunks are associated directly with equivalent AVOW diagrams, we can observe the effect of its circuit decomposition in the breakdown of the AVOW diagram's construction into stages. We now show how the decompositions can be affected by learning, how the participants' data provide reliable decompositions, and how well the participants' data are matched by the model.

## Decompositions are Due to Learning

The precise number and content of chunks used by CHREST+ is governed by its experience with the previously encountered problems. By providing different sets of problems, CHREST+ extracts different familiar chunks when decomposing the same sample circuit diagram. For example, presenting the sample problem after initial training on a circuit containing only a single resistor leaves CHREST+ with little choice but to decompose the problem into 9 distinct resistors; a more extensive training sequence allows CHREST+ to identify just 2 sub-circuits within the sample circuit. With the training sequence used, CHREST+ therefore makes a two-fold prediction: that four chunks are identified in the circuit, and that their form is as illustrated in Figure 2(c).

## Specifying a Decomposition

Returning to the graphs of the participants' drawing actions shown in Figure 4, we can consider how the decompositions provided by the peaks in the drawing actions compare across participants, and also whether they compare with the model's predicted chunks.

The first participant, CP, in Figure 4(a) has clearly begun from the right-hand side of the figure, then worked out the central triplet of resistors. These stages are preceded by longer pauses between the drawing times (marked by asterisks), and their correspondence to the chunks given by the model is clear: we highlight the stages with vertical divisions, illustrating the current state of the solution at the end of each stage. Similarly with the second participant, SG, who instead begins from the left-hand side; note also that SG requires considerably more time than CP. Note the different order in which the diagram is tackled, although the

Table 1 : The number of actions each participant made when completing the sample problem, classified as follows: NP – non-peaks; P – peaks; PC – pre-chunk; SB – start+bounding box; L – labels; E – error; Mi – missed chunks;  $\Sigma$  – totals.

Participant							
	CP	DJ	EF	HA	RH	SG	$\Sigma$
NP	19	17	7	16	8	8	75
P	5	5	4	7	3	5	29
PC	3	3	2	3	2	4	17
SB	2		1	1	1	1	6
L		2		3			5
E			1				1
Mi	1	1	1		2		5

overall stages are the same. Finally, the third participant, EF, used whole rectangles when constructing the solution; these rectangles were laid out in sequential fashion, beginning from the right-hand side, and then top to bottom. However, from the pauses evident in the times between drawing actions, we can see that this sequence of boxes was divided into the four stages corresponding to individual chunks. The remaining three participants show a similar pattern, but are not illustrated here.

## Matching Observed Decompositions

We can now directly compare the stage-wise output of the sample circuit's AVOW diagram by CHREST+ with its solution by the participants. We quantify the correspondence between CHREST+'s prediction and the participants' behaviour by counting how many of CHREST+'s chunk boundaries correspond with the participants' peaks. For this analysis, a peak is a time between drawing actions prominently larger than the preceding and succeeding times: the peaks used are highlighted in the figure with asterisks.

For example, the graph for CP shows five peaks. The first and second peaks correspond to CP beginning the problem and creating a bounding box for the entire circuit, as illustrated in the diagram before the first dividing line. Between the third and fourth peaks, CP completes the part of the diagram which corresponds with chunk 4 in CHREST+'s output, and hence we count the third peak as a pre-chunk boundary. Similarly, between the fourth and fifth peaks, chunks 2 and 3 are completed in the diagram. Note that there is no peak corresponding to a retrieval of the 3<sup>rd</sup> chunk, against the model's prediction. Finally, after the fifth peak, CP completed chunk 1 and then added the labels to all the AVOW boxes; the simplicity of this process is reflected in the low times between these operations.

We therefore explain the five peaks as follows: the first two are for the start and bounding box, and the next three are pre-chunk boundaries. One chunk boundary seems to have been missed. Table 1 summarises the analysis for all six participants.

The analysis shows that nearly all of the peaks correspond with stages in the drawing which we would explain by the use of chunks. Note that, out of the 24 predicted chunks (6 participants and 4 predicted chunks), 17 peaks were clearly identifiable chunk boundaries, and there were only 5 missed chunks. This leaves unaccounted 2 chunks, which were instead created directly from the start, and thus are included with the starting times of the participant: this analysis therefore identifies 19 peaks in the participants' behaviour which precede chunk boundaries out of 24 predicted chunks. These results demonstrate that CHREST+ predicts the decomposition of problems evident in the participants' data, and so support our claim about the role of learnt perceptual chunks in problem decomposition.

## Conclusion

This paper has used the perceptual chunking theory, as implemented in CHREST+, to predict specific perceptual chunks learnt from a given sequence of instruction. We have presented results from a study of six participants solving electric-circuit problems using AVOW diagrams. The predictions from the model have been shown to correlate with the stages in problem solving evident in the participants' performance.

To fully understand the participants' problem decomposition and learning pattern, we need to consider more closely what is happening during the peaks in their output timings. Looking at the CHREST+ model, the participants' peaks correspond to the processes of pattern recognition and retrieval. In addition, processes of planning along with some lookahead must be going on. This lookahead and planning probably explains the missed chunks in the preceding analysis. At present, CHREST+ always outputs its solution AVOW diagram as soon as it is found. Through a small modification in its output strategy, CHREST+ could instead retain more than one chunk for solution, and output several together. This would provide CHREST+ with the potential for lookahead, making it a more plausible problem solver, as well as capture the pattern of missed chunks.

The interesting result from this paper is that the chunks used by learners within such domains may be predicted using an established computational model. Further work with this domain should aid in refining the model and its predictions, and also extend it into other domains. In particular, this use of perceptual chunks in decomposing diagrams has already been shown to occur more generally (Cheng, McFadzean & Copeland, 2001). In the longer-term, one of the important applications for this research is likely to be the design of effective computer-based learning environments (Gobet & Wood, 1999).

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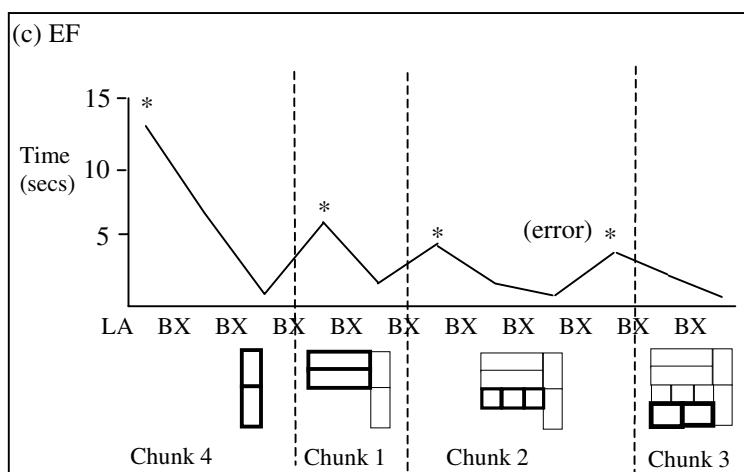
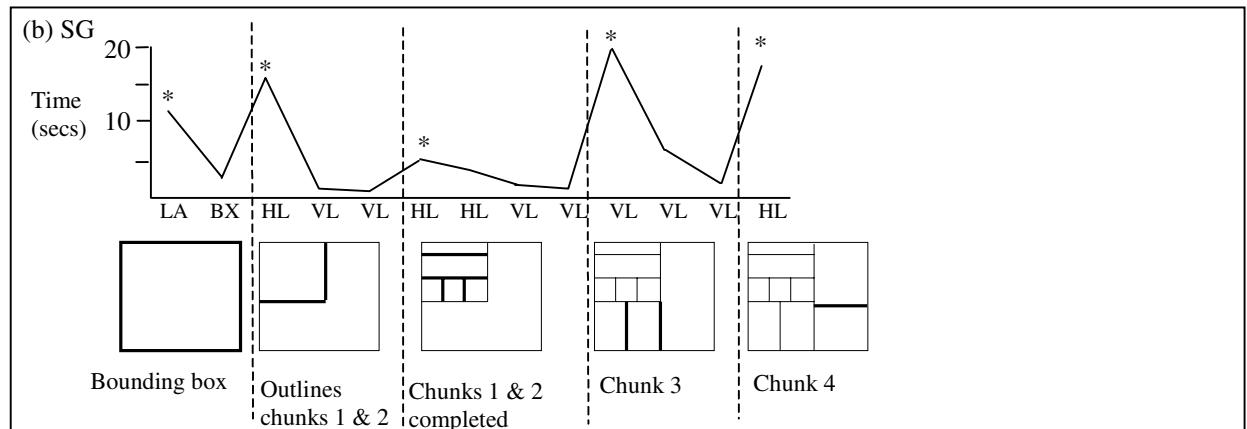
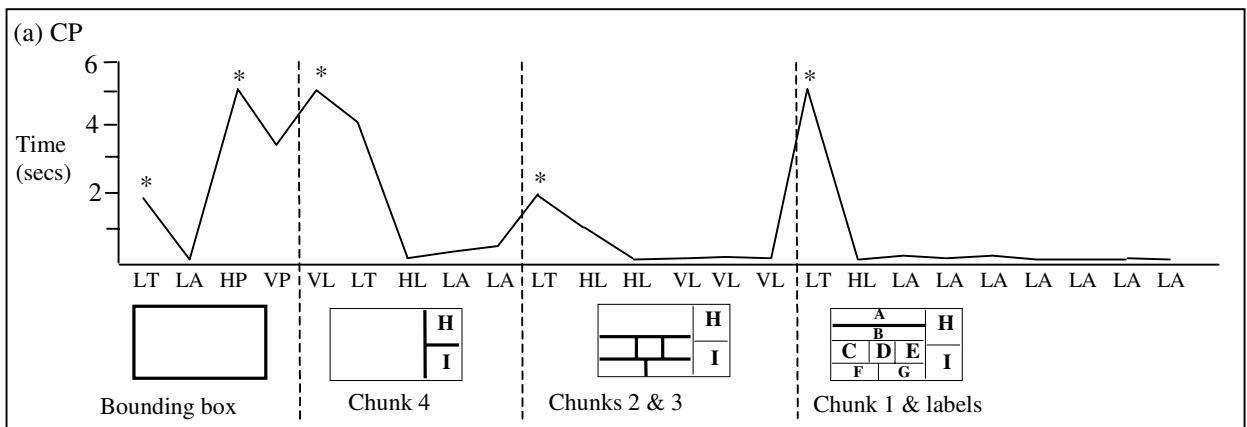
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**Key to drawing operations:**

BX	Box
HL	Horizontal Line
HP	Horizontal Parallel Lines
LA	Add Text Labelling
LT	Change Line Thickness
VL	Vertical Line
VP	Vertical Parallel Lines

Bold lines indicate the features added during each stage.

Figure 4 : The performance of three participants when solving the sample problem. The \* indicates peaks used in the analysis. Under the graphs, information is given on the specific drawing action performed, the stage of the diagram just prior to the peaks, and the correspondence with the chunks predicted by CHREST+.