The Hot Hand in Basketball: Fallacy or Adaptive Thinking?

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Abstract

In basketball, players believe that they should "feed the hot hand," by giving the ball to a player more often if that player has hit a number of shots in a row. However, Gilovich, Vallone & Tversky (1985) analyzed basketball players' successive shots and showed that they are independent events. Thus the hot hand seems to be a fallacy. Taking the correctness of their result as a starting point, I suggest that if one looks at the hot hand phenomena from Gigerenzer & Todd's (1999) adaptive thinking point of view, then the relevant question to ask is does belief in the hot hand lead to more scoring by a basketball team? By simulation I show that the answer to this question is yes, essentially because streaks are predictive of a player's shooting percentage. Thus belief in the hot hand may be an effective, fast and frugal heuristic for deciding how to allocate shots between member of a team.

The Hot Hand as Fallacy

Gilovich, Vallone and Tversky (1985) defined the "hot hand" in basketball as the belief that during a particular period a player's performance is significantly better than expected on the basis of a player's overall record. Gilovich et al. found that 91% of fans agreed that a player has "a better chance of making a shot after having just made his last two or three shots" and 68% said the same for free throws; 84% of fans believed that "it was important to pass the ball to someone who has just made several (two, three, or four) shots in a row." Numerical estimates reflected the same belief in streak shooting, and most players on a professional team endorsed the same beliefs. Thus belief in the hot hand appears to be widespread, and Gilovich et al. suggest that may it affect the selection of which player is given the chance to take the next shot in a game. This implication is captured by a phrase heard in basketball commentary: "feed the hot hand."

To test if the phenomena described by the hot hand actually exist, Gilovich et al. (1985) analyzed a professional basketball team's shooting over a season in order to see if streaks occur more often than expected by chance. They found that for each individual player, the proportion of shots hit was unrelated to how many previous shots in a row he had either hit or missed. Analysis also showed that the number of runs of hits or misses for each player was not significantly different from the expected number of runs calculated from a player's overall shooting percentage and assuming that all shots were independent of each other. The same independence was found for free-throws, as the probability of hitting a free-throw was the same after a hit as after a miss for a group of professional players. A controlled study of college players found the same independence between shots and found that observers could not predict which shots would be hit or missed. Thus the hot hand phenomenon appears to be a fallacy.

Why do fans and players believe in the hot hand if the empirical evidence shows that successive shots are independent? Gilovich et al. (1985) suggest that the persistence may be due to memory biases (streaks are more memorable) and misperception of chance, such as belief that small as well as large sequences are representative of their generating process (Tversky & Kahneman, 1974). Falk and Konold (1997) see the hot hand as a cognitive illusion that is another example of people's inability to perceive randomness. Again we see people inventing superfluous explanations because they perceive patterns in random phenomena.

Gilovich et al.'s (1985) result has been cited over 100 times in journals. Many of these citations are in the decision making literature, but it is also widely cited across a variety of fields. There are many citations in sports science (Vergin, 2000) and economics (Pressman, 1998), but it has also been cited in literature on law (Hanson & Kysar, 1999) and religion (Chaves & Montgomery, 1997).

There have been some challenges to Gilovich et al.'s (1985) conclusion that there are no more streaks than expected by chance in basketball, or at least to the finding's generalizability. Gilden and Wilson (1995) found some evidence of more streaks than expected in golf putting and darts, although they explain this as due to fluctuations in performance producing more streaks than expected rather than a real dependence between events. Miyoshi (2000) used simulations to suggest that Gilovich et al.'s analysis may not have been sensitive enough to detect the hot hand if hot-hand periods are relatively infrequent. However, in this paper I will assume Gilovich et al.'s (1985) conclusion that successive shots in basketball are independent events, in fact, my analysis will depend on it.

One reason for the wide interest in Gilovich et al.'s result may be the implications it appears to have for behavior. As Gilovich et al. (p. 313) state "...the belief in the 'hot hand' is not just erroneous, it could also be costly." This is because it may affect the way shots are allocated between members of a team. However, I will argue in this paper that this implication does not

necessarily follow from their analysis, rather belief in the hot hand may actually be adaptive.

The Hot Hand as Adaptive Thinking

Gigerenzer and Todd (1999) emphasize that humans and animals make decisions about their world with limited time, knowledge, and computational power. So they propose that much of human reasoning uses an adaptive tool-box containing fast and frugal heuristics that make inferences with limited time, knowledge and computation. Their viewpoint is based on a conception of bounded rationality. They contrast this with an assumption of unbounded rationality, which leads to a focus on the question: what is the normatively correct answer? Gigerenzer and Todd instead argue that one should ask: what is adaptive? That is, what behavior will meet the person's goals and uses a process that stays within the bounds of their resources?

From the point of view of basketball, whether successive shots are independent may not be the most relevant question to ask. What is adaptive for them is to maximize the number of points their team scores, so the question to be asked is does belief in the hot hand lead to more scoring than would disbelief in the hot hand?

The practical effect of belief in the hot hand is that it affects distribution of the ball. This is reflected in the statement that Gilovich et al. (1985) presented to fans and players, "it is important to pass the ball to someone who has just made several (two, or three, or four) shots in a row." Who should take the next shot is a question faced by the members of a team every time they have possession of the ball. In the absence of a time-out, it is a decision that each member of the team have to make by himself or herself in, at most, 30 seconds. Every player on a professional team is probably aware of the shooting percentage (i.e., what percentage of a players total number of shots a player hits) for each member of the team. However, knowing that one player has a 55% and another a 60% shooting percentage, does not tell one how often to give the ball to each player, given that one cannot simply give the ball to the player with the higher shooting percentage every time. Players are unlikely to be able to do a calculation to determine the optimal distribution, so fast and frugal heuristics for deciding who should take the next shot are likely to be exploited if they are effective in increasing scoring.

I propose that belief in the hot hand is such a heuristic. The basic argument is straight forward: if one accepts Gilovich et al.'s (1985) finding that successive shots are independent events, then the higher a player's shooting percentage is, the larger the number of runs of hits a player will have. Therefore, a bias to give the ball to players with the hot hand is equivalent to a bias to give the ball to players with higher shooting percentages. Giving the ball to the player with the hot hand requires no calculation, it requires only remembering the most recent shots, and it can be decided fast. Thus belief in the hot hand could be an example of adaptive thinking.

I will support this analysis with computer simulations testing whether a team that believes in the hot hand will outscore one that does not. However, I will first show empirically that players with higher shooting percentages experience more runs of hits.

Empirical Analysis

Gilovich et al. (1985, Table 1) presented the probabilities of players making a shot after runs of hits or misses of length one, two and three, as well as the frequencies of each run for players. The statistics came from analysis of the 48 home games of the Philadelphia 76ers during the 1980-81 season. In Table 1, I have re-analyzed this data to calculate for each player the proportions of his total number of shots (excluding a player's first shot in a game) which were parts of runs of hits or misses of length 1, 2, and 3.

Table 1: Proportions of players total shots that were parts of runs of 1, 2, or 3 hits, or 1, 2, or 3 misses. Correlations are between these proportions and the players' season long shooting percentage (all significant at $p < .01$)

Player	Shooting percentage	Total shots	3 misses	2 misses	1 miss	1 hit	2 hits	3 hits
Lionel Hollins	.46	371	.11	.25	.54	.46	.18	.07
Andrew Toney	.46	406	.08	.22	.53	.47	.19	.07
Caldwell Jones	.47	225	.09	.21	.52	.48	.16	.05
Clint Richardson	.50	206	.06	.16	.49	.51	.22	.10
Julius Erving	.52	836	.11	.23	.49	.51	.25	.12
Bobby Jones	.52	310	.06	.17	.47	.53	.25	.11
Steve Mix	.54	386	.06	.17	.46	.54	.25	.09
Maurice Cheeks	.56	292	.04	.13	.43	.57	.26	.11
Daryl Dawkins	.62	358	.02	.09	.38	.62	.31	.15
Correlations with shooting percentage:			-.804	-.874	-.993	.993	.954	.899

If shots are independent events, then the higher a player's shooting percentage, the larger the number of runs of hits he should have and the fewer number of runs of misses he should have. Table 1 presents the correlations between a player's shooting percentage and the proportion of his shots that were parts of runs of each length. As can be seen, all runs of misses are highly negatively correlated with shooting percentage, and all runs of hits are highly positively correlated with shooting percentage (all $p < .01$). This supports Gilovich et al.'s (1985) argument that successive shots are independent events, and also the consequence of my argument that runs are predictive of a player's shooting percentage.

Design of the Computer Simulations

In creating the simulations I strove to make them as transparent as possible and to utilize as few free parameters as possible. The basic simulation of basketball shooting had two parameters for each player: an allocation and a shooting percentage. The *allocation* is the probability of a player being given the next shot, whereas the *shooting percentage* is how often the player hits a shot. The sum of allocation parameters for all players must be 1.0 and was used to represent some underlying bias to give the ball to a player. No assumption was made regarding the source of these biases, but it was fixed for the length of a simulation. Shooting percentage reflects a player's ability to hit shots, and was also fixed for the length of a simulation, as it was for Gilovich et al.'s (1985) analysis.

The program simulated basketball shooting, with one shot per trial. On each trial a player was randomly selected to be given the next shot, with each player having the probability of being given the shot indicated by their allocation. The player given the shot then randomly hits or misses the shot with a probability indicated by the player's shooting percentage.

To simplify the simulations, rather than represent all five members of a normal basketball team, only two will be included. This reduces the number of parameters and how many players are represented should not matter with regard to the conclusions I wish to draw from the simulations. To further reduce the number of free parameters the allocation and shooting percentages were only varied for one player, and the other player's parameters were simply one minus each of the first player's parameters. Thus a whole simulation was described by just two free parameters, allowing the entire parameter space to be explored.

To simulate belief in the hot hand, a simple rule was used that determine who should be given the next shot:

1) Give the next shot to a player which has the longest run of hits (in effect, the one who hits its most recent shot), then keep giving it to that player until a miss.

2) If both players have missed their last shot, then the allocations parameters were used to select a shooter randomly.

The hot-hand could be simulated in other ways, but this seems a simple, easily understandable version, and it is parameter-less. More complicated ways of calculating who has the hot-hand would involve arbitrary parameters but produce the same pattern of results.

To test the effect of belief in the hot-hand, two simulations were run with each combination of the two parameters. Simulations with a given combination of parameters were run in pairs. In one run, the hot hand rule was turned on, and in the other it was turned off so the player to take the shot was always determined randomly using the allocation parameter. All parameter combination for allocation values from 0.01 to 0.99 were run in increments of 0.01, and shooting percentage values from 0.50 to 0.99 in increments of 0.01 (0.00 to 0.49 would simply repeat the other combinations). Thus 4851 pairs of simulations were run.

Results of the Simulations

Each combination of parameters was run for 1,000,000 trials with the hot-hand rule, and 1,000,000 without it. Each simulation produced a score, which was how many of the trials were hits. To determine the effect of belief in the hot-hand, for each parameter pair the score for the simulation without the hot-hand rule was subtracted from the score from the simulation with the hot-hand. This difference was divided by the total number of trials to yield an advantage score (adv):

$$adv = \frac{\text{(score with hot hand) - (score without hot hand)}}{\text{total number of trials}}$$

Figure 1 presents a contour graph for the 4852 (49x99) pairs of simulations (the 0.50 shooting percent parameter is excluded because when there is no difference between players, there is nothing for the hot hand to exploit). This graph represents three dimensions of information: the allocation percentage, the shooting percentage, and the adv score in favor of the hot hand simulation for that combination of parameters. The numbered contours define boundaries in the distribution of adv scores found for parameter pairs. So, for example, for every combination of parameters above the line labeled "0.2" the hot hand simulation scores at least 0.2 points per trial of the simulation (on every trial the players score 0 or 1). The areas at the bottom of the graph labeled "0.0" indicate regions in which the hot hand lost in this set of simulations. (To creates these plots I used Sigma graph, which tried to "smooth" contours resulting in these odd shapes.)

Almost all the pairs of parameters yielded positive advantage scores. Not surprisingly, the greatest advantage occurs when shooting percentage is high and allocations are low, as in effect the hot hand rule increases the allocation for the player with the higher shooting percentage. As Figure 1 shows, only when the shooting percentage is low, which is when the two players differ little in shooting percentage, does the hot hand sometimes lose. Figure 1 also shows that there is no systematic relationship between allocation and shooting percentage which results in the hot hand doing worse. When the hot hand does worse is essentially random and only occurs when there is little difference between the players for the hot hand to exploit. The most negative advantage score obtained was -0.0018 points per trial.

Table 2 shows for how many of the simulations with each shooting percentage (99 as the allocation varies from .01 to .99) the hot hand wins. When shooting percentage is equal to .50 then it should be random which simulation wins because the hot hand cannot help the better shooter when the two players do not differ. As the shooting percentage increases, and thus the difference between players increases, loses by the hot hand simulation become rarer. There were no losses by the hot hand for shooting percentages in excess of .60. There was no systematic relationship between allocation parameters and loses by the hot hand, except for a few extra loses at allocations of 0.99.

Table 2: The number of allocation values (99) that the hot hand simulation wins or loses for each shooting percentage parameter less than .61.

Shooting percentage	Hot hand loses	Hot hand wins
.50	44	55
.51	48	51
.52	21	78
.53	10	89
.54	2	97
.55	2	97
.56	1	98
.57	2	97
.58	2	97
.59	1	98
.60	1	98

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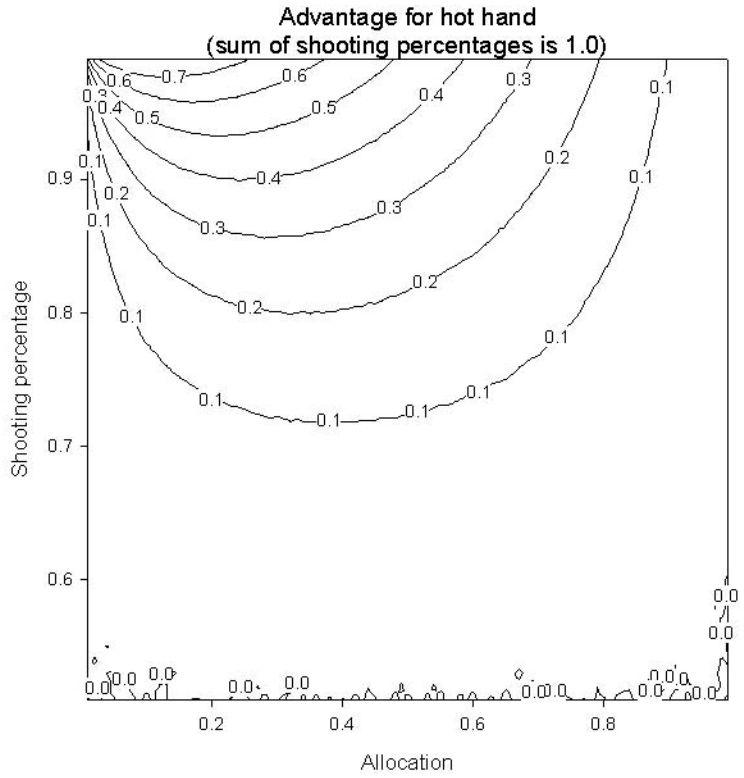

Figure 1: Contour graph showing how many points per trial the simulation using the hot hand rule came out ahead of the simulation not using the hot hand rule, for every pair of shooting percentage and allocation parameters. The lowest shooting percentage is 0.51 and highest is 0.99. The range for allocation parameters were 0.01 to 0.99.

One question that may be raised is whether the results are due to one player having a shooting percentage below 50% and the other above 50%. To check this, a set of simulations were run which were identical to the above set, but the sum of the two shooting percentages for the two players equaled 0.50. So the shooting percentage parameter was varied from 0.25 to 0.49. Very similar results were obtained with the hot hand only recording loses when the two shooting percentages were very close. Simulations of defenders acting on belief in the hot has also shown that it helps them.

Why the hot hand wins is quite clear. The hot hand rule has no effect on the percentage of shots hit by a player and does not introduce any dependencies between successive shots by the same player, instead it leads to the player with the higher shooting percentage being given more shots. In effect, the hot hand increases the allocation parameter for the player with the higher shooting percentage.

These simulations are of course not complete simulations of basketball games. Realistic simulations of ten players interacting on a basketball court are not possible, and thus this simulation cannot be compared directly to real data from basketball players. This however is not the point of the simulations, instead they are intended as an instantiation of the thought experiment conducted above, that belief in the hot hand should increase scoring by a team if successive shots are independent events. The belief increases the likelihood that the best scorers will take more shots, thus it is adaptive. The simulations are actually unnecessary if one already accepted the basic argument. However if one accepts this argument then it changes the interpretation of the data of Gilovich et al. (1985) and any other belief regarding streaks when the events making up the streaks are independent.

Fallacy and Adaptation

Is the hot hand a fallacy or adaptive thinking? In a sense it can viewed as both, it depends on what question one thinks is the most relevant one to ask.

The question that Gilovich et al. (1985) sought to answer was whether basketball players produce more streaks of hits or misses than expected by chance given their underlying shooting percentage. Their analysis showed that the answer to this question was "no", for an individual. The analysis presented here in no way challengers this result, in fact it is built into the simulations as an assumption. It may seem obvious that if a belief in the hot hand as defined for an individual is erroneous, then it must also be erroneous when applied to a team. Gilovich et al. (1985) make this quite understandable connection without comment, and thus make statements about the supposedly negative consequences for a team of passing the ball to the player with the hot hand.

The ease with which one can slip from referring to an individual's behavior to a team's behavior is reflected in the statements Gilovich et al. (1985) asked basketball fans and players to consider. Gilovich regard both of the following two statements as indicators of an erroneous belief in the hot hand:

1) "[a player] has a better chance of making a shot after having just made his last two or three shots than he does after having missed his last two or three shots"

2) "it is important to pass the ball to someone who has just made several (two, three, or four) shots in a row"

Statement 1 refers to an individual's streaks, and Gilovich et al. (1985) show empirically that it is incorrect. However, Statement 2 is about a team's decisions about how to allocate shots between players. Gilovich et al.'s data does not address this question, but the arguments and simulations presented here show that Statement 2 is correct. It is adaptive rather than an erroneous belief. From this conclusion, it is interesting to note that Statement 2 was the only statement given to the professional players by Gilovich et al. that was endorsed by every one of them.

The alternative question regarding the hot-hand is suggested by Gigerenzer and Todd's (1999) approach: is belief in the hot hand adaptive? Whether there actually are streaks in individual players' shooting is irrelevant from this point of view. The basketball players' primary goal when his or her team has possession of the ball is to maximize the number of points that the team scores (notwithstanding the behavior of some current NBA stars), as that is what determines the outcome of the game. If belief in the hot hand (as defined as giving the ball to the player experiencing a streak) tends to increase point scoring as compared to when the hot hand is disregarded, then the hot hand is adaptive thinking rather than a fallacy. "Feed the hot hand" can be seen as a fast and frugal heuristic for making a critical decision: who should get the next shot? Belief in the hot hand provides a quick and easily assessable answer to this question. (This is not to imply that the hot hand is the only way, or always the best way, to make this decision. Like any heuristic, it may fail.)

If there were fluctuations in a player's underlying shooting percentage, which could arise for various reasons, then the hot hand provides a further advantage over any calculations based on a shooting percentage or some other product of the history of a player. The hot hand is immediately sensitive to fluctuations because if a player's shooting percentage changes then his or her expected number of streaks will be affected immediately. The impact of fluctuations on a player's season long shooting percentage, or any other statistics,

will be delayed. Gilden and Wilson (1995) argue that such fluctuation could create streaks despite independence between successive events. Whether Gilovich et al.'s analysis was sensitive enough to detect such streaks is a question raised by the analysis of Miyoshi (2000) who points out that it would depend on the frequency and duration of such events. However, even if there are no fluctuation driven streaks in basketball, there may be in other multi-trial tasks, and thus belief in the hot-hand may be a general heuristic that people learn is effective in a variety of situations.

It could be argued that even if the belief in the hot hand is adaptive then it may originate and be sustained by a fallacy regarding the streaks of individuals. Thus basketball players may have just got lucky that their fallacy helps rather than hinders them. I have presented no evidence regarding the origin of the belief in the hot hand, and I doubt that players are consciously using the analysis I present here to support their belief in giving the ball to the player with the hot hand. However, it could be argued that what sustains belief in the hot hand is simply that players have learned that giving the ball to the player experiencing streaks has a positive outcome. The work on implicit learning shows that people may not necessarily know what they have learned. Nisbett and Wilson (1977) review evidence that people may make appropriate decisions without conscious awareness of the true reasons why they made that decision, and then they may make up plausible sounding reasons for their behavior. The erroneous beliefs fans and players make about the consequences of streaks by individual players may simply be an attempt to rationalize a behavior they have learned is adaptive. Thus belief in streaks for individuals may be a misanalysis of the reasons for an accurate perception of the hot hand as it applies to a team play, rather than a misperception of sequences which appears to be the basis of the gambler's fallacy. The connection between the gambler's fallacy and the hot hand, which may be related but describe the opposite behavior (i.e., go with the streak, verse go against the streak), may be a fruitful area for future research.

In summary, the final answer to the question posed by the title depends on which question one prefers to ask. Either, what is a normatively correct way of describing the performance of an individual basketball player, or what may lead to a higher score in a game? Even though the relevance of both questions can be seen, to an adaptive organism the later question should be more important on the basketball court.

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